Introducing chaotic circuits in an undergraduate electronic course

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Abstract

For decades, the engineering undergraduate education in the area of circuit and design has been mainly focused in linear models. The reason is that linear system theory has been thoroughly developed and mathematical tools are available to analyze such systems. This philosophy has led to disregard many observed phenomena such as unusual complicated waveforms, commonly known as strange attractors. Therefore, it is important for today's students to be exposed to this complex chaos phenomenon.

In this paper several chaotic circuits are presented. Electronics Workbench [1] is used to simulate circuits and show the presence of chaos. Complete circuits implementation and oscilloscope plots are all given.

Introduction

For decades, the engineering undergraduate education in the area of circuit and design has been mainly focused in linear models. The reason is that linear system theory has been thoroughly developed and mathematical tools are available to analyze such systems. This philosophy has led many scientists and experimentalists to disregard many observed phenomena because linear system theory cannot explain them. In the last decade, there is a strong interest in exploring systems that display unusual complicated waveforms, commonly known as strange attractors. These attractors have been increasingly observed in several nonlinear deterministic systems [1-9].

Therefore, it is important for today's students to be exposed to these complex chaos phenomena. From the educational aspect, students need to learn not only how to control and avoid chaos but also how to design chaotic circuits and develop applications, which explore these phenomena. Realizing the educational value to introduce undergraduate students to the phenomena of chaos, Lonngren [1] describes an interesting electronics experiment to illustrate the existence of chaos. The described laboratory experiment with the accompanying theory is a good start for the student to grasp and understand chaos. As a continuation of this goal and to enhance the student understanding of chaos, Hamill [2] presented a collection of ten chaotic circuits simulated using PSpice. These circuits, some being quite simple, illustrate how chaos can be generated.

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In this paper, two chaotic circuits are presented for the student and the practicing engineer to study and experiment with. It is worthwhile to mention that one of the presented circuits, such as the Colpitts oscillator, is taught to students in electronics courses. Yet, there is no mention that chaotic behavior may occur. We have chosen to stay away from circuits that require a degree of mathematical sophistication beyond the undergraduate level. We have selected to use Electronics Workbench (EW) to simulate circuits since it provides an interface as close as to the real implementation environment. In addition, complete circuits implementation and oscilloscope graphical plots are all presented.

It is our hope that this paper will raise many questions in the minds of students and practicing engineers.

Example of autonomous chaotic circuit

Colpitts oscillator: The first example is the Colpitts oscillator [10,11] shown in Fig. 1(a).

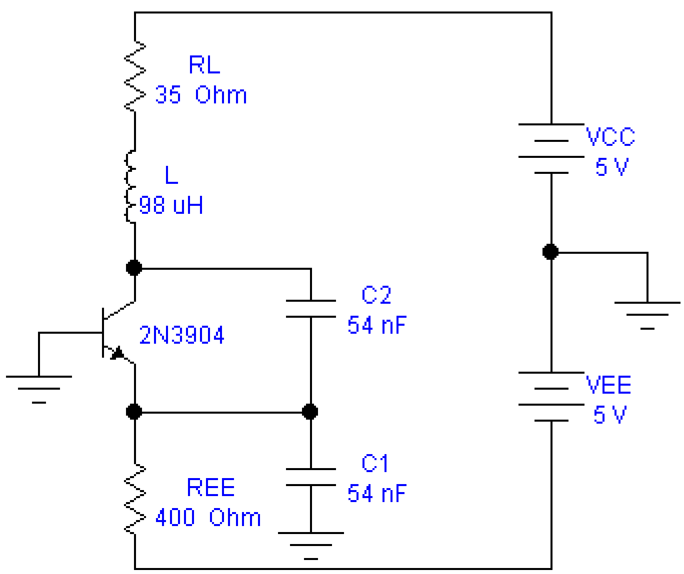

Fig. 1(a). Colpitts oscillator.

It is described by a system of three state equations:

$$
\begin{aligned}
C_1 \frac{dV_{CE}}{dt} &= i_L - i_C \\
C_2 \frac{dV_{BE}}{dt} &= -\frac{V_{EE} + V_{BE}}{} - i_L - i_B \\
L \frac{di_L}{dt} &= V_{CC} - V_{CE} + V_{BE} - i_L R_L
\end{aligned}
\tag{3}
$$

The transistor is modeled as follows

$$i_B = \begin{cases} 0 & if \quad V_{BE} \le V_{TH} \\ \dfrac{V_{BE} - V_{TH}}{R_{ON}} & if \quad V_{BE} > V_{TH} \end{cases}$$

$$i_C = \beta_F i_B$$

Where, V_{TH} is the threshold voltage (≈0.75V), R_{ON} is the small signal on-resistance of the base-emitter junction, and β_F is the forward current gain of the device. In most undergraduate electronics books, this circuit is shown to oscillate but yet it can be driven to chaos. Fig. 1(b) shows the chaotic phase portrait. By changing the value of resistance R_L we obtain different trajectories.

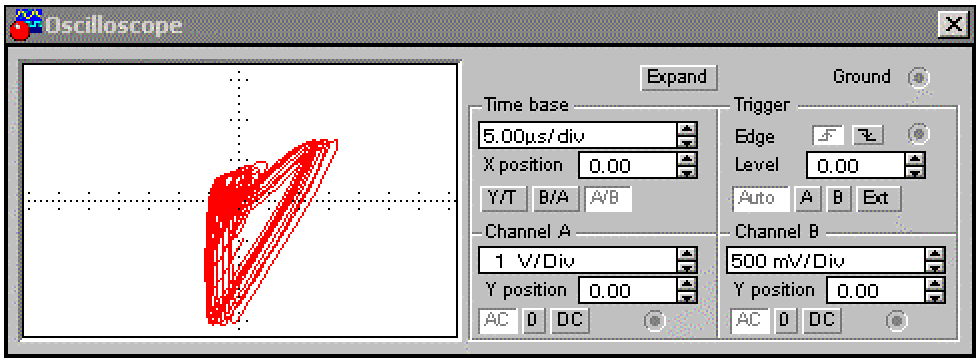

Fig. 1(b). Phase portrait of V_C versus V_E

The Colpitts oscillator was implemented and its phase portrait scroll is shown in Fig. 1(c). The voltage $V_C(t)$ and its spectrum are shown in Fig. 1(d)

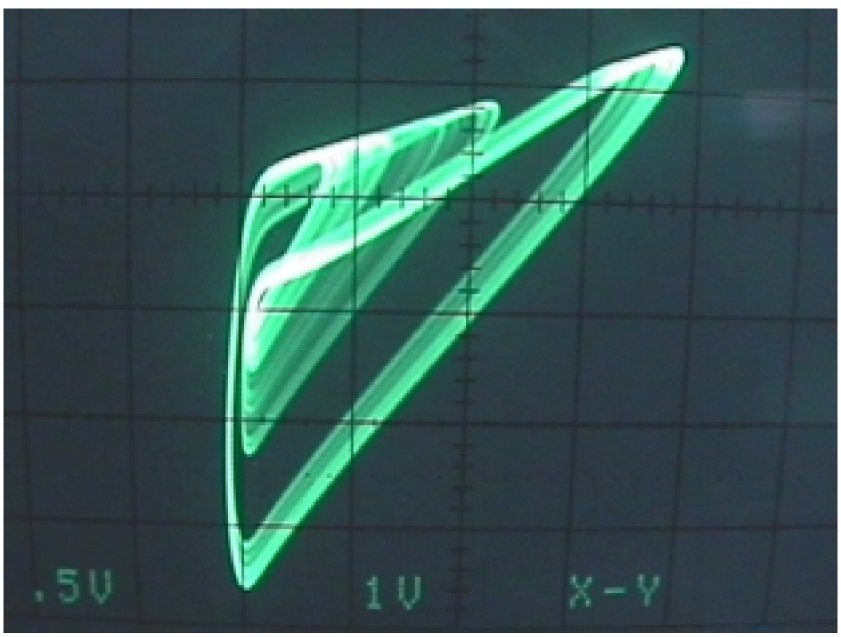

Fig.1(c). Phase portrait of V_C versus V_E for R_{EE}=466Ω

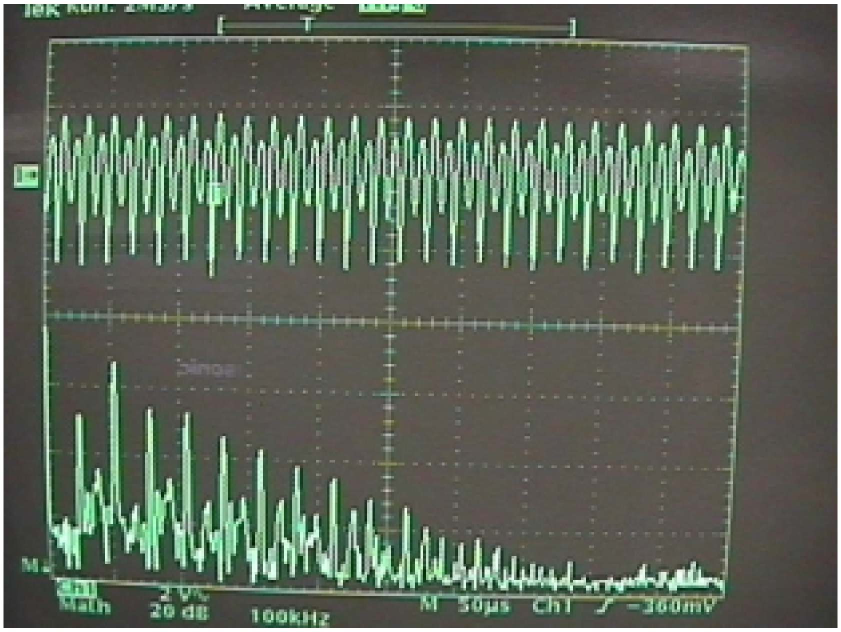

Fig.1(d). Top: $V_C(t)$. Bottom: its corresponding spectrum

Example of nonautonomous chaotic circuit

RL-Diode circuit: The second example is a nonautonomous chaotic circuit referred to as the driven RL-diode circuit [12,13] shown in Fig 2(a).

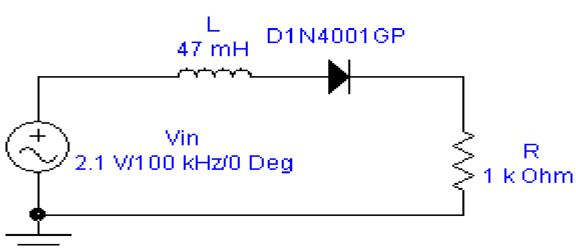

Fig. 2(a). RL-Diode chaotic circuit

It consists of a series connection of an ac-voltage source, a linear resistor, a linear inductor and a diode that is the only nonlinear circuit element. The state equations describing this circuit is:

$$V_{in} = Ri + L\frac{di}{dt} + V_D$$

$$i = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right)$$

Where, V_D is the voltage across the diode, I_S is the diode saturation current, n is a constant which has a value between 1 and 2 depending on the material and the physical structure of the diode, and V_T is the thermal voltage. The i-v characteristic of a diode is shown in Fig. 2(b).

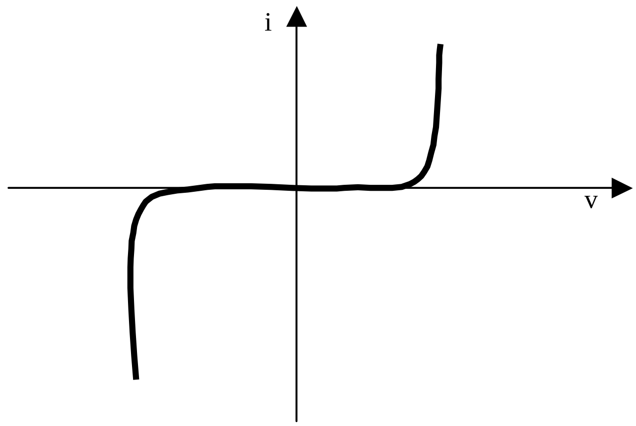

Fig. 2(b). I-V diode characteristic.

An important feature of this circuit is that the current i (or the voltage across the resistor R) can be chaotic although the input voltage V_{in} is nonchaotic. The results of the Electronic Workbench simulation, are shown in Fig. 2(c) for R=1kΩ.

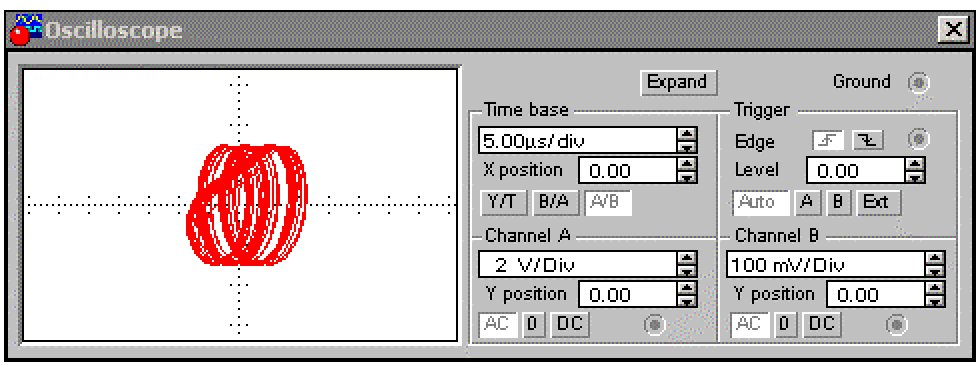

Fig. 2(c). Phase portrait of Vin versus V_R

The RL-diode was implemented and its phase portrait is shown in Fig. 2(d). The voltage across the resistance R and its spectrum are shown in Fig. 2(e). The reader is encouraged to experiment with different values of R to obtain different phase portraits.

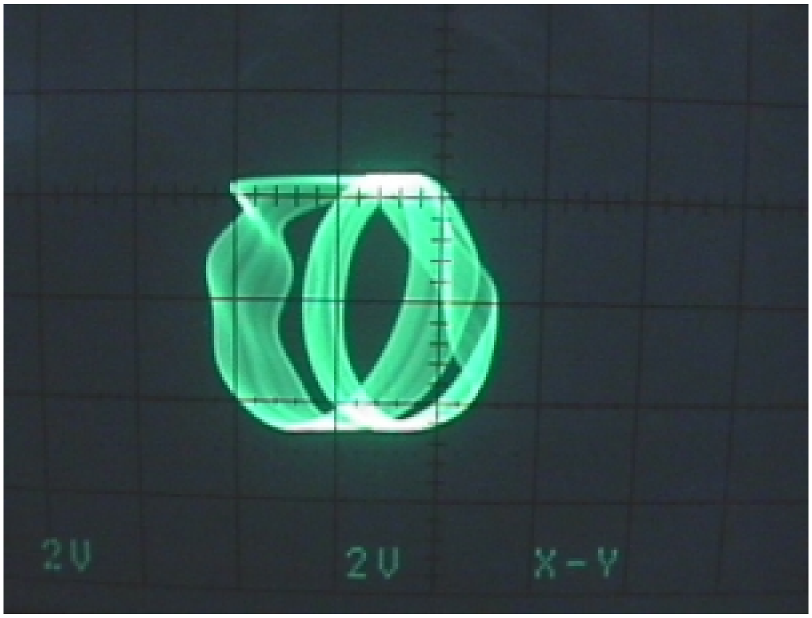

Fig.2(d) Plot of Vin versus VR for the input frequency = 130KHz Vin peak-peak=6.5V and R=26KΩ.

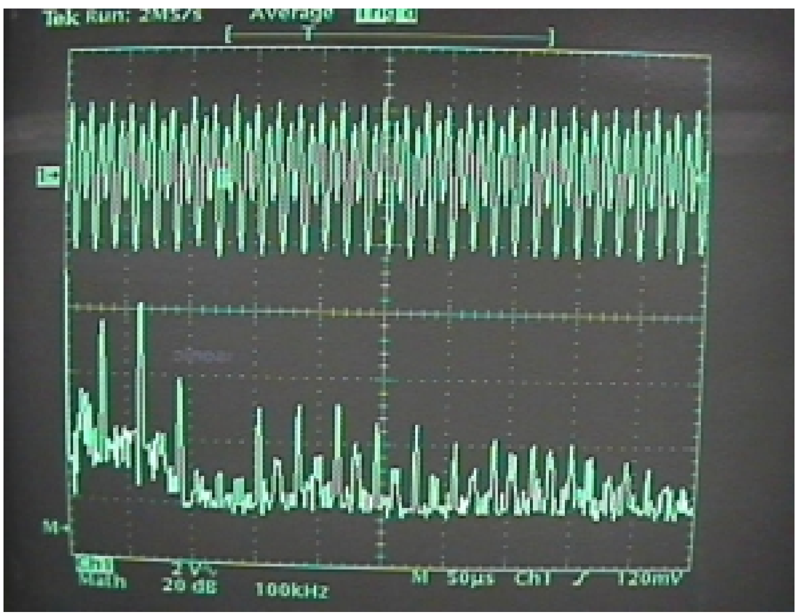

Fig.2(e). Top:$V_R(t)$. Bottom: Its corresponding spectrum

Conclusion

Introducing the phenomena of chaos to students and practicing engineers is very important not only to investigate its existence even in simple nonlinear circuits but also to explore it and build some sophisticated applications. The circuits presented here are selected because of their simplicity from the mathematical point of view. Electronics Workbench simulation provides a virtual electronic lab environment to experiment with chaos. This will enhance the learning process by being able to make all circuit changes before purchasing any component to implement actual circuits. Complete circuits implementations are presented to show the existence of chaos behavior.

It is our hope that this paper will entice the reader to experiment with these nonlinear phenomena.

References

1. K. E. Lonngren, "Notes to accompany a student laboratory experiment on chaos," *IEEE Transactions on Education*, Vol. 34, No.1, February 1991.

2. D. C. Hamill, "Learning about chaotic circuits with Spices," *IEEE Transactions on Education*, Vol. 36, No.1, pp. 123-128, February 1993.

3. R. Madan, Chua's Circuit: A Paradigm for Chaos. Singapore: World Scientific, 1993.

4. M.P. Kennedy, "Three Steps to chaos-Part I: Evolution," *IEEE Transaction on Circuits and Systems*, Vol.40, no.10, pp.640-656, October 1993.

5. M.P. Kennedy, ``Three Steps to chaos-Part II: A Chua's Circuit Primer," *IEEE Transaction on Circuits and Systems*, Vol.40, no.10, pp.657-674, October 1993.

6. L.O. Chua and G.N. Lin, "Canonical realization of Chua's circuit family," *IEEE Transaction on Circuits and Systems*, Vol.37, no.7, pp.885-902., 1990.

7. L.O. Chua, et Al, "A Universal Circuit for Studying and Generating Chaos -Part I : Routes to Chaos," *IEEE Transaction on Circuit and Systems*, Vol.40, no10, October 1993, pp.732-744.

8. L.O. Chua, et Al, "A Universal Circuit for Studying and Generating Chaos -Part II : Strange Attractors," *IEEE Transaction on Circuit and Systems*, Vol.40, no10, October 1993, pp.745-762.

9. C.Aissi and F. Fadul, "Chaos in conditionally reset linear systems," *Journal of Circuits, Systems and Computers*, Vol. 7, No. 3, pp. 177-189, 1997.

10. M.P. Kennedy, "Chaos in the Colpitts Oscillator," *IEEE Transaction on Circuits and Systems*, Vol.41, pp.771-774, 1994.

11. M.P. Kennedy, "On the relationship Between the Chaotic Colpitts Oscillator and Chua's Oscillator," *IEEE Transaction on Circuits and Systems I*, Vol.42, pp.376-379., June 1995.

12. T. Matsumato, L. Chua and S. Tanaka, "Simplest Chaotic Nonautonomous Circuit," *Phys. Rev A*, Vol. 30, pp. 1155-1157, August 1984.

13. A. Azzouz and M. Hasler, "Orbits of the R-L-Diode Circuit," *IEEE Transaction on Circuits and Systems*, Vol.37, pp.1330-1339, November 1990.

14. K. Murali, M. Lakshmanan and L.O. Chua, ``The Simplest Dissipative Nonautonomous Chaotic Circuit," *IEEE Transaction on Circuits and Systems I*, Vol.41, no.6, June 1994, pp. 462-463.

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Dr. Aissi currently serves as an Associate Professor of Electronics in the Industrial Technology Department at the University of Louisiana at Lafayette. His research interests are in the areas of design of microprocessors-based systems, intelligent microcontrollers, analysis of nonlinear dynamics of complex systems and design of chaotic circuits.