APPLICATION OF VECTOR ANALYSIS IN ELECTRIC ENERGY CONVERSION USING MATLAB^{TM}

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Abstract

In electrical engineering, specifically in Electric Power Systems, complex numbers are used widely throughout. More specifically in Electric Energy Conversion there is a concept called the "revolving magnetic field", which simply describes how the stator of an electrical machine generates a rotating magnetic field. This concept is modeled with a set of differential equations and with some manipulations we arrive to a set of vectors. These vectors in turn indicate the rotating field in space. The manipulation of such theory is simple yet hard to present to the students. It is for this reason that the use of MATLAB^{TM} and/or MatCad^{TM} is recommended. This paper presents a mathematical manipulation applied to electrical engineering of a "revolving magnetic field". Furthermore it takes the mathematical approach to a MATLAB^{TM} simulation.

Three-phase analysis of an stator

Three phase induction machines are the work-horse of industry, and these machines have a "rotating magnetic field". We will give a brief physical description of the stator of these machines. Consider a sequence a-b-c and a symmetric distribution of the phases by 120^0 electrical degrees each in space and around the air gap. The basic three-phase machine will have three coils that we consider to have the following terminals:

$$a \qquad a' \qquad b \qquad b' \qquad c \qquad c'$$

These coils are excited by a time dependent sinusoidal alternating current producing a sinusoidal magneto-motive force (mmf) wave at the center of the magnetic axis of particular phase. Therefore the three-space sinusoidal mmf waves are displaced 120^0 electrical degrees in space. Figure 1 shows how we determine the magnetic axis of a coil. Figure two indicates the three magnetic-axis of three coils placed in space around the stator. A three-phase system requires three coils to create three magnetic

fields that will interact among each other to obtain a resulting magnetic component. Furthermore, this resultant component will rotate in space around the air gap of the electric machine.
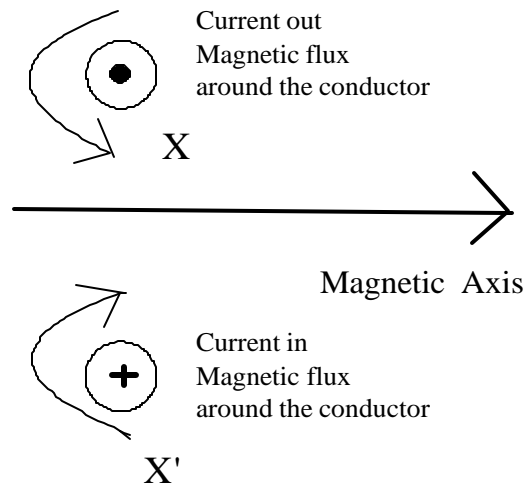

Figure 1. Magnetic axis of a single coil

Figure 1 shows a coil with current coming out conductor X with a dot and a conductor X' with current getting into the conductor. Using the right hand rule [3], we determine the direction of the magnetic flux. The magnetic axis is, therefore, created perpendicular to the coil. Figure 2 shows three coils placed in space around the stator of a primitive three-phase machine. Notice that the magnetic axis of each coil is perpendicular to their corresponding plane.
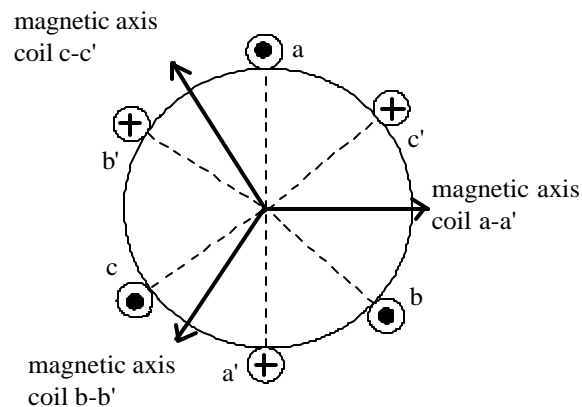

Figure 2. Magnetic axis of three coils displaced symmetrically

For balanced conditions the instantaneous currents, for each coil, are:

$$i_a = I_m \sin wt \qquad (1)$$

$$i_b = I_m \sin(wt - 120^0) \qquad (2)$$

$$i_c = I_m \sin(wt - 240^0) \qquad (3)$$

In vector form, equations 1 thru 3 become:

$$\overline{I}_a = I_m e^{j0^0} \qquad (4)$$

$$\overline{I}_b = I_m e^{-j120^0} \qquad (5)$$

$$\overline{I}_c = I_m e^{-j240^0} \qquad (6)$$

Where I_m is the maximum value of current.

From electromagnetic theory [4] $B \propto \Phi$ indicating that the magnetic flux density is proportional to the magnetic flux. If $I \propto \Phi$ then $B \propto I$ and consequently equations 4, 5 and 6 become:

$$B_a = B_m \sin(wt) \qquad (7)$$

$$B_b = B_m \sin(wt - 120^0) \qquad (8)$$

$$B_c = B_m \sin(wt - 240) \qquad (9)$$

The magnetic flux densities of equations 7, 8 and 9 are located in space as indicated in figure 2. Then we have the following vectors to solve in order to determine the resultant magnetic flux density.

$$\overline{B}_a = B_m \sin(wt) \, e^{j0^0} \qquad (10)$$

$$\overline{B}_b = B_m \sin(wt - 120^0) \, e^{j120^0} \qquad (11)$$

$$\overline{B}_c = B_m \sin(wt - 240) \, e^{j240^0} \qquad (12)$$

Finally the resultant magnetic flux density is obtained by adding the vectors from equations 10, 11 and 12. Then :

$$\vec{B}_R = \overline{B}_a + \overline{B}_b + \overline{B}_c \qquad (13)$$

As example, let ωt be equal to zero, then $\overline{B}_r$ becomes:

$$\overline{B}_r = B_m \sin(0) \, e^{j0^0} \quad + \quad B_m \sin(0 - 120^0) \, e^{j120^0} \quad + \quad B_m \sin(0 - 240) \, e^{j240^0}$$

$$\overline{B}_r = \frac{3}{2} B_m \, e^{j90^0}$$

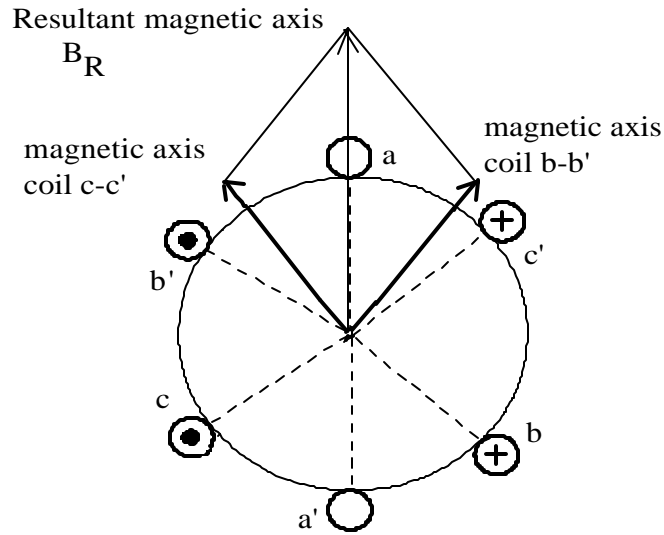Figure 3 indicates the vector addition and the resultant vector at zero degrees.

Figure3. Resultant magnetic flux at ωt=0

At ωt =90° the resultant magnetic flux is equal to $1.5e^{j0^o}$. Figure 4 shows the vector diagram for this resultant magnetic flux.
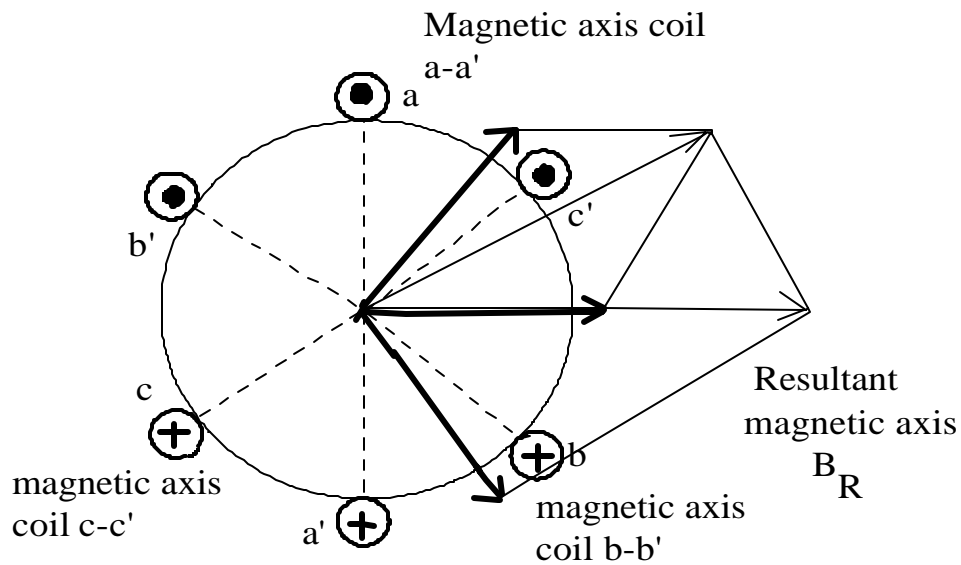
Figure4. Resultant magnetic flux for $\omega t = 30^o$

Simulation

This a program developed using Matlab^{TM} that performs the calculations for B_r for 360 degrees. The animation part of it and the graphics indicate very clearly the vector rotation of electric magnetic fields in an electric three-phase machine.

```
%------------------------------------------------------------------
-----
% CALIFORNIA STATE UNIVERSITY NORTHRIDGE
% ROTATING MAGNETIC FIELD
% JANUARY 2002
%   -------------------------------------------------------------
-----
% CALCULATE NET MAGNETIC FIELD PRODUCED BY A THREE-PHASE STATOR

% BMAX = 1
% FREQUENCY = 60 HZ
% GENERATE THREE COMPONENTS MAGNETIC FIELD

hz = 60
t  = 0:1/6000:1/60
omega = hz*2*3.14159
j = sqrt(-1)

Bx = sin(omega*t)*(cos(0) + j*sin(0))
By = sin(omega * t - 2*pi/3)*(cos(2*pi/3)+ j*sin(2*pi/3))

Bz = sin(omega*t + 2*pi/3)*(cos(-2*pi/3)+ j*sin(-2*pi/3))

Br = Bx + By + Bz

circle = 1.5*(cos(omega*t)+ j*sin(omega*t))
% plot the magnitude and direction of the resultant magnetic field Br
1.5 * Bmax*angle


ii = 1:length(t)
k = 1

%subplot(3,1,1)
%plot([ 0 real(Bx(ii))],[0 j*(Bx(ii))],'k','linewidth',2)

%subplot(3,1,2)

%plot(circle, 'k')
```

```
%subplot(3,1,3)
%plot (Br,t)
iii = 59

for iii=1:101

k(1)=Bx(iii)


k(2)=By(iii)

k(3)=Bz(iii)

k(4)=Br(iii)

%subplot(1,2,1)
compass(k)
pause(.1)
end

%subplot(1,2,2)
%polar(angle(Br),abs(Br))

%compass(Br)
```
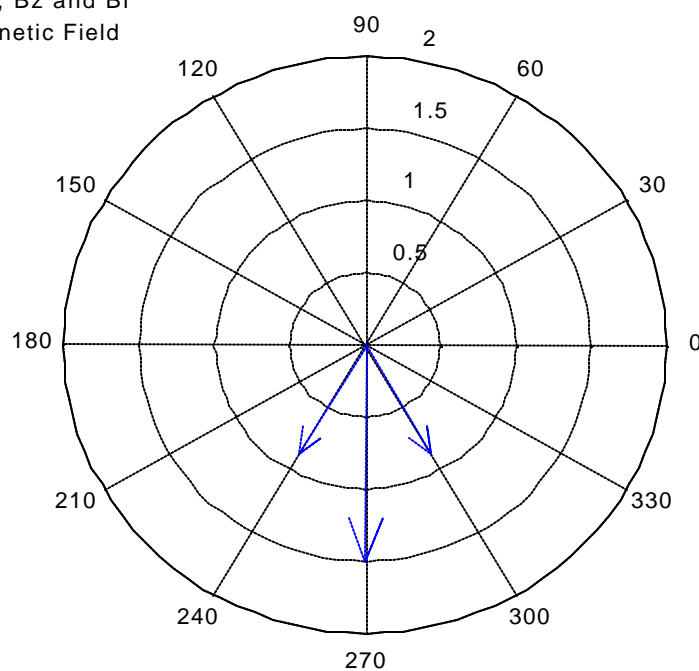
Output

The graphical output is shown below. This graph is an animated graph that can only be shown if we run the program. In the meantime we show only the last calculated point. Observe that the maximum value of the resultant phasor is 1.5 as it is indicated in the circle.

Plot of Bx, By, Bz and Br
Rotating Magnetic Field

Conclusions

MATLAB is a very powerful simulation package that we are trying to include in our energy conversion curriculum. As it can be seen in the output of the simulation the numerical part is quite clear, yet, the graphical part emphasizes the result and "brings home" the concept of having a constant vector magnitude of1.5 B for the resultant magnetic flux rotating in space around the stator.

The effect that this simulation has had in our students is great, due to the fact that they get very motivated to continue studying our electric machines and looking forward to perform simulations that would match their homework or concepts explained in class.

For further research we will look into the rotating phasors for synchronous machines operating at different power factors.

Bibliography

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BRUNO OSORNO, is a professor of electrical and computer engineering at California State University Northridge. He is the lead faculty member in the Power Systems and Power Electronics program. Professor Osorno has written over 20 technical papers. His current interest in research is Fuzzy Logic applications in power electronics and electric motor drives.