Josephson Junction Under Microwave Radiation

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Abstract

In this research we have studied the behavior of a single Josephson junction under microwave radiation. A Josephson junction consists of two slices of superconducting material, separated by a small oxide layer. The two superconductors used for Josephson junctions are usually made of niobium (Nb) and the thin insulator layer is made of aluminum oxide (AlO). In this research we have solved the dynamical system equations for a single Josephson junction under microwave radiation. We have used Runge-Kutta fourth order to solve these differential equations. In our system, we used microwave radiation amplitude (Γ) as a control parameter to vary the dynamical state of the Josephson junction for a constant microwave radiation frequency (ω_d). The system demonstrates periodic and chaotic behavior for certain values of radiation amplitude. To determine the system's behavior we have examined the phase portrait in two dimensions and three dimensions along with power spectrum analysis. For a value of 1.0 for microwave radiation amplitude we see one single frequency and no noise background, instead when a value of 3.8 is used it shows a noisy background for chaotic behavior. Also we have plotted a bifurcation diagram which shows the possible long-term values a variable of a system can obtain in function of a parameter of the system, which in our case is the amplitude of the microwave radiation. For bifurcation diagram we gradually increased the amplitude of the microwave radiation from 0.9 to 1.5 with a step size of 0.005. The bifurcation diagram further confirms our previously obtained results for the periodic and chaotic regions. We were able to observe how the systems behavior bifurcates from one cycle to two cycle and four cycle behavior to chaotic behavior.

Introduction

Josephson junction devices have been used in many applications, such as ultrahigh sensitive detectors, superconducting quantum interference devices (SQUID), and voltage standards. This remarkable two-terminal device, exhibits extremely rich dynamics and displays a wide variety of exotic nonlinear phenomena. Since the oscillation frequencies of chaos in Josephson junction are potentially of hundreds of gigahertz, Josephson-junction devices could be useful for ultrahigh-speed chaotic generators for applications of code-generation in spread-spectrum communications and random key generation in secure communications.[1] Josephson Junctions are made of two superconductors separated by a thin insulator layer. A superconductor is an element, inter-metallic alloy, or compound that will conduct electricity without resistance below a certain temperature.[2]The two superconductors used for Josephson junctions are usually made of niobium (Nb) and the thin insulator layer is made of aluminum oxide (AlO). The Josephson effect is a phenomenon named for a Cambridge graduate Brian Josephson who predicted that electrons would "tunnel" through a narrow non-superconducting region, even in the absence of an external voltage.[2] Below is a theoretical model of the Josephson junction.(Fig.1)

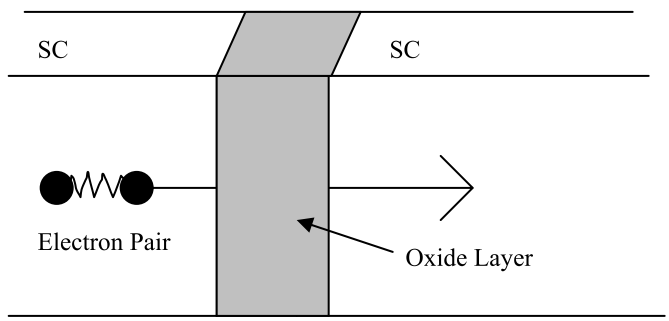

Fig.1 Josephson junction theoretical model

Materials and Methods

The research started by evaluating the necessary tools and techniques to develop a simulation of the Josephson junction's behavior under microwave radiation. We observed that for this research mathematical background was of the essence, so we proceeded to investigate differential equations and some methods to solve these equations. In order to develop a model and due to the amount of calculations behind these differential equations our main tool would be a powerful mathematical simulator. We decided to use software designed by Mathworks named MATLAB version 7.0. This software has amazing capabilities performing complicated tedious calculations and has and exceptional ability to perform high detail high resolution graphs for simulation. The technique of main focus used to solve our particular type of differential equations of interest was the Runge-Kutta method of fourth order. We became familiar with this technique by analyzing all the necessary equations involved with it. The Runge-Kutta method became of importance because the Josephson junction's behavior is described by a differential equation that is conveniently solved using this method.
The Josephson junction has an equivalent circuit of the following form: (Fig. 2)

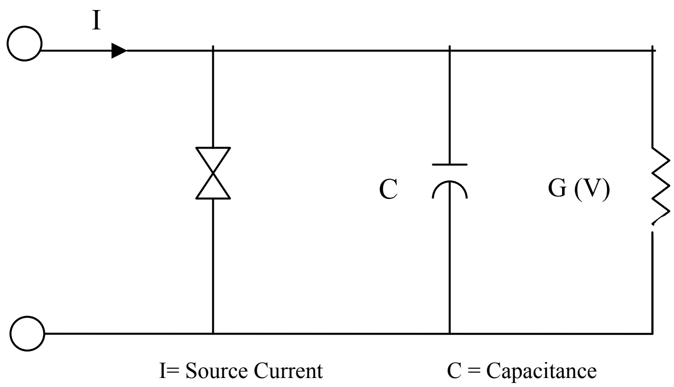

Fig.2 Josephson junction circuit equivalent

This circuit consists of a Josephson junction, a capacitor c that stands for the capacitance in the system and a resistor G that stand for the resistance. An applied signal would be the microwave radiation signal.

The equation that describes the behavior of the Josephson junction is the following:
(Eq. 1)

$$\frac{d^2x}{dt^2} + \frac{\alpha}{\tau}\frac{dx}{dt} + \Omega^2 \sin x = \Gamma \cos \omega_p t \quad \text{(Eq. 1)}$$

$$\frac{dx}{dt} = y \quad \text{(Eq. 2)}$$

We solved this equation using the Runge-Kutta method of fourth order and using the generated MATLAB code. The parameters fed into the system were 'x' is the phase difference between quantum states, 'α/τ' is the resistance in the system, 'Ω^2'is the critical current, 'Γ' is the amplitude of the microwave frequency, 'y' is the voltage, and 'ω_p' is the microwave frequency.

After investigating and familiarizing with the mathematical methods to be used we investigated the software to be utilized. Our first step was to become acquainted with the programming language used in MATLAB. We studied its mathematic and graphic capabilities and functions and became familiar with them in order to instruct MATLAB on how to create and structure the desired model. After acquiring and developing all the necessary tools and knowledge in order to fluently continue the research, we then structured and generated a MATLAB code that solved the differential equation that describes the behavior of the Josephson junction using the Runge-Kutta method. This code calculated the outputs for the Josephson junction. The calculations were done by feeding the system some initial parameters. The model then ran in a loop where for the first cycle it calculated the outputs using the given initial parameters, then proceeded to calculate the outputs in the following cycle using the results from the previous one and doing this for ten thousand times. We then determined we needed some background information on Josephson junctions in order to know more or less what to expect as graphic result of the junction's periodic and chaotic behavior. The system's behavior would be observed using specific graphs. The graphic methods used here to observe the systems behavior are its phase portrait in 2D and 3D, time series, bifurcation diagram, and the system's power spectrum.

Results

For our purposes in this research, when having MATLAB calculate the 10,000 iterations we will ignore the first 4,000 results. The reason for ignoring the first 4,000 iterations is because the system is not stable in this region and is not of particular interest. This amount of calculations to be ignored will be called transients, and will be represented by the variable M. The variable N will represent the amount of total iterations to be done in this case 10,000. Throughout this research N = 10,000, M = 4,000, ω_p = sqrt(Ω^2)*wdww0, wdww0=1.12, x = 1, y = -1, $\alpha / \tau = 0.5$, $\Omega^2 = 1$, and the step size for the Runge-Kutta method will be h=0.1.

The variable of main focus for this case of study will be **Γ**, which will vary in value in order to be able to observe how the microwave radiation affects the Josephson junction. We will observe how the microwave radiation makes model enter its chaotic and periodic regions.

We will start by demonstrating the periodic cycle of the Josephson junction. Observe figure (Fig. 3) which is the phase portrait for the Josephson junction output in a regular periodic cycle.

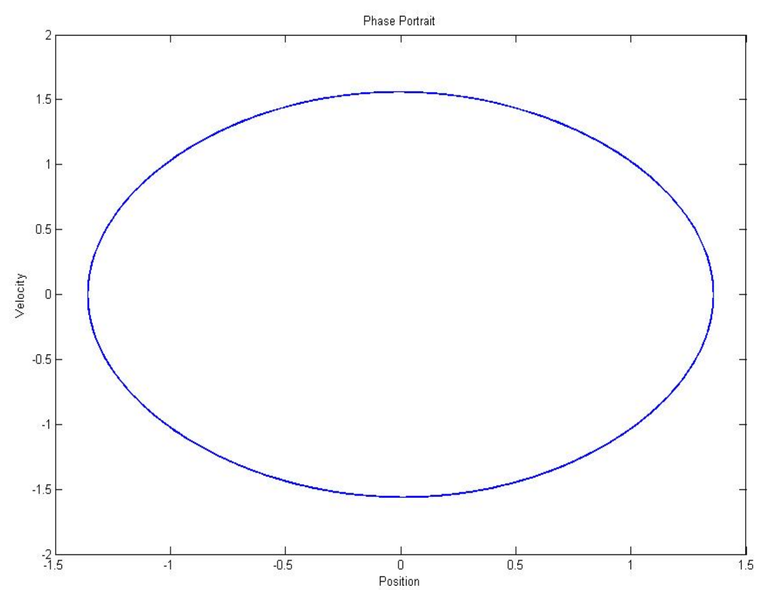

Fig.3 Josephson Junction Phase portrait periodic one cycle behavior (Γ=1.0).

From figure (Fig. 3) we can observe that under the given conditions the systems output is stable and periodic. We can determine that the system is periodic because the output, graphically is a closed curve. From figure (Fig.4) which is the same phase portrait but in a 3D image we can observe that the result previously showed is consistent.

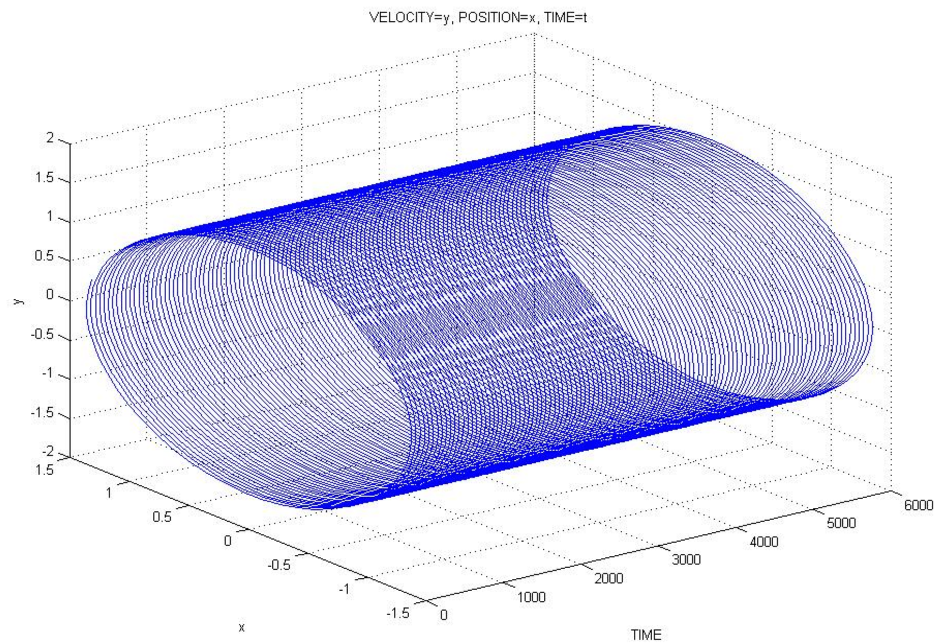

Fig.4 Josephson junction Phase portrait 3D, periodic one cycle behavior (Γ=1.0)

From this graph we can observe that the one cycle output just displays a smooth cylinder which is because it is still the same closed curve with the variant that it is plotted versus time. In the following graph you will be able to observe that since the system is in a periodic one cycle behavior, it outputs one single frequency which you can observe in the figure below. (Fig.5). For this time series the amount of iteration has been changed from 10,000 to 6,000 in order provide a better view of the signal. The amount of transients still remains the same.

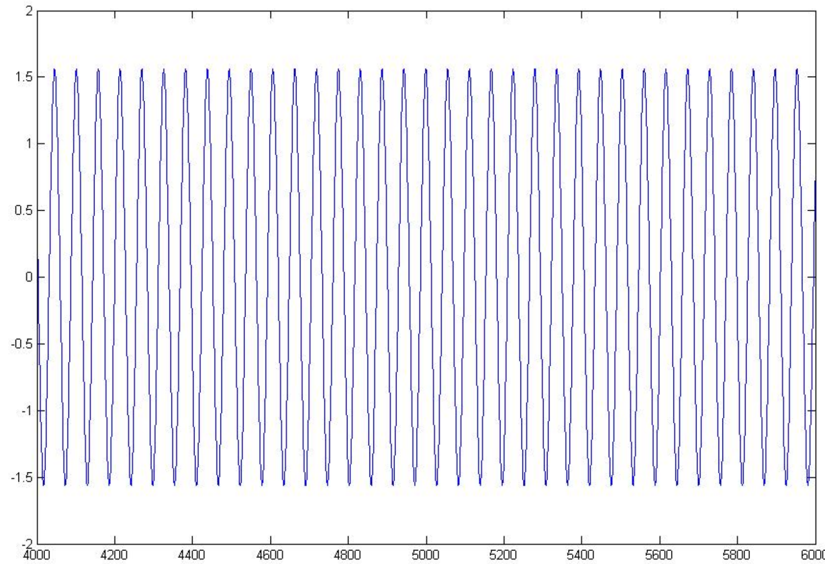

Fig. 5 Josephson junction Time Series, periodic one cycle behavior (Γ=1.0)

One other method that we are using in which you can observe the systems output is the power spectrum. The power spectrum shows only the frequency content of the output. The following graph shows the system's power spectrum. (Fig.6)

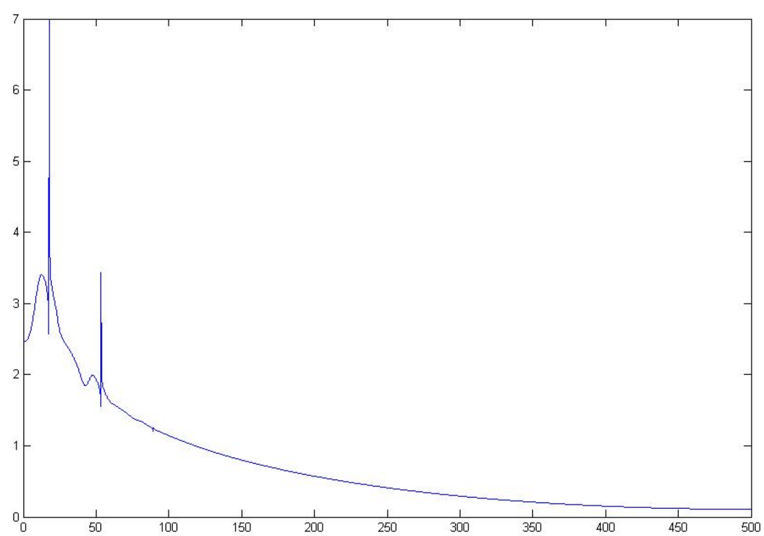

Fig.6 Josephson junction Power Spectrum, periodic one cycle behavior. (Γ=1.0)

We now proceed to observe the same Josephson junction system, but now in its chaotic region. The Josephson junction enter in its chaotic region when the intensity of the microwaves is increased in this case to Γ=3.8. The system's chaotic region is distinguished by observing the output; the output is totally unpredictable and irregular. The systems output is now inconsistent and unpredictable for long terms of time.

We will now observe the phase portrait for the Josephson junction in its chaotic region, observe figure below: (Fig.7)

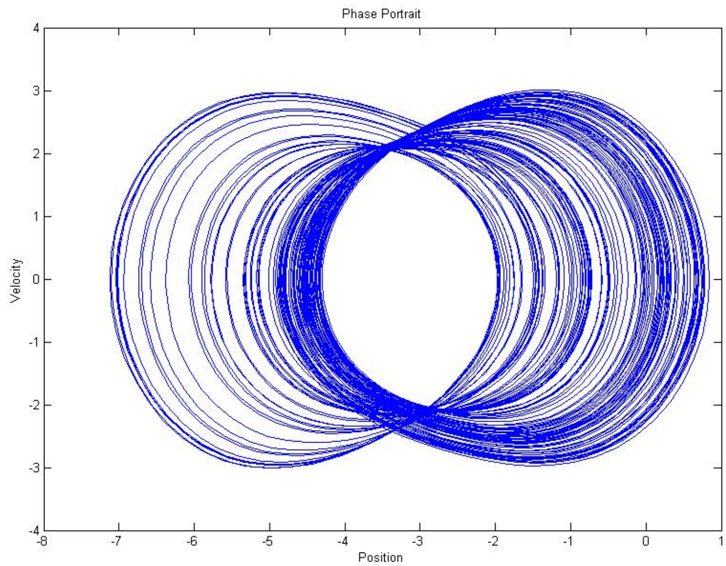

Fig.7 Josephson Junction Phase portrait chaotic behavior (Γ=3.8).

We can determine from the previous graph that the system is in its chaotic region because as we can see the system's output is different for every iteration. This chaotic behavior is better observed when it is plotted versus time. When the phase portrait above is graphed versus time it becomes a 3D graph where we can observe how the system's output behave through time. The following figure is the 3D phase portrait. (Fig.8)

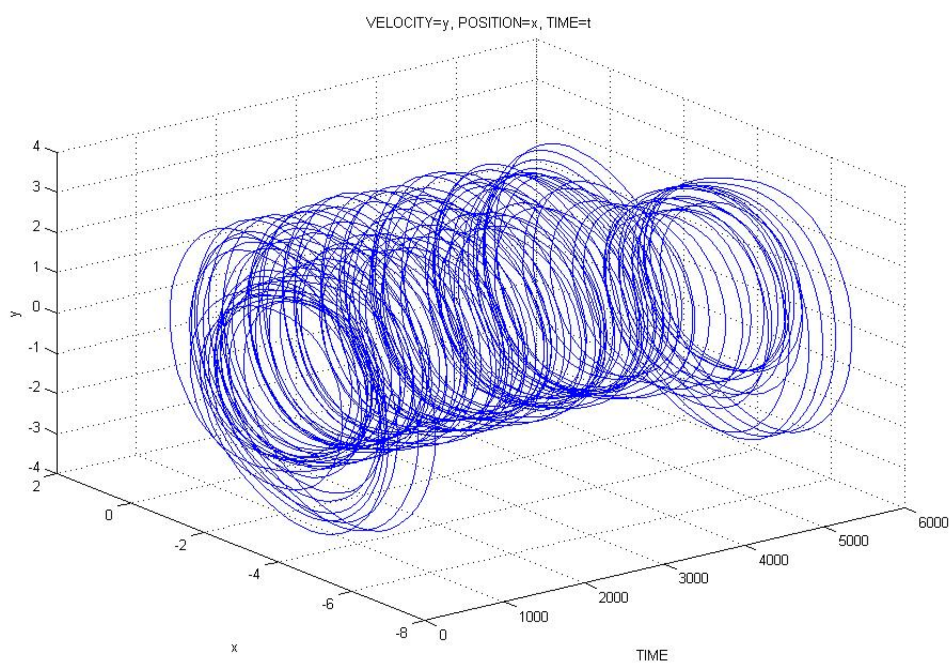

Fig.8 Josephson junction Phase portrait 3D, chaotic behavior (Γ=3.8)

As mentioned earlier from this graph we can see that the system's output is completely different for every calculation. Chaotic behavior may be observed from the irregularity of the output values. Although the system is in its chaotic region when observed closely the system has windows of periodic behavior. Another method we use to observe the systems chaotic behavior is its time series to observe the output's frequency. The time series is provided below. (Fig.9). For this chaotic behavior time series the amount of iteration has been reduced from 10,000 to 6,000 to provide a better view of the signal. The value of transients still remains the same.

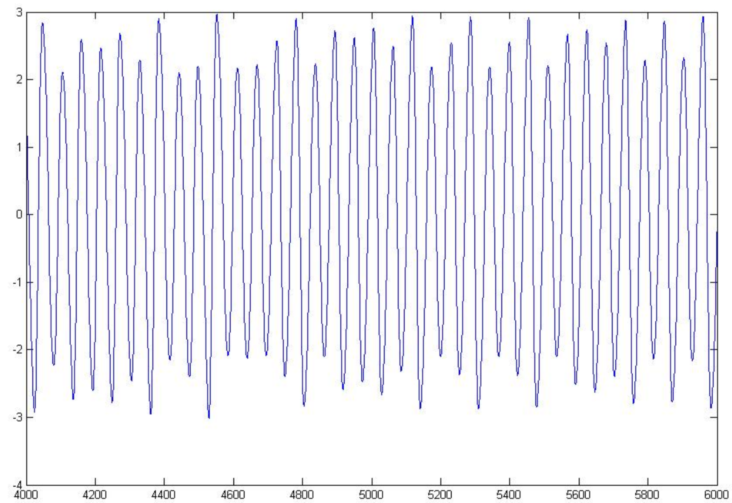

Fig.9 Josephson junction Time Series, chaotic behavior (Γ=3.8)

As you can see from the time series the signal is in its chaotic region and has the appearance of what is known as noise. We can also observe this noise appearance in the power spectrum graph which is shown below. (Fig.10) We can determine from the power spectrum that the system's frequency content has the appearance of noise.

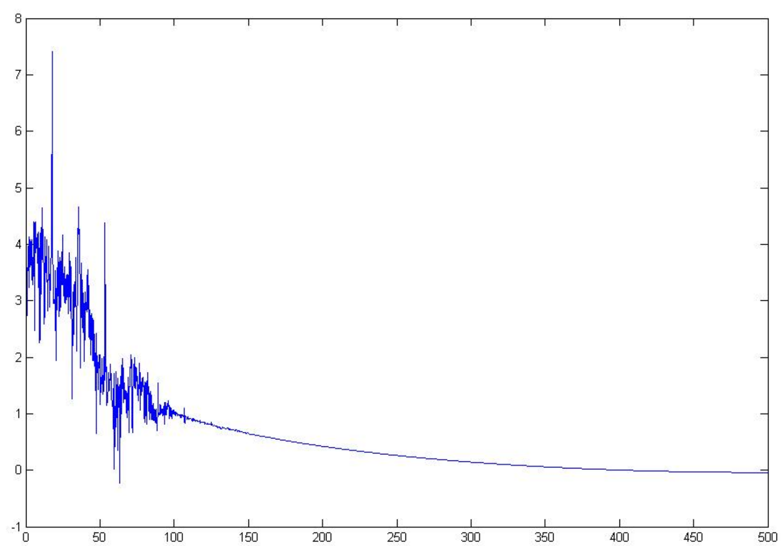

Fig.10 Josephson junction Power Spectrum, chaotic behavior (Γ=3.8)

Our next step is to observe the systems bifurcation diagram for which MATLAB was indicated to calculate 50,000 iterations ignoring the firs 10,00 for a variation of $0.9<\mathbf{\Gamma}<1.5$ with increments for $\mathbf{\Gamma}$ of 0.005 This diagram makes it possible to observe the system's behavior over a range of variation of the intensity in the microwaves. In the bifurcation diagram we can see at what values the system enters its chaotic behavior and its periodic behavior. The bifurcation diagram from the Josephson junction under microwave radiation is provided below. (Fig.11)

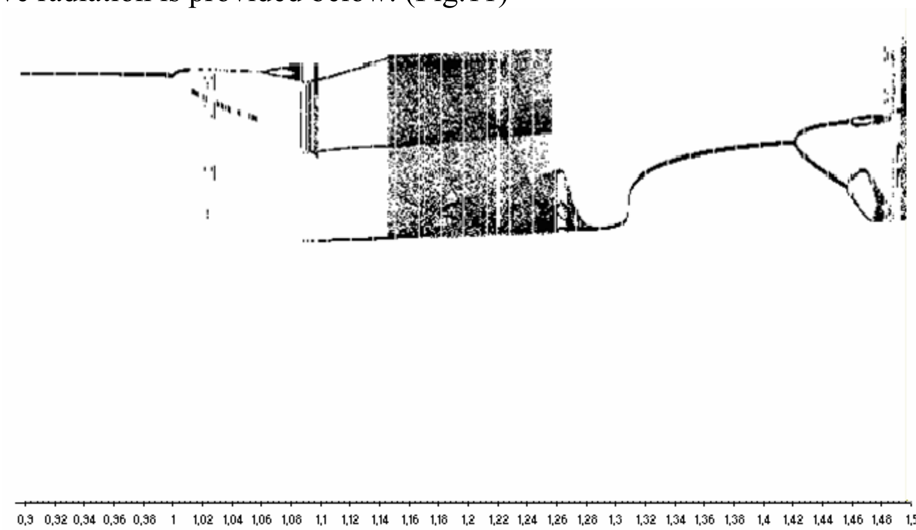

Fig.11 Josephson junction bifurcation diagram ($0.9<\Gamma<1.5$)

From this diagram we can observe that when $\Gamma= 1.0$ the system is in its periodic behavior. When $\Gamma= 1.2$ the system is in its chaotic behavior. When $\Gamma= 1.48$ the system is periodic four cycle.

Discussion and Conclusions

Upon completion of this research, we observed a single Josephson junction's behavior under microwave radiation. We have shown the Josephson junction's behavior in the regular periodic and chaotic region. The junction's behavior was demonstrated using a 2D and 3D phase portrait, time series, bifurcation diagram, and power spectrum. We showed how a short range of variation of microwave radiation intensity in the system behaved periodically and chaotically. We demonstrated how no matter what initial parameter is fed into the system the output in periodic region is always the same. In chaotic region initial conditions matter because it affects the output. The Josephson junction becomes unstable and unpredictable under certain condition under microwave radiation. The output signal of the system becomes noise like in the chaotic region. The noise like signal in some cases may be desired depending on the application of the device. We were able to observe how any device built with Josephson junctions would behave under microwave radiation. This is helpful because it helps determine the acceptable functional range. One application in which the noise like output may be desired is in the case of decoders or high speed noise generators. Another application of the Josephson junction is SQUIDs (Superconducting Quantum interfering Devices) which are used as sensitive magnetic field detectors.

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References

1. Atsushi Uchida, Hidetoshi Iida, Natsuko Maki, Masayuki Osawa, and Shigeru Yoshimori, "Chaotic Oscillations in Josephson Tetrode." IEEE Transactions on Applied Superconductivity, Vol. 14, No. 4, December 2004, p2064-2070
2. SUPERCONDUCTORS>ORG, http://superconductors.org/INdex.htm
3. B. A. Huberman, J. P. Crutchfield, N. H. Packard, "Noise phenomena in Josephson junctions." Appl. Phys. Lett. Vol. 37, No. 8, 15 October 1980, p750-752
4. Lynch, Stephen, "Dynamical Systems with Applications using MATLAB." 2004
5. University of Aberdeen, Department of Physics http://www.abdn.ac.uk/physics/case/squids.html
6. Francis C. Moon, Chaotic and Fractal Dynamics, John Wiley and Sons, 1992.

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Herman is a senior undergraduate student at Southern University A&M college. He is majoring in Electronics Engineering and Technology. This research was preformed for the "Strengthening Minority Access to Research and Training" (SMART) program.