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Annuities as a Good Course Example

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Abstract

Annuities are widely recommended and purchased as a low-risk retirement investment, guaranteeing income for life. As an example, an immediate or a deferred annuity would be purchased for $100,000 at age 62, and then monthly benefits would start at any time until age 85. In these annuities, once monthly income starts it continues until death, but nothing is received if death occurs before income starts.

We discuss why and how to include immediate and deferred annuities in a course's early present worth coverage. Now that inflation is again significant, annuities provide a needed example of accounting for inflation through the interest rate, rather than assuming constant value dollars and real interest rates. Like other personal finance examples this prepares students for personal and professional use of engineering economy. It also helps motivate students by using real-life scenarios.

Continued use of the annuity example in rate of return, breakeven, uncertainty, and inflation material links topics and extends student understanding. Possible coverage in more advanced courses includes case studies comparing results for different interest rates and different purchase dates, starting ages, and expected death ages. In more advanced courses inflation-adjusted annuities and mortality distributions merit coverage.

Introduction

Annuities are an investment where a person invests a sum of money (either all at once or over time), then is guaranteed a set income for life. The annuity income may start paying right away (an immediate annuity) or income may be delayed to later (a deferred annuity). A simple annuity has no other features, but a vast array of options is usually available—at a price.

While annuities are mentioned in engineering economy textbooks and literature, the topic is largely ignored even though annuities are the source of one of our cash flow symbols, *A*. This paper opens a discussion exploring the use of how best to use annuities as a class example. More broadly, we are examining how annuities can be best evaluated as an investment.

Annuities are often sold by insurance companies but are also widely offered through brokerage firms [1]. They are widely recommended as a low-risk investment, primarily as guaranteed retirement income [2]. The people who buy annuities are usually looking for guaranteed income for life, as a hedge against outliving their savings and investments [3]. Social Security benefits fill a similar need, and a good class exercise is asking how large an annuity purchase needs to be to match those benefits.

Annuities can be used as a continuity example when teaching engineering economics. Annuity income estimates are publicly available and can be looked up by students. This is real world data, part of personal finance, which students usually find engaging and useful [4]. Annuities can be used to demonstrate concepts such as uniform cash flow, equivalence, present worth, annual worth, and rate of return. They can also be used to demonstrate more involved concepts such as uncertainty, probability, expected value, and risk. Using annuities as a class example provides skills in both quantitative analysis and financial literacy for better personal decision making [5].

Literature review

Annuities were not in the table of contents nor index for leading introductory engineering economy texts [6, 7, 8, 9, 10, 11]. Newnan's Appendix 9A is the most complete coverage of investing for retirement, but it does not include annuities. There may be end-of-chapter problems on annuities, but none were found. Coverage is more likely to be found in personal finance texts which were not reviewed for this paper.

Proceedings of ASEE, ASEM, and IISE conferences were reviewed without finding results for teaching engineering economy using annuities. In addition, very little was found regarding use of personal finance in engineering economy teaching with the exception of [12].

Introducing Annuities in the Present Worth Chapter

The first reason to include annuities with present worth (PW) material is to reinforce and extend coverage of the basic engineering economy factors and spreadsheet financial functions. The PW formula for an *immediate* annuity purchased for \$100K and delivering the monthly benefits in Table 1 is Eq. (1)—typically the first formula covered after the (P/F) factors. This assumes all benefits are received as end-of-year cash flows and that death occurs at the end of the year.

$$P = -\$100\text{K} + 12(\text{M})(P/A, i, N) \tag{1a}$$

where

M = monthly annuity income
i = annual interest rate
N = number of years until death

This can be expressed as a spreadsheet function

$$P = -\$100\text{K} + \text{PV}(i, N, -12*\text{M}) \tag{1b}$$

These equations represent annual compounding. Monthly compounding could be used, but annual compounding matches annual mortality data. Monthly compounding would increase the PW; but a mid-year average death date in year N would decrease the PW. Using monthly periods is more likely to produce errors than insight.

Delayed or *deferred* annuities are specifically covered in [9 (Eq. 2a) and 13 (Eq. 2a & 2b)]. Other introductory texts may include examples or problems, but no examples were found. These equations assume purchase at the beginning of the "buy" year, and the monthly benefits are received at the end

$$P = -\$100\text{K} + 12(\text{M})(P/A, i, N - D)\,(P/F, i, D) \tag{2a}$$

$$P = -\$100\text{K} + 12(\text{M})[(P/A, i, N) - (P/A, i, D)] \tag{2b}$$

where

D = number of years between buying the annuity and starting monthly income

Equation (2a) can be expressed as a spreadsheet function.

$$P = -\$100\text{K} + \text{PV}(i, N-D, -12*\text{M})*\text{PV}(i, D, 0,-1) \tag{2c}$$

Table 1 shows the monthly benefits of a $100,000 annuity purchased at age 62 and starting annuity income at various ages. Note that annuities can legally consider gender as part of rate setting. Auto insurance illustrates that gender, age, and driving record can also legally be part of rate-making. The higher monthly payments for males are because females live longer on average.

TABLE I.
MONTHLY ANNUITY INCOME (M) FOR $100K ANNUITY PURCHASED AT AGE 62 [2].

Starting Age	Monthly Benefit	
	Male	Female
62	$446	$425
66	553	524
70	722	674
75	1048	969
80	1697	1519
85	3110	2574

As described in [6 & 7] and exemplified in many textbook loan and savings examples and problems, including personal finance applications of engineering economy can be more engaging and motivating than industrial examples. Annuities are one such personal finance example, but annuities are particularly good demonstrations of inflation and of the role of risk and insurance in decision-making.

Inflation is part of many engineering economy applications. It is currently increasing from the reported low levels that many countries have had since the financial crisis. Textbook problems and examples implicitly assume constant value dollars and real interest rates for the time value of money until late-in-text coverage of inflation. Sullivan [9] covers inflation earlier and Newnan [7, p. 110] covers why constant-value cash flow patterns of A, G, and g are so often assumed.

We suggest that it is appropriate and important to stress early in a course that the level of inflation must be estimated as small—before it can be ignored. Even a seemingly small rate of 2% can be significant since many engineering economy applications have a time horizon of decades. Introducing this early in the course and reinforcing it will better prepare students for the transition from their text to real-world application.

The consensus of faculty in engineering economy expressed in textbooks is that detailed coverage of inflation early in a course would be confusing. However, providing students with data from Table 1 provides an example where cash flow amounts do not change with inflation—but their value does. When calculating the PW, the easy approach is to use an interest rate that includes both the time value of money and inflation—Eq. (3).

$$\text{Interest rate} \approx \text{real interest rate} + \text{inflation rate} \tag{3}$$

The mathematically correct formula that adds the cross-product term (often noted as $i*f$) is normally presented in inflation chapter coverage, but it would complicate the presentation of inflation at this stage (and not greatly impact the results at the currently forecasted inflation rates). We observe that the uncertainty in inflation estimates is far larger than the cross-product. We also note that published interest rates for savings, loan, credit card accounts, and even the weighted average cost of capital include inflation as part of the rate. These are the interest rates that students understand before they take engineering economy.

That leads to the question of what rate for the time value of money and for inflation. For this example, we choose to use data available from Social Security. The data has the advantages of public availability, clarity, and clear applicability. The yearly cost of living adjustments (COLA) shown in Figure 1 from 1985 to 2020 average 2.50%. Note that the COLA adjustment for 2022 is 5.9%. The real interest rate that is most used in Social Security analyses is 3%. Adding 2.5% inflation to the real rate of 3% means the correct rate to calculate the PW of an annuity is 5.5%, not 3%.

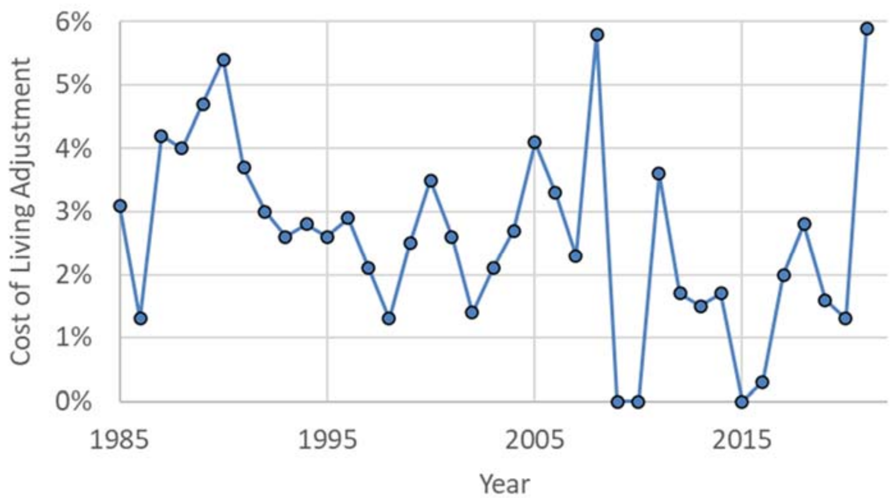

Fig. 1. Social Security cost of living adjustment [14].

The results shown in Figure 2 show that using the correct interest rate matters. At the correct rate of 5.5%, the annuity benefits do not have a positive PW—no matter how long the buyer lives. We can also observe that all PW curves flatten as the time horizon increases, as is typical.

Examples for Use in Early Chapters

When applying Equations 1 and 2 to annuity problems, the age when the annuity is purchased becomes *time* 0, and it is subtracted from other ages to determine the other years. For example, purchasing at 62, starting at 66, and dying at 85 equates to a D of 4 years and an N of 23 years.

A good initial example would be finding the present worth for a female, funding an immediate annuity with \$100,000 at age 62, and starting annuity income at age 62. Assume the person lives to an expected age of 85 and i = 5.5%. From Table 1 the monthly benefit is \$425, and from the solid blue line in Figure 2 the result should be about −\$35,000. Substituting values into Eq. (1a):

$$\text{PW} = -100\text{K} + 12*425(P/A, 5.5\%, 85 - 62) = -\$34{,}337$$

(Exact with a spreadsheet or financial calculator and close with interpolation between tabulated factors.) Or using Eq. 1b in a spreadsheet:

PW = −100,000 + PV(5.5%,85 − 62, −425*12) = −$34,337

Note that the data values should be entered into a data block and not hard coded into the formula.

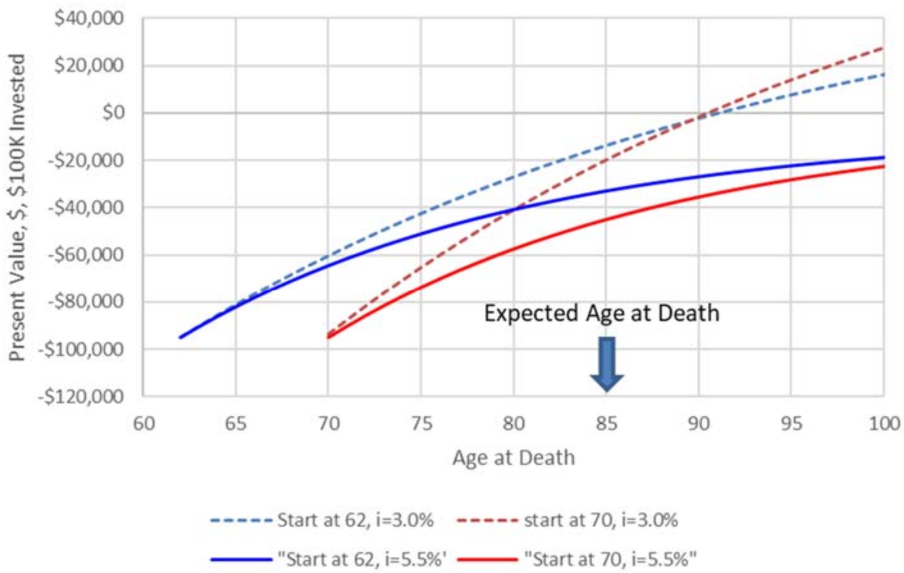

Fig. 2. Annuity PW vs. age at death for female.

Table 1 supports many variations by changing the age when income begins, the gender, interest rate, or the expected age at death. While the 5.5% interest rate used here is best supported by spreadsheets, if tabulated factors are the planned tool then an integer value for the inflation rate is suggested.

Another example matched to PW coverage is finding the number of years until the PW is worth 80% of the investment at i = 5.5%. From Table 1, assuming a male invests $100K at age 62 and begins annuity income at age 62, the monthly benefit is $446. Using spreadsheets,

years =NPER(5.5%, 12*446, −100000*0.8) = 32.25 years

If this example is changed to a deferred annuity with income beginning at age 66, it is best solved by modifying Equation (2c). The change is to nest the P/F factor =PV(rate, N, 0, −FV) within the NPER function and add the 4 year deferral period.

years = 4 + NPER(5.5%, 12*553*PV(5.5%,4,0, −1), −100000*0.8) = 36.17 years

We have no example for annual worth material—because we recommend against using annuities there. Uniform annual cash flows are nearly always assumed to be constant value dollars. This applies to evaluating social security [15] and other similar benefits that include adjustments for inflation. Much of the value of the annuity examples is that they represent a clear example of cash flows that are not adjusted for inflation.

As a rate of return example, at what interest rate is the PW = 0 for a female dying as she ends her 85^{th} year? Assume an annuity is purchased for $100,000 at age 62 and begins providing income at age 66. Using Eq. 2a:

$PW = 0 = -100K + 12*524(P/A, i, 19)*(P/F, i, 4)$

$(P/A, i, 19)*(P/F, i, 4) = 15.903$

at $i = 1.25\%$, $(P/A, i, 19)*(P/F, i, 4) = (16.849)(0.9515) = 16.03$

at $i = 1.50\%$, $(P/A, i, 19)*(P/F, i, 4) = (16.426)(0.9422) = 15.48$

interpolating, $i = 1.31\%$

Using a spreadsheet, two approaches are possible. The most commonly used is to list the cash flows in a table, and use the IRR block function, starting with the investment.

$=IRR(CF_{Investment}:CF_{Final}) = 1.30\%$

The second approach uses Eq. 2c for the PW, and uses the GOAL SEEK (under What-If Analysis) tool to find the interest rate (cell that changes) that sets the PW = 0 (value for goal cell).

$= -100,000 + PV(i, 85 - 66, -12*524)*PV(i, 66 - 62, 0, -1) = 1.30\%$

The 1.30% is below the expected inflation rate of 2.5%. Applying Equation (3), the inflation rate of 2.5% is subtracted for a real return of

$\approx -1.20\%$. This is a poor investment.

Continuity Example for Later Material

Dealing with uncertainty through sensitivity analysis and/or use of probabilities is usually covered in later chapters of most textbooks and may or may not be included in undergraduate courses. Figure 2, which shows the present worth of an immediate and a deferred annuity over different ages at death, is an example of the many sensitivity possibilities. Other factors that may be analyzed include different purchasing ages, different ages when annuity income begins (from age 62 to 85), and interest rates. A presentation at a recent ASEM conference [16] provided preliminary results for a case study in using annuities.

However, the age at death is the primary uncertainty in all cases. The simplest way to approach this uncertainty using probabilities is to analyze annuity values at an average age at death. Searches should be phrased as “life expectancy at age 62” for example. Searching for “average age at death” includes data for those who died before age 62.

Annuities are a good example of decision-making when faced with uncertainty. As shown in Figure 2, annuities have a present worth less than zero when we use average ages at death and appropriate interest rates. This makes annuities poor investments but valid insurance policies. What is insured is the continued income stream in case of a long life. This is longevity risk, or the risk of outliving one’s savings and investments. Annuities are often sold as products addressing longevity risk.

Insurance premiums in general must cover expected losses and many other costs. These include costs to market and sell policies, the cost of managing accounts and processing claims (some fraudulent), and an expected profit for the firm. It is no surprise that most annuities are sold by insurance companies. A typical customer expects to lose money on insurance, and hopes that claims do not need to be made on the policy. Similarly, annuities are insurance policies that benefit the buyer in case a person outlives other sources of income.

If a later inflation chapter is covered, then revisiting annuities is certainly appropriate. They still provide a good example of actual or nominal dollars versus the constant value dollars used throughout engineering economy texts. This is the time to build tables showing the relationship between rates for inflation, nominal interest, and real return. In addition, inflation adjusted annuities may be purchased. For example, a person can purchase an annuity whose annual income increases by 2% each year. The initial monthly benefit is lower than shown in Table 1, but the inflation adjusted annuity increases annually while the Table 1 values continue unchanged.

Other Uses and Applications of Annuities

More advanced classes can extend using average life expectancies to using mortality statistics on the probabilities at death at each age for a variety of demographic groups. Sources such as the National Vital Statistics System (NVSS) provide annually updated Life Tables which may be used. As noted earlier, care needs to be taken; the information is based on a person being alive at birth. The data needs to be modified to create conditional probabilities assuming the person is alive at 62 (or other age). The conditional probabilities may be used to determine the expected value (EV) and the standard deviation (a key measure of risk) of an annuity. A study of this type was published in *The Engineering Economist* about calculating risk and standard deviation in evaluating social security starting ages [17].

Annuities can be compared with Social Security. Many (probably nearly all) undergraduates think that SS is only for old people. Learning about benefit amounts and survivor's benefits for spouses and dependents is an important element of personal finance. Here is a possible assignment that could be used in an introductory class with other inflation material.

You have had a lucrative career and you are now eligible to receive $3113 per month, which was the maximum monthly social security benefit at the full retirement age in 2021. Social security payments are indexed for inflation annually, thus the comparison should be made with inflation adjusted annuities. A $100,000 annuity, with a 2% annual inflation adjustment, will initially pay a male $471 per month or a female $432 per month. This assumes that the annuity is funded at age 62 and will begin payments at the same time as the social security full retirement age. How large an annuity purchase is needed to match your social security benefit?

Male: (3113/471)*100,000 = $660,934 required as principal
Female: (3113/432)*100,000 = $720,602 required as principal

Hundreds of thousands of dollars must be invested in an annuity to equal the social security benefit, with females needing to invest considerably more than males. These annuities also lack the spousal and survivor's benefits offered by Social Security.

Material presented in this paper may be used in a variety of ways. These may be examples in lectures or used for in-class exercises. The annuity information is publicly available and changes almost continuously as interest rates change. Homework can be assigned, and it can be broken out many ways, including the amount invested, gender, interest rate, when an annuity is scheduled to begin payments, or with different age at death assumptions. If probabilistic mortality data are used, results for different demographics can be compared.

As problems become more complex (such as incorporating mortality data), case studies are recommended. These can be done by teams or individuals, with different people (or teams) analyzing different scenarios. The same databases may be used in a wide variety of applications, providing different results.

Conclusions

Different class uses of immediate and deferred annuities are analyzed in this paper. Information regarding investments and payments are publicly available, representing real and timely data. This is personal finance, where students tend to be more engaged and motivated. Examples can be used as a continuity example across a course, using the same database to explain many engineering economy topics. Because annuities pay out over many years, the interest rate assumption is very important, and can be used as an early introduction to inflation. We believe that constant-value dollars should be contrasted with dollars whose values are shrunk by inflation. We believe that the *fixed* dollars of annuities are a good starting point.

The analyzed data is real. It can be supplied by the instructor or looked up by the students. The topic is good for lectures, in-class assignments, homework, and case studies. Multiple versions of any assignments can be made, providing students with alternative versions of the same basic problem.

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