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3D Printed Composite Body Illustrating Area and Mass Moment of Inertia with Mohr's Circle and Pole Method

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Abstract

A 3D printed composite body connects the math intensive concept of area moment of inertia to the real world. When studying area moment of inertia, students calculate moment of inertia for an area composite body. A 3D printed composite body of constant thickness and density has a resistance to rotation quantified by the mass moment of inertia corollary to the area moment of inertia. Holes pass through the model in such a way that the model can be spun about the horizontal and vertical centroidal axes. Students can use the product of inertia and pole method to identify the principal area moments of inertia and the corresponding axes. The 3D printed composite body has holes allowing for rotation about these axes as well. A 3D printed composite shape allows students to see and experience the otherwise math intensive and abstract concept of area moment of inertia.

This paper is submitted as a non-technical paper to the Civil Engineering Division "Best in 5 Minutes: Demonstrating Interactive Teaching Activities" session.

Notation

$A =$	total area of the shape
$dA =$	differential area
$dF =$	the differential force
$dm =$	differential mass
$dx =$	differential width in the horizontal direction
$dy =$	differential width in the vertical direction
$I =$	mass moment of inertia about an arbitrary axis
$I_c =$	mass moment of inertia about the centroidal axis for a particular shape
$I_{comp} =$	composite mass moment of inertia about an arbitrary axis
$I_{max} =$	maximum area principal moment of inertia
$I_{min} =$	minimum area principal moment of inertia
$I_x =$	area moment of inertia about an arbitrary x axis
$I_{xc} =$	area moment of inertia about the centroidal x axis for a particular shape
$I_{xcomp} =$	composite area moment of inertia about an arbitrary x axis
$I_{xy} =$	area product of inertia about an arbitrary x and y axes
$I_{xycomp} =$	area product of inertia for a composite body about an arbitrary x and y axes
$I_{x_c y_c} =$	area product of inertia about the centroidal x and y axes
$I_y =$	area moment of inertia about an arbitrary y axis
$I_{yc} =$	area moment of inertia about the centroidal y axis for a particular shape
$I_{ycomp} =$	composite area moment of inertia about an arbitrary y axis
$I_z = J_o =$	area polar moment of inertia about the z axis
$m =$	mass of a body
$M_y =$	moment about the y axis from a force

$P = slope \cdot x =$ the applied pressure (see Figure 1)
$POLE_x =$ x axis coordinate of the pole
$POLE_y =$ y axis coordinate of the pole
$r =$ perpendicular distance from an arbitrary axis to the center of the differential area, differential mass, or the center of mass of a particular shape
$r_m =$ mass radius of gyration about an axis
$r_x =$ area radius of gyration about the x axis
$r_y =$ area radius of gyration about the y axis
$slope =$ the slope of the applied pressure
$t =$ thickness of a prismatic shape
$x =$ perpendicular distance from an arbitrary y axis to the centroid of a differential area or the centroid of a particular shape
$x_c =$ perpendicular distance from the y axis to the centroid of the whole area
$\tilde{x} =$ perpendicular distance from the y axis to the centroid of one part of the area
$y =$ perpendicular distance from an arbitrary x axis to the centroid of a differential area or the centroid of a particular shape
$\rho =$ density of a prismatic shape
$\theta_p =$ angle of the principal strong axis

Introduction

Engineering students often first encounter engineering mechanics through a Statics course (Yay! Nothing moves) before progressing to Dynamics (Yay! Everything moves!), Mechanics of Materials (Yay! Things move a little bit!), and Fluids Mechanics (Yay! Things flow!). Though Statics itself focuses on systems in equilibrium, Statics also introduces foundational pre-requisite concepts including moment of inertia. Though moment of inertia has concrete applications in Fluids (pressure on submerged surfaces), Mechanics of Materials (bending stiffness and strength in beams), and Dynamics (rotational movement), in Statics the topic is often taught with minimal context and abstract mathematics [1], [2].

The 3D printed composite body presented in this paper provides students with the opportunity to calculate the area moment of inertia about its centroid for a 2D area composite body using the parallel axis theorem and relate the area moment of inertia to the mass moment of inertia and rotation of a 3D prismatic object. The 3D model can also be used to explore product of inertia, principal moments of inertia and the principal axes from Mohr’s circle and the pole method.

Literature Review

Mass moment of inertia is easier than area moment of inertia to present as a physical model. Some instructors have used qualitative examples [3] while others have used quantitative exercises [4]. Such demonstrations are natural extensions of models intended to demonstrate center of gravity [5]–[8] or other statics concepts [9]. Instructors developing area moment of inertia demonstrations are typically focused on some other connected behavior [10]. Textbooks typically make only cursory connections about area moment of inertia and its use in future classes [1], [2]. The 3D printed composite body and examples presented here attempt to provide context to an abstract problem, expanding on work in equivalent systems, moment calculations, and centroids [11].

Application

The following sections illustrate theory, examples, and demonstrations from three lectures on moment of inertia. The first example is used to derive the integral solution for area moment of inertia from a calculation of the moment generated by a linearly increasing load. This definition is tangentially connected to centroids and demonstrates the relationship between area moment of inertia and mass moment of inertia. In the second lecture, students calculate the area moment of inertia about the vertical and horizontal centroidal axes for a composite body and observe the connection between the calculated area moment of inertia and the observable mass moment of inertia by spinning the 3D printed body about those axes. The third demonstration introduces students to product of inertia, Mohr's circle, and the pole method showing students how to identify the principal area moments of inertia and connecting the numerical solution to qualitative analysis of the rotation of the 3D printed composite body about the principal axes.

Area and Mass Moment of Inertia: Context and Derivation

Main Idea

For a linearly increasing load acting on surface, the calculation of a moment about a particular axis can be conveniently calculated using the geometric property called the area moment of inertia. Area moment of inertia can be normalized by area to identify the radius of gyration. The relationship between area moment of inertia and mass moment of inertia is analogous to the relationship between area centroid and center of mass for prismatic, homogeneous shapes.

This discussion and classroom demonstrations should prepare students to:

- Describe…
  - Area moment of inertia.
  - Area radius of gyration.
  - Polar moment of inertia.
  - Mass moment of inertia.
  - Mass radius of gyration.

Context

The calculation of a moment generated by a linearly increasing pressure is a common occurrence in the study of fluid hydrostatics and bending behavior in mechanics of materials. Equation 1 shows the calculation for the moment (M_y) about the axis of zero pressure (consider the y axis in Figure 1) as a function of the slope of the applied pressure.

$$M_y = \int \vec{r} \times d\vec{F} = \int x \cdot P \cdot dA = \int x \cdot slope \cdot x \cdot dA = slope \int x^2 \cdot dA = slope \cdot I_y \tag{1}$$

where:		
	$\vec{r} = x =$	the perpendicular distance from the y-axis
	$d\vec{F} = P \cdot dA =$	the differential force
	$P = slope \cdot x =$	the applied pressure (see Figure 1)
	$slope =$	the slope of the applied pressure

For a given situation, the slope of the linearly increasing load is constant and can be removed from the integration. The remaining terms of integration are based on the geometry alone. This final geometric property is the area moment of inertia.

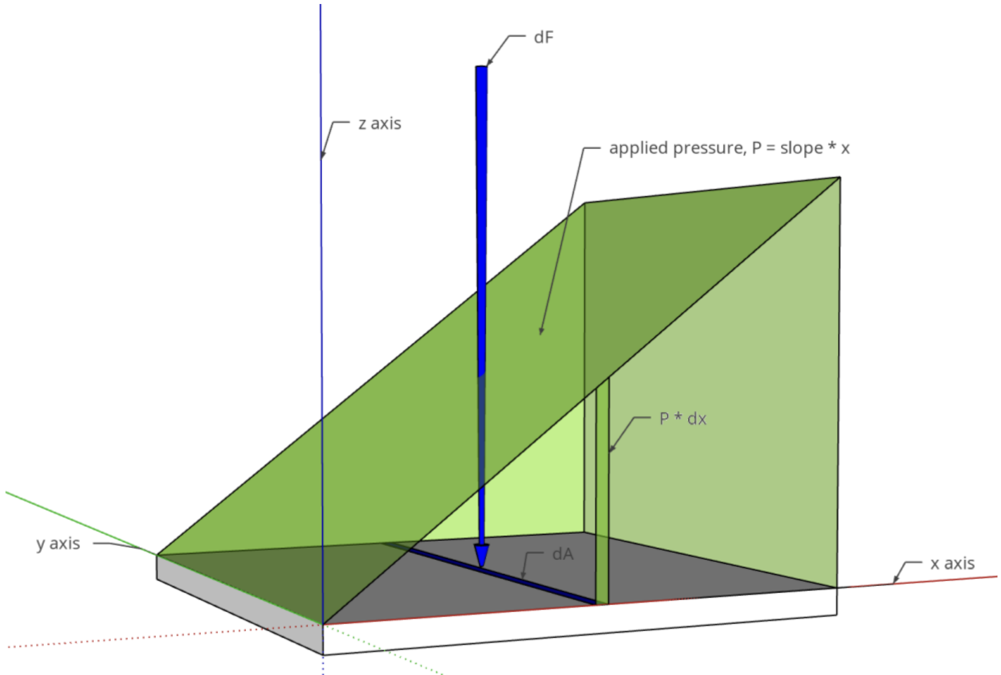


Figure 1. Surface with a linearly increasing load.

Area moment of inertia and radius of gyration

Table 1 compares the first moment of area and centroid equations to the area moment of inertia (or the second moment of area) and the radius of gyration. Area moment of inertia has units of length to the fourth. The radius of gyration with units of length, normalizes the area moment of inertia by the total area.

Table 1. Comparison of first moment of area and centroid to area moment of inertia and area radius of gyration.

first moment of area	second moment of area or area moment of inertia about the y axis	second moment of area or area moment of inertia about the x axis
$\int x \, dA = x_c A$	$I_y = \int x^2 \, dA = A \, r_y^2$	$I_x = \int y^2 \, dA = A \, r_x^2$
centroid	radius of gyration about the y axis	radius of gyration about the x axis
$x_c = \frac{\int x \, dA}{\int dA} = \frac{\sum \tilde{x} \, A}{\sum A}$	$r_y = \sqrt{\frac{\int x^2 \, dA}{\int dA}} = \sqrt{\frac{I_y}{A}}$	$r_x = \sqrt{\frac{\int y^2 \, dA}{\int dA}} = \sqrt{\frac{I_x}{A}}$

The mathematical expressions for area moment of inertia communicate that the more area that is farther from the axis of interest the higher the area moment of inertia. The radius of gyration communicates the distance from an axis to where an equivalent area could be placed to generate the same area moment of inertia. The larger the radius of gyration, the further the area is from the axis.

Area polar moment of inertia and polar radius of gyration

The polar moment of inertia is used to analyze problems involving torsion, twist around the z axis. The polar moment of inertia is equivalent to the area moment of inertia about the z axis coming out of the page. As seen in Figure 2, the perpendicular distance to the z axis is the radius from the origin. The Pythagorean Theorem is used in Equation 2 to show that the polar moment of inertia, J_o, is the sum of two perpendicular area moment of inertias.

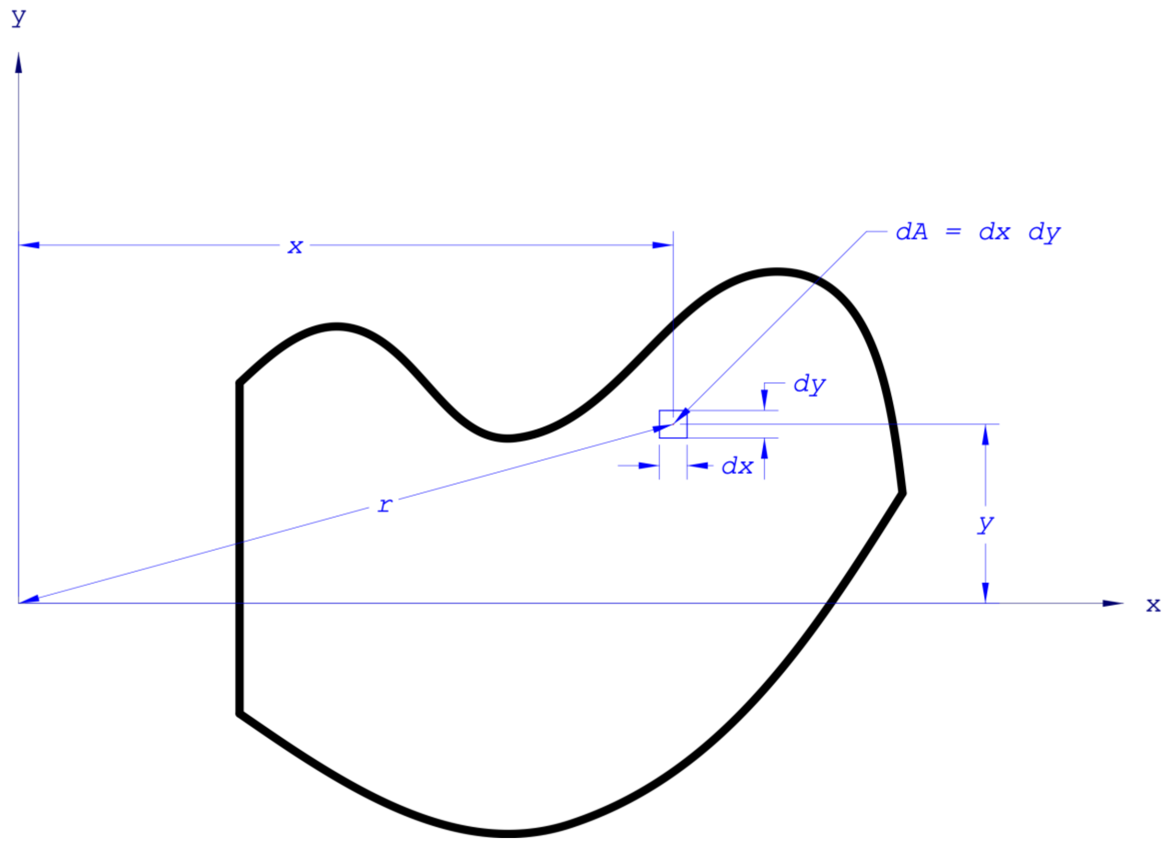


Figure 2. Position of differential area for the calculation of polar moment of inertia.

$$I_z = J_o = \int r^2 \, dA = \int (x^2 + y^2) \, dA = \int x^2 \, dA + \int y^2 \, dA = I_y + I_x \tag{2}$$

Mass moment of inertia and radius of gyration

Area moment of inertia and mass moment of inertia share a corollary relationship with one another, like the relationship between area centroid and center of mass for a body. Where the area moment of inertia supports the analysis of linear increasing pressures in fluids and mechanics of materials, the mass moment of inertia relates to the resistance to rotation about a particular axis. Table 2 shows the equations of mass moment of inertia and mass radius of gyration about an arbitrary z axis. By corollary analysis, for a homogeneous, prismatic shape, the mass moment of inertia about the z axis can be found by multiplying the polar area moment of inertia (Figure 2 and Equation 2) by the body's density, ρ, and thickness, t, as seen in Table 2. The units of mass moment of inertia are mass times length squared.

Table 2. Comparison of first moment of mass and center of mass
to mass moment of inertia and mass radius of gyration.

first moment of mass	second moment of mass or mass moment of inertia
$\int r\,dm = \rho t \int r\,dA$	$I = \int r^2\,dm = m\,r_m^2 = \rho t \int r^2\,dA$
center of mass	radius of gyration
$r_c = \frac{\int r\,dm}{\int dm} = \frac{\rho t \int r\,dA}{\rho t \int dA}$	$r_m = \sqrt{\frac{\int r^2\,dm}{\int dm}} = \sqrt{\frac{I}{m}}$

For rotations about other axes, the general equation for remains the same, but the influence of the thickness will be non-linear, i.e., the integration must consider both the thickness of the body and the cross-sectional area. Never-the-less, for a homogeneous prismatic body, a larger cross-sectional area moment of inertia will create a larger mass moment of inertia; as the thickness of the member increases, the more the mass moment of inertia about the x and y axes are governed by the thickness (or length), rather than the cross-sectional area.

Context

The mass moment of inertia is mathematically like the area moment of inertia. The farther the mass is from the axis of interest the larger the mass moment of inertia. The mass moment of inertia is a measure of resistance to the rotation of a body about a particular axis. The larger the mass moment of inertia, the slower the angular acceleration associated with a particular applied moment or the slower the angular velocity (rate of rotation) associated with a particular kinetic energy.

Composite Body Moment of Inertia

Main Idea

The parallel axis theorem allows for the calculation of moment of inertia about any axis related to the moment of inertia about a parallel centroidal axis. The parallel axis theorem can be used to calculate the area moment of inertia for a composite shape consisting of simpler shapes.

This discussion and classroom demonstrations should prepare students to:

- Calculate area moment of inertia for composite bodies using the parallel axis theorem.

Parallel Axis Theorem

The parallel axis theorem shown in Table 3 allows to the calculation of moment of inertia about different parallel axes with reference to the centroidal axis. The moment of inertia about a particular axial direction is smallest about the parallel axis passing through the centroid. The farther the axis of interest is away from the centroid the larger the moment of inertia.

Table 3. Parallel axis theorem equations.

area moment of inertia about an arbitrary x axis	area moment of inertia about an arbitrary y axis	mass moment of inertia about an arbitrary axis
$I_x = I_{xc} + y^2 A$	$I_y = I_{yc} + x^2 A$	$I = I_c + r^2 m$

Composite Body Theory

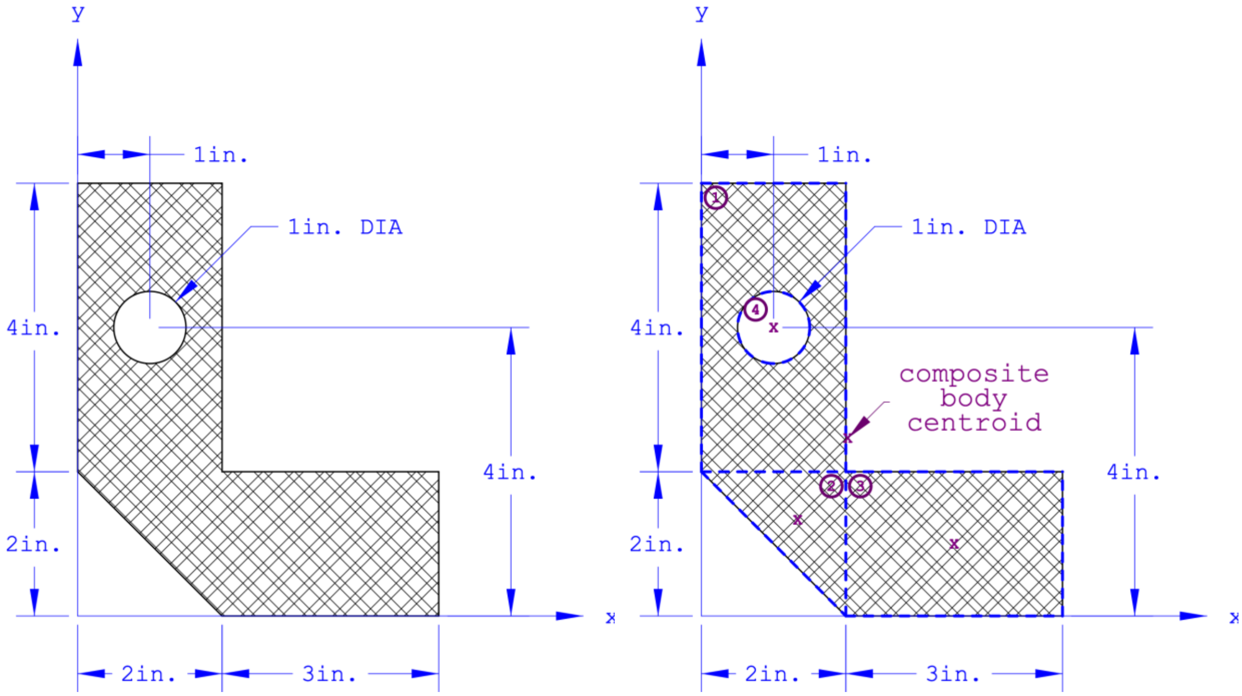
Composite body theory uses the parallel axis theorem to calculate the moment of inertia for a complex area made up of simple shapes with known moment of inertia properties. By summing the centroidal moment of inertia (moment of inertia about an axis passing through the centroid) for simple shapes and the second moment components of the parallel axis theorem, the moment of inertia for a composite shape can be calculated using the equations in Table 4.

Table 4. Composite body moment of inertia equations.

area moment of inertia about an arbitrary x axis	area moment of inertia about an arbitrary y axis	mass moment of inertia about an arbitrary axis
$I_{xcomp} = \sum I_{xc} + \sum y^2 A$	$I_{ycomp} = \sum I_{yc} + \sum x^2 A$	$I_{comp} = \sum I_c + \sum r^2 m$

Example

Figure 3.a shows a dimensioned cross section of a 3D printed composite body. In Figure 3.b, the composite body is broken into two rectangles, a triangle and a hole passing through the shape. Using the process described elsewhere, the composite body centroid location is at $(2.03, 2.47)$in. [11]



(a) (b)

Figure 3. The 2D cross section of a prismatic object (a) with total dimensions, and (b) in four segments: (1) a tall rectangle, (2) a right isosceles triangle, (3) a wide rectangle, and (4) a circular hole [11].

Table 5 presents the segment properties for each shape in the composite body. The last row sums the various columns useful for calculating the composite body area moment of inertias.

Table 5. Section properties for the composite prismatic object in Figure 2.

Segment	Area	Centroids		Parallel Axis Theorem Components		Centroidal Moments of Inertia	
	A (in.^{2})	x (in.)	y (in.)	x^2A (in.^{4})	y^2A (in.^{4})	I_x (in.^{4})	I_y (in.^{4})
1	8.00	1.00	4.00	8.00	128.00	10.667	2.667
2	2.00	1.33	1.33	3.56	3.56	0.444	0.444
3	6.00	3.50	1.00	73.50	6.00	2.000	4.500
4	-0.79	1.00	4.00	-0.79	-12.57	-0.049	-0.049
Sums:	15.21			84.27	124.99	13.06	7.56

Equations 2 show the calculation of the area moment of inertia based on the equations in Table 4. These area moments of inertia are about the original x and y axes.

$$I_{xcomp} = \sum I_{xc} + \sum y^2 A = 13.06\text{in.}^4 + 124.99\text{in.}^4 = 138.05\text{in.}^4 \tag{2.1}$$

$$I_{ycomp} = \sum I_{yc} + \sum x^2 A = 7.56\text{in.}^4 + 84.27\text{in.}^4 = 91.83\text{in.}^4 \tag{2.2}$$

To find the area moment of inertia about the centroidal axes, Equations 3 rearrange the parallel axis theorem from Table 3. The area moment of inertia about the x centroidal axis, I_{xc}, is larger than the area moment of inertia about the y centroidal axis, I_{yc}, because the area is farther from the x centroidal axis that the y centroidal axis as seen in Figure 4.

$$I_{xc} = I_x - y_c^2 A = 138.05\text{in.}^4 - (2.47\text{in.})^2(15.21\text{in.}^2) = 45.50\text{in.}^4 \tag{3.1}$$

$$I_{yc} = I_y - x_c^2 A = 91.83\text{in.}^4 - (2.03\text{in.})^2(15.21\text{in.}^2) = 29.15\text{in.}^4 \tag{3.2}$$

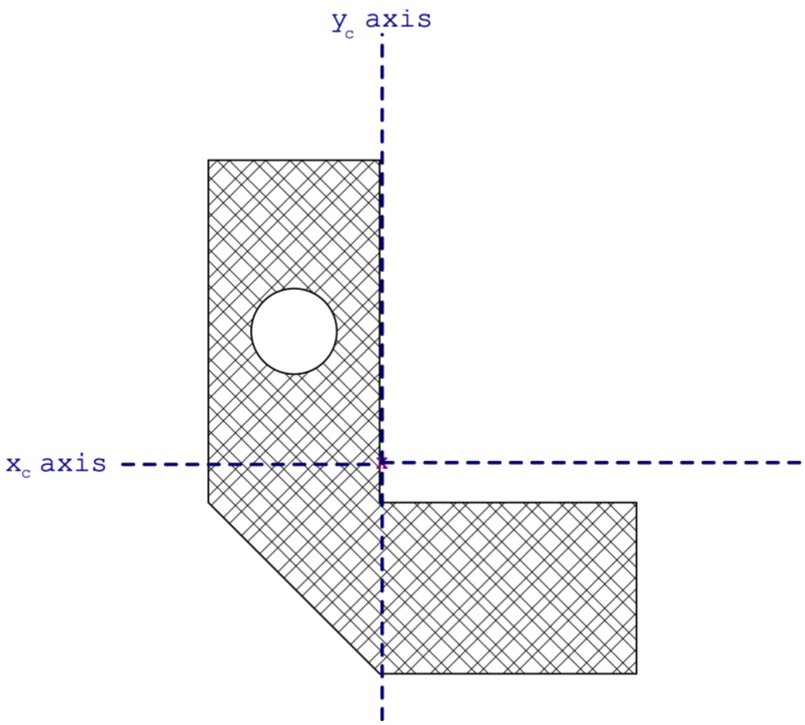
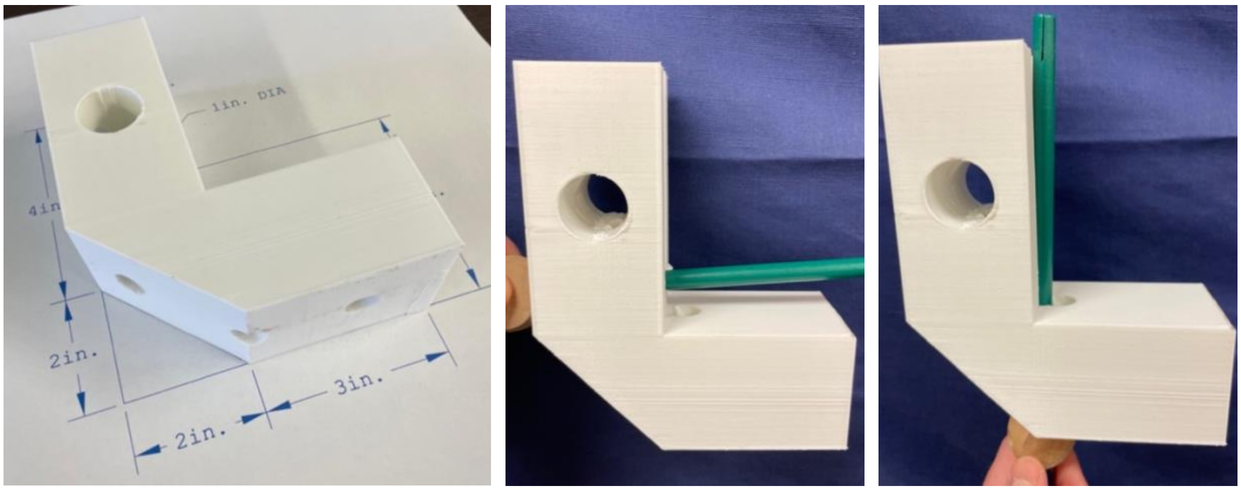


Figure 4. Cross section with centroidal axes shown. Similar to previous work [11]

In the classroom, these calculations are illustrated with the 3D printed composite body shown in Figure 5. By connecting area moment of inertia for a prismatic body to the mass moment of inertia (Table 2), students can observe the resistance to change in angular rotation about both the x (Figure 5.b) and y (Figure 5.c) axes by spinning the object about a rod run along those axes. Though the mass moment of inertia about the centroidal x and y axis for the given shape is not directly calculated from the area moment of inertia, by assuming roughly the same frictional condition, the rotation about the x axis will come to a stop more slowly than the rotation about the y axis due to the higher mass moment of inertia.



(a) (b) (c)

Figure 5. 3D printed composite body (a) dimensioned and spun about the centroidal (b) x axis, and (c) y axis.

Product of Inertia, Mohr's Circle and Principal Moments of Inertia

Main Idea

The product of inertia quantifies the degree of symmetry of a body about two perpendicular axes. The product of inertia can be used to create a Mohr's Circle and calculate the area moment of inertia about various axes.

This discussion and classroom demonstrations should prepare students to:

- Describe product moment of inertia for…
  - Continuous 2D areas.
  - Composite bodies using the Parallel Axis Theorem.
- Describe principal moments of inertia using…
  - Analytical equations.
  - Mohr's Circle for moment of inertia.

Product of Inertia

The product of inertia has the same units as area moment of inertia (length to the fourth power) but is calculated by multiplying each of the two perpendicular distances together as seen in Equation 4. As a result, the product of inertia will be zero if either axis is as axis of symmetry. While the area moment of inertia will always be positive, the product of inertia can be positive or negative depending on the quadrant containing most of the area.

$$I_{xy} = \int x\, y\, dA \tag{4}$$

Product of Inertia Parallel Axis Theorem

The product of inertia also has its own parallel axis theorem shown in Equation 5.

$$I_{xy} = I_{x_c y_c} + xyA \tag{5}$$

Product of Inertia of a Composite Body

Calculating the product of inertia for a composite body uses simplified shapes with known product of inertia properties and the product of inertia parallel axis theorem shown in Equation 6.

$$I_{xycomp} = \sum I_{x_c y_c} + \sum xyA \tag{6}$$

Mohr's Circle for Moment of Inertia with the Pole Method

Mohr's Circle is a convenient graphical method for determining the area moment of inertia for an area about rotated axes. Mohr's circle for area moment of inertia is plotted in the area moment of inertia vs product of inertia space using the following procedure:

1. Construct Mohr's Circle for moment of inertia
   1.1. Plot the location points (I_x, I_{xy}) and $(I_y, -I_{xy})$.
   1.2. Draw the diameter of the circle between the first two points.
   1.3. Draw a circle where the center is at the area moment of inertia intercept (x axis intercept) of the diameter and the radius extended to the first two points.

This circle describes every combination of area moment of inertia and product of inertia for all axes passing through the original origin.

2. Locate the Pole
   2.1. Draw a line parallel to the y axis through the $(I_y, -I_{xy})$ point.
   2.2. Draw a line parallel to the x axis through the (I_x, I_{xy}) point.
   2.3. These two lines should intersect on the circle at the pole.

Any line drawn parallel to an axis and passing through the pole will intersect Mohr's circle at that axis' moment of inertia and product of inertia. Where the circle intercepts the horizontal axis defines the principal moments of inertia. The strong axis is associated with the maximum moment of inertia where the shape's area is as far as possible from the axis. The weak axis is always perpendicular to the strong axis, has the minimum moment of inertia, and places the area of the shape as close to the axis as possible. Equations 8 calculate the intercept points based on the geometry of the center and radius of Mohr's circle.

$$center = \frac{I_x + I_y}{2} \tag{7.1}$$

$$radius = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2} \tag{7.2}$$

$$I_{min} = center - radius = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2} \tag{8.1}$$

$$I_{max} = center + radius = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2} \tag{8.2}$$

The angle of the principal axis can be found by drawing a line from the pole to the maximum intercept on the moment of inertia. The angle of this line is the angle of the maximum principal axis. Equation 9 shows the various trigonometric relationships that lead to the principal angle.

$$\theta_p = 2\ tan^{-1}\left(\frac{-2\ I_{xy}}{I_x - I_y}\right) = tan^{-1}\left(\frac{POLE_y}{I_{max} - POLE_x}\right) = tan^{-1}\left(\frac{I_{xy}}{I_{max} - I_y}\right) \tag{9}$$

Example

Table 6. Additional section properties for the composite prismatic object in Figure 2.presents the segment properties for each shape in the composite body. The last row sums the various columns useful for calculating the composite body area moment of inertias. The following equations calculate the product of inertia about the original axes and the centroidal axes.

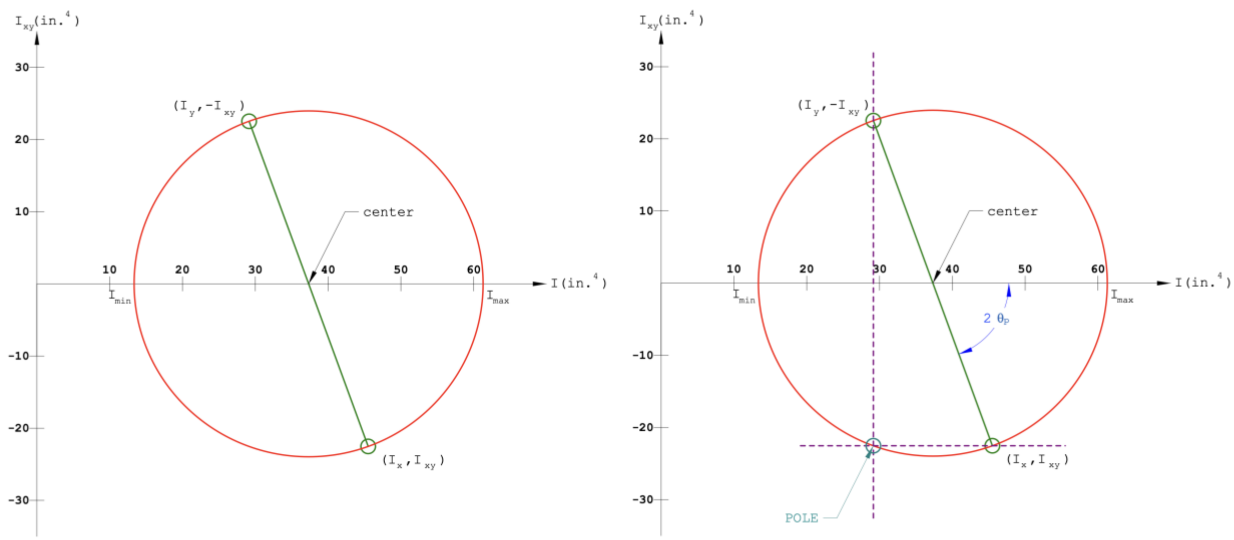
Table 6. Additional section properties for the composite prismatic object in Figure 2.

Segment	Area	Centroids		Parallel Axis Theorem Component	Centroidal Product of Inertia
	A (in.^{2})	x (in.)	y (in.)	xyA (in.^{4})	I_{xy} (in.^{4})
1	8.00	1.00	4.00	32.00	0.000
2	2.00	1.33	1.33	3.56	0.222
3	6.00	3.50	1.00	21.00	0.000
4	-0.79	1.00	4.00	-3.14	0.000
Sums:	15.21			53.41	0.22

$$I_{xycomp} = \sum I_{x_c y_c} + \sum xyA = 53.41 in.^4 + 0.22 in.^4 = 53.64 in.^4 \tag{6}$$

$$I_{x_c y_c} = I_{xy} - xyA = 53.64 in.^4 - (2.47 in.)(2.03 in.)(15.21 in.^2) = -22.53 in.^4 \tag{5}$$

Using the data for the 3D printed composite body, the Mohr's circle can be constructed using the process above (Figure 6.a) and the pole can be identified (Figure 6.b). From Mohr's circle, the center, radius, and principal moments of inertia can be calculated using Equations 7 and 8.



(a) (b)

Figure 6. Mohr's circle for moment of inertia for the 3D printed composite body: (a) constructed and (b) with pole identified.

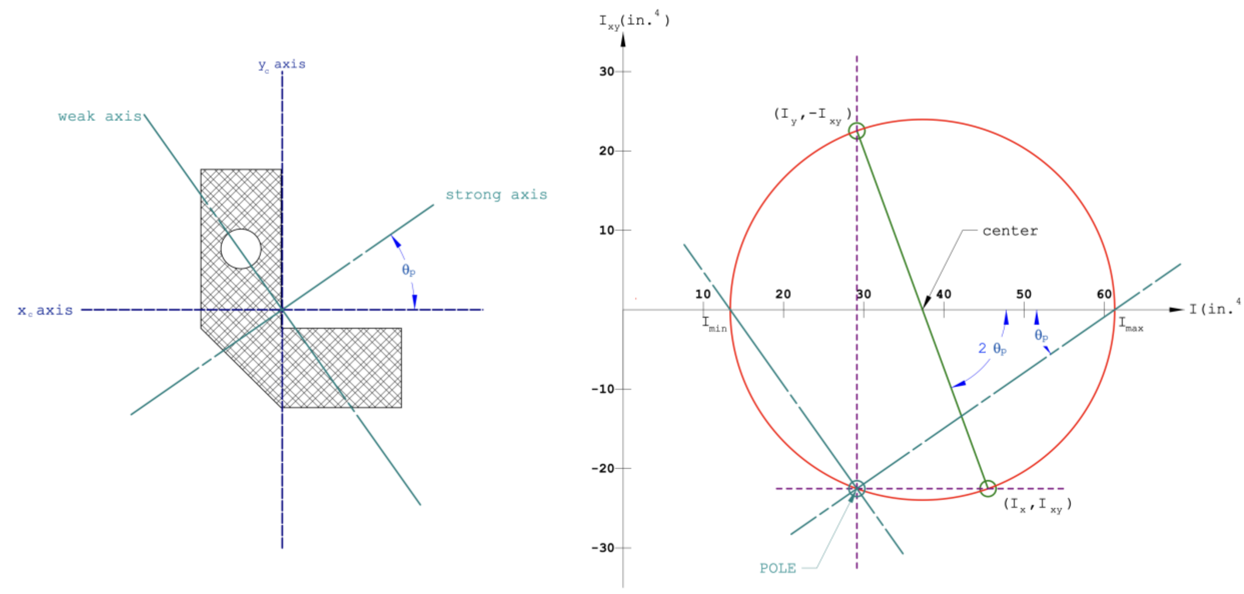
$$center = \frac{I_x + I_y}{2} = \frac{45.40\text{in.}^4 + 29.15\text{in.}^4}{2} = 37.33\text{in.}^4 \tag{7.1}$$

$$radius = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2} = \sqrt{\left(\frac{45.04\text{in.}^4 - 29.15\text{in.}^4}{2}\right)^2 + (22.53\text{in.}^4)^2} = 23.97\text{in.}^4 \tag{7.2}$$

$$I_{min} = center - radius = \frac{I_x + I_y}{2} - \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2} = 37.33\text{in.}^4 - 23.97\text{in.}^4 = 13.36\text{in.}^4 \tag{8.1}$$

$$I_{max} = center + radius = \frac{I_x + I_y}{2} + \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + {I_{xy}}^2} = 37.33\text{in.}^4 + 23.97\text{in.}^4 = 61.29\text{in.}^4 \tag{8.2}$$

Figure 7 shows the lines connecting the pole to the principal area moments of inertia. These lines directly correlate to the strong and weak axes. Though the principal angle can be calculated using Equation 9, they can also be measured directly with a protractor.

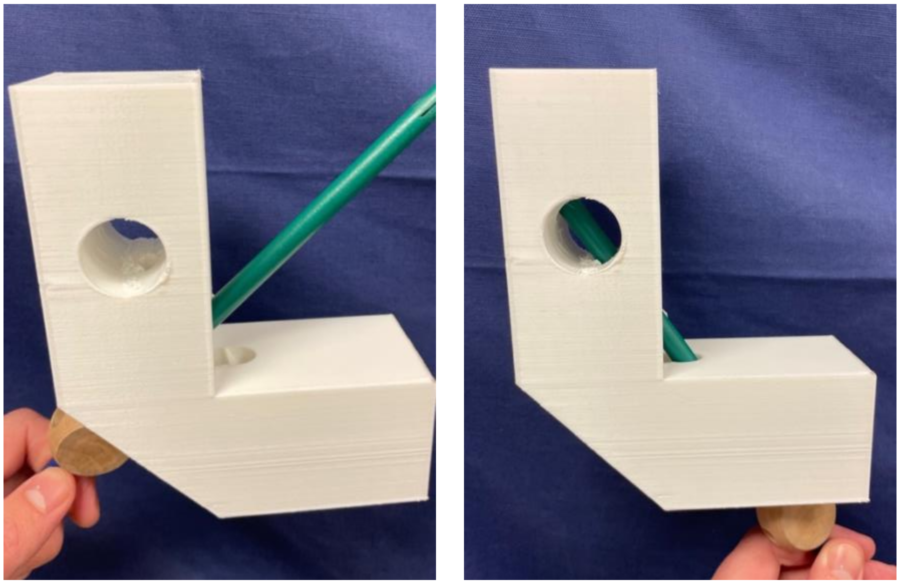


(a) (b)

Figure 7. Principal moment of inertia and the weak and strong axis on (a) the composite area and (b) Mohr's circle.

$$\theta_p = 2\ tan^{-1}\left(\frac{-2\ I_{xy}}{I_x - I_y}\right) = tan^{-1}\left(\frac{POLE_y}{I_{max} - POLE_x}\right) = tan^{-1}\left(\frac{I_{xy}}{I_{max} - I_y}\right) = 35° \tag{9}$$

Finally, though the mass moment of inertia about the x and y axis is not directly proportional to the area moment of inertia, the principal mass and area moments of inertia will fall along the same axes. The 3D prismatic composite body can be spun around the weak and strong axes as seen in Figure 8. Given an initial rotation and constant frictional force against the rod, the body will stop more quickly when spun around the weak axis due to the minimum moment of inertia. It will lose rotational speed more slowly when spun around the strong axis due to the largest moment of inertia and the mass as far from the strong axis as possible.



(a) (b)

Figure 8. 3D printed composite body rotated about (a) the strong axis and (b) the weak axis.

Student Response

Students often struggle to create a functioning mental model for area moment of inertia. A rotating physical model allows students to connect pre-existing knowledge of mass moment of inertia phenomena (like the spinning figure skater) to area moment inertia. Students appear to better comprehend the mathematics, when they see the behavior of the physical model at the end of a set of calculations. Rotated moment of inertia is particularly powerful as students can see the dramatic difference between strong and weak axis rotations.

Conclusion

Area moment of inertia can be taught within the context of applied loads and mass moment of inertia. A 3D printed prismatic composite body can be used to illustrate moment of inertia about horizontal and vertical axes. Mohr's circle for moment of inertia uses product of inertia to identify the principal moments of inertia. The pole method allows for a direct graphical identification of the weak and strong axes. The strong and weak axis can be illustrated using the 3D printed composite body to help students form connections between area moment of inertia, mass moment of inertia and pre-existing knowledge about rotation.

References

[1] R. C. Hibbeler, *Engineering Mechanics: Statics*, 14 edition. Hoboken: Pearson, 2015.

[2] F. Beer, E. R. Johnston, D. Mazurek, P. Cornwell, and B. Self, *Vector Mechanics for Engineers: Statics and Dynamics*, 11 edition. New York, NY: McGraw-Hill Education, 2015.

[3] J. Hanus and A. Estes, “Keep Them On The Edge Of Their Seats: Bringing Drama Into The Engineering Classroom,” Jun. 2002, p. 7.777.1-7.777.13. Accessed: Feb. 09, 2022. [Online]. Available: https://peer.asee.org/keep-them-on-the-edge-of-their-seats-bringing-drama-into-the-engineering-classroom

[4] B. Newberry, “Experimentally Determining Mass Moment Properties,” Jun. 2008, p. 13.594.1-13.594.9. Accessed: Feb. 09, 2022. [Online]. Available: https://peer.asee.org/experimentally-determining-mass-moment-properties

[5] S. C. MacNamara and J. V. Dannenhoffer, “First Encounters: Statics as a Gateway to Engineering,” Jun. 2013, p. 23.600.1-23.600.15. Accessed: Apr. 07, 2021. [Online]. Available: https://peer.asee.org/first-encounters-statics-as-a-gateway-to-engineering

[6] R. Al-Hammoud and K. Ghavam, “Engaging Engineering Students in Lectures Using Anecdotes, Activities, and Games,” presented at the 2018 ASEE Annual Conference & Exposition, Jun. 2018. Accessed: Apr. 07, 2021. [Online]. Available: https://peer.asee.org/engaging-engineering-students-in-lectures-using-anecdotes-activities-and-games

[7] D. Raviv and D. R. Barb, “A Visual, Intuitive, and Engaging Approach to Explaining the Center of Gravity Concept in Statics,” presented at the 2019 ASEE Annual Conference & Exposition, Jun. 2019. Accessed: Apr. 08, 2021. [Online]. Available: https://peer.asee.org/a-visual-intuitive-and-engaging-approach-to-explaining-the-center-of-gravity-concept-in-statics

[8] S. C. M. Namara, “The Design Competition as a Tool for Teaching Statics,” Jun. 2012, p. 25.1283.1-25.1283.13. Accessed: Apr. 08, 2021. [Online]. Available: https://peer.asee.org/the-design-competition-as-a-tool-for-teaching-statics

[9] A. K. T. Howard, “Work in Progress: 3-D Models with Lesson Plans,” presented at the 2019 ASEE Annual Conference & Exposition, Jun. 2019. Accessed: Apr. 07, 2021. [Online]. Available: https://peer.asee.org/work-in-progress-3-d-models-with-lesson-plans

[10] A. Choudhury, C. Ochei, and R. Rice, “Simulation For The Study Of Beam Deflection In Strength Of Materials,” Jun. 2001, p. 6.877.1-6.877.10. Accessed: Feb. 09, 2022. [Online]. Available: https://peer.asee.org/simulation-for-the-study-of-beam-deflection-in-strength-of-materials

[11] T. A. Wood, “3D Printed Composite Body Illustrating Composite Body Centroid and Center of Gravity,” presented at the 2021 ASEE Virtual Annual Conference Content Access, Jul. 2021. Accessed: Aug. 10, 2021. [Online]. Available: https://peer.asee.org/3d-printed-composite-body-illustrating-composite-body-centroid-and-center-of-gravity