Comparison of the Genetic Algorithm to Simulated Annealing Algorithm in Solving Transportation Location-allocation Problems With Euclidean Distances

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Abstract

This paper describes a two-phase stochastic procedure based on the Genetic Algorithm, to minimize the total transportation cost in transmitting power from sources to destinations. The Genetic Algorithm portion minimizes the total cost by modifying the source locations, and the Linear Programming technique optimizes the power distribution from the proposed source locations to each destination. The proposed algorithm was compared to a similar two-tiered heuristic procedure based on the Simulated Annealing Algorithm. A suite of 19 small test problems (using 2 to 4 sources and 4 to 8 destinations), and two large test problems (8 X 16 and 12 X 16) were tested. The problems were constructed in such a way that the exact solution were known. In all cases, the algorithm based on Simulated Annealing was superior to the other techniques.

Introduction

A transportation location-allocation problem is a problem in which both optimal source locations and the optimal amounts of shipments from sources to destinations are to be found. In recent years, several researchers have attempted to solve these types of multi-modal objective problems. Some approaches solving these problems are outlined below.

Cooper [2] formulated the transportation-location problem which was a generalization of both the Hitchcock "Transportation Problem" and the "Location-Allocation" problem with unlimited constraints. He proposed an exact algorithm, which is considered to be exact and relatively simple in concept, but its use is limited to relatively small problems.

A heuristic algorithm called the "Alternating Transportation-Location Heuristic" was also developed by Cooper [1]. This algorithm involved an iterative search technique to find the optimum. The steps are iterated until the amount of improvement in the objective value is reduced to within some tolerance. Even this algorithm had its limitations, one being that it would sometimes end in a local optimum.

The need for short computation time, coupled with increased complexity of many optimization problems, has prompted a search for more efficient methods. One approach is to use heuristic algorithms, such as Simulated Annealing (SA) and the Genetic Algorithm (GA), which produce more nearly global optima.

Liu et al.[4] have applied Simulated Annealing to solve large-scale Location-Allocation problems with rectilinear distances. The results showed high solution quality and computation time. Gonzalez–Monroy et al.[3] have compared the use of simulated annealing with the Genetic Algorithm for optimization of energy supply systems. The results inferred that for small problems, the Genetic Algorithm was more efficient than Simulated Annealing, but for large problems, the reverse was true.

The present work is also a comparison of Genetic Algorithms to that of Simulated Annealing in solving Transportation-Location problems. The two features of comparison are the quality of solution and the computation time. The present work builds upon the work of Chowdhury et al. [5], by using the Genetic Algorithm in place of Simulated Annealing and comparing the results for their efficiency.

Problem Statement

Although the general transportation-location problem refers simply to "sources" and "destinations," for clarity, the algorithm will solve a particular example of a transportation-location problem, namely, identifying the optimal location of new powerplants to supply the new (or future) energy demands of a number of cities. The objective of this problem is to minimize the total power distribution cost. The power distribution cost is the sum of the products of the power distribution cost (per unit amount, per unit distance), the distance between the plant and the city, and the amount of power supplied from the plant to the city, for all plants and all cities. For each city, the total amount of energy supplied by all plants is made equal to the total demand of that city. And for each plant, the total amount of energy supplied by the plant is to be less than or equal to the total capacity of the plant.

The mathematical form of the problem can be written as,

$$Min.Cost\,(C) = \sum_{i=1}^{n}\sum_{j=1}^{m}\phi.\delta_{ij}.v_{ij} \qquad \text{Eqn. 1}$$

subject to;

$$\sum_{i=1}^{P} v_{ij} = d_j \qquad \text{for} \quad j = 1, m$$

$$\sum_{j=1}^{C} v_{ij} \le c_i \qquad \text{for} \quad i = 1, n$$

Where

ϕ =	*transportation cost per unit amount per unit distance*
δ_{ij} =	*distance from source i to destination j*
v_{ij} =	*amount supplied from source i to destination j*
n =	*number of plants*
m =	*number of cities*
x_i , y_i =	*X & Y coordinates of the source i*
x_j , y_j =	*X & Y coordinates of the destination j*
d_j =	*demand of the destination*
c_i =	*source capacity*

It is noticed that the Euclidean distance term, δ_{ij}, can be calculated using Eqn. 2 below.

$$\delta_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \qquad \text{Eqn. 2}$$

Method

A Two-Phase method is implemented to the solve location – allocation problem. Phase 1 involves the Genetic Algorithm technique, which is used to minimize the transportation cost by varying the source locations. Phase 2 includes a Linear Programming technique to allocate the power from the sources to the destinations in accordance with the constraints.

Phase 1

1. The locations and demands for each city; the lower and upper limits for the plant locations; the plant capacities; the population; and the number of generations are specified. The upper and lower limits are used to create the initial random population of the source locations.

2. The objective function (Eqn. 1) is evaluated for the random population of plant locations by calling the phase 2 subroutine, which optimally allocates power from the plants to the demand points, and insures that the constraints are satisfied.

3. The X and Y locations of all of the plants of the initial population are converted to base 10 integers and converted to their binary forms. From the objective function values, the probabilities and the cumulative probabilities for each individual in the population are calculated.

4. Parent selection is made on the basis of fitness function. Individuals having higher fitness values are chosen more often. The greater the fitness value of an individual the more likely that the individual will be selected for recombination. The selection of mating parents is done by roulette wheel selection, in which a probability to each individual , i ,

   $P_i = f_i/f_1+f_2+f_3$…….. where P_i = Probability of individual , f_i = fitness values

   is computed. A parent is then randomly selected, based on this probability.

5. The parents thus selected are made to mate using a single-point crossover method. The offspring thus obtained form a new population of plant locations. The binary version of the new population are converted to base-10 integers and then to real values.

6. Steps 2 - 5 are repeated until the desired number of iterations have been performed.

7. In order to maintain diversity in the population two operators, viz., mutation and elitism are included. Mutation is the random change of a gene from 0 to 1 (or 1 to 0). Elitism is the procedure by which the weakest individual of the current population is replaced by the fittest individual of the immediately-preceding population. The mutation, and elitism operators offer the oppurtuinity for new genetic material to be introduced into the population.

8. The final cost and final (X and Y) location of the plants are reported.

Phase 2

In Phase 2 the random locations of the plants are received from Phase 1 and are solved as a linear transportation problem using the simplex algorithm. The Simplex Algorithm optimizes the cost for allocation of power from the plants to the cities, to a minimum. The optimal cost value, which is the objective function value in the Genetic Algorithm, is passed back to Phase 1.

A sample of 20 problems is solved using the above Genetic Algorithm and the results are displayed and analyzed.

Results

The method described above was applied to the sample problems given in Cooper (1972), and the efficiencies of both the Genetic Algorithm and Simulated Annealing Algorithm were compared. These results are shown in the tables below. The algorithms are compared for two features:

a. Quality of the solution (The efficiency is compared for the same number of cycles.)
b. Computation Time (The number of cycles is compared to obtain the same efficiency.)

A set of eighteen small problems and two large problems were tested. All twenty problems were designed in such a way that the optimal values were known in advance. Since the method described in this paper involves random perturbations, each of the small sample problems was solved ten times each, and the average result is reported below.

Quality of the solution

Running the algorithms for same number of cycles compares the efficiency.

Prob No.	Src X Destn	Exact Solution	SA Solution	GA Solution	Computation Cycles	%Difference SA Vs Exact	%Difference GAVs Exact
1	2 x 7	50.450	50.450	50.465	15000	0.0000%	1.1377%
2	2 x 7	72.000	72.010	72.033	9000	0.0144%	1.6152%
3	2 x 7	38.323	38.323	38.334	12500	0.0000%	2.6420%
4	2 x 7	48.850	48.850	48.850	8000	0.0000%	0.7389%
5	2 x 7	38.033	38.037	38.398	8000	0.0116%	2.4084%
6	2 x 7	44.565	44.565	44.565	6500	0.0000%	1.4809%
7	2 x 7	59.716	59.717	59.921	15000	0.0008%	2.6442%
8	2 x 7	62.204	62.209	62.380	9000	0.0079%	0.9742%

Table 1: Results Obtained for the Above Set of Eight Problems; Reflects the Quality of Solution.

Prob No .	Src X Destn	Exact Solution	SA Solution	GA Solution	Computation Cycles	%Difference SA Vs Exact	%Difference GA Vs Exact
1	2 x 4	54.14246	54.14315	54.16013	12500	0.00129%	0.03265%
2	2 x 5	65.78167	65.78545	66.83248	15000	0.00575%	1.59742%
3	2 x 6	68.28538	68.28678	68.78933	12500	0.00205%	0.73800%
4	2 x 7	44.14334	44.14334	44.17555	25000	0.00000%	0.07296%
5	2 x 8	93.65978	93.66392	95.48586	20000	0.00442%	1.94969%
6	3 x 3	40.00267	40.00331	40.28115	15000	0.00159%	0.69615%
7	3 x 4	40.00020	40.00092	40.50941	10000	0.00180%	1.27301%
8	3 x 5	60.00000	60.00672	60.74852	10000	0.01120%	1.24753%
9	3 x 6	54.14263	54.14266	54.47150	15000	0.00006%	0.60741%
10	4 x 4	10.00000	10.00083	11.06346	15000	0.00797%	10.6346%

Table 2: Results Obtained for the Above Set of Ten Problems; Reflects the Quality of Solution.
Large problems

Prob No .	Src X Destn	Exact Solution	SA Solution	GA Solution	Computation Cycles	%Difference SA Vs Exact	%Difference GA Vs Exact
1	8 X 16	216.549	224.10744	502.9196	25000	3.48%	132.2%
2	12 X 16	160.000	160.24570	444.1291	25000	0.15 %	177.5%

Table 3. Results Obtained for the Above Set Two Large Problems; Reflects the Quality of Solution.

Computation time

The number of cycles is compared to obtain the same efficiency. For small problems Simulated Annealing took less than 15,000 cycles. Genetic Algorithm took 300,000 cycles to reach nearly the same.

Problem No.	Source X Destination	Exact Solution	SA Solution	GA Solution	%Difference SA Vs Exact	%Difference GAVs Exact
1	2 x 7	50.450	50.450	50.465	0.0000%	0.0297%
2	2 x 7	72.000	72.010	72.033	0.0144%	0.0458%
3	2 x 7	38.323	38.323	38.334	0.0000%	0.0287%
4	2 x 7	48.850	48.850	48.850	0.0000%	0.0000%
5	2 x 7	38.033	38.037	38.398	0.0116%	0.9597%
6	2 x 7	44.565	44.565	44.565	0.0000%	0.0000%
7	2 x 7	59.716	59.717	59.921	0.0008%	0.3432%
8	2 x 7	62.204	62.209	62.380	0.0079%	0.2836%

Table 4. Results Obtained for the Eight Small Problems; Reflects the Computation Time & the Level of Accuracy.

Problem No.	Source X Destination	Exact Solution	SA Solution	GA Solution	% Difference SA Vs Exact	% Difference GA Vs Exact
1	2 x 4	54.14246	54.14315	54.14248	0.00129%	0.00005%
2	2 x 5	65.78167	65.78545	65.80696	0.00575%	0.03844%
3	2 x 6	68.28538	68.28678	68.29348	0.00205%	0.01186%
4	2 x 7	44.14334	44.14334	44.14421	0.00000%	0.00197%
5	2 x 8	93.65978	93.66392	93.66516	0.00442%	0.00574%
6	3 x 3	40.00267	40.00331	40.00626	0.00159%	0.00897%
7	3 x 4	40.00020	40.00092	40.00634	0.00180%	0.01534%
8	3 x 5	60.00000	60.00672	60.00212	0.01120%	0.00353%
9	3 x 6	54.14263	54.14266	54.14834	0.00006%	0.01055%
10	4 x 4	10.00000	10.00083	10.01878	0.00797%	0.18780%

Table 5. Results Obtained for the Ten Different Small Problems; Reflects the Computation Time & the Level of Accuracy.

Large Problems

Prob No .	Src X Destn	Exact Solution	SA Solution	GA Solution	Computation SA,GA	%Difference SA Vs Exact	%Difference GA Vs Exact
1	8 X 16	216.549	224.107	243.975	25K,300K	3.48%	12.5%
2	12 X 16	160.000	160.246	217.592	25K,300K	0.15 %	35.99 %

Table 6. Results Obtained for the Two Large Problems; Reflects the Computation Time & the Level of Accuracy.

Discussion of Results

In the case of quality of solution, for small problems, as it can be seen from Tables 1 and 2, Simulated Annealing converged very nearly to the exact solutions (within 0.01 %); whereas Genetic Algorithm only converged to within 10% of the exact solutions. For the large problems, results in Table 3 shows that Simulated Annealing converged to within 3.5% of the exact solution for the first problem, and to 0.15% for the second problem; whereas Genetic Algorithm incurred an error of 132.2% for the first large problem and 177.5% for the second problem. With regard to computation time, as it can be seen from Tables 4, 5 and 6 to reach the same level of accuracy Simulated Annealing reached the solution in 25,000 cycles, whereas the Genetic Algorithm took 300,000 cycles.

Conclusions

The results illustrate that, the two tiered hybrid Simulated Annealing and Linear Programming method is better than the two tiered hybrid Genetic Algorithm and Linear Programming method, for solving Transportation-Location problems.

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