Managing Variability in Monte Carlo

James E. Schueler, Zuhdi Al-Jobeh, and Gerald R Seeley
Valparaiso University

Introduction

In the past, for the most part, calculations were done using discrete values for variable quantities. Factors of safety were imposed to manage inherent uncertainties. However, with the advent of modern computer hardware and software, it is now possible to move away from this archaic mode of thinking. Using modern computational tools, our graduates will have the ability to estimate the probability of their designs satisfying applicable criteria. Thus, in order to prepare our students for practice in the 21st Century, the Valparaiso University Department of Civil Engineering is infusing its curriculum with computer-assisted Monte Carlo simulations. This paper presents the rationale and several examples using two different software packages. The presentation will give additional examples of homework which has been done this semester.

Managing Variability Through "Factors of Safety"

It is probable that the earliest civil engineers, practicing from intuition and experience, were painfully aware of the inherent variability of the physical quantities and properties with which they dealt. The desire to make their predictions more reliable led them to apply the developing principles of science to their art. Practice became ever more algorithmic and computational, always tempered by experience and judgment.

Until a few decades ago, civil engineers made their calculations "by hand" using slide rules or mechanical/electrical calculators. They were, therefore, generally limited to performing calculations once, and they were forced to select a single value for each variable involved in the algorithm. Engineers, generally aware of the uncertainties inherent in the numbers used, included a "factor of safety" to achieve what they hoped would be a "safe design." This computational history led to a mind set in which engineers consider physical quantities to be representable by a single number (i.e., E = 29,000 KSI). This nearly ubiquitous mind set has generally prevented engineers from viewing and evaluating their projects as systems of interrelated random variables. Additionally, the "factor of safety" approach to managing the variability inherent in all physical quantities and properties precludes quantitative estimates of the chances of "failure."

The “Monte Carlo Simulation” Alternative

Spreadsheet and statistical software are now available which readily permit each variable in the mathematical system model (algorithm) to be described as a distribution of values. For each variable in the algorithm, the type of distribution (normal, log-normal, uniform, etc.) and parameters of the distribution (usually the central tendency and dispersion) must be established based on experience or available data. The software will automatically execute the algorithm thousands of times, randomly selecting a different value from each variable distribution for each computation. Because the algorithm is solved thousands of times, there are thousands of results. This large, but finite, set of varying results can be used to estimate the frequency distribution of all possible outcomes (the population). One method of achieving repeated computational cycling and random selection of variable values is the “Monte Carlo Simulation.”

Monte Carlo Simulations Provide Two Crucial Advantages

Monte Carlo Simulations require the engineer to estimate the shape, central tendency, and dispersion appropriate for each variable. This process requires the engineer to visualize every physical quantity or property as a random variable. The result, and one of the principal advantages of Monte Carlo Simulation, is a far more realistic modeling of physical phenomena. Developing a perception of the physical world which includes dispersion, or degree of variability, will also help engineers avoid unfounded conclusions. The simple comparison of means of two groups of data to determine which is “better” is a common error. Expressing predictions to an unwarranted number of significant figures would occur much less frequently if outcomes were visualized as distributions.

The other principal advantage of Monte Carlo Simulation is the ability it provides to estimate the chances of exceeding or falling below certain critical values. Available software typically presents simulation results in a histogram showing the frequency at which outcome values fall within certain intervals. The software usually allows one to specify an outcome of particular interest (i.e., Factor of Safety = 1.00), and calculates the percentage of the outcome values falling below that value. It is far more instructive to estimate that “the chances of failure are one in one thousand” than to conclude that “failure is very unlikely because the customary factor of safety of 2.0 was applied.”

Examples

Two simple examples, each using different software, of Monte Carlo Simulation follow. They are intended to demonstrate the basic concepts and procedures. The authors are aware that many statistical software packages are available, that nuances of variable type and interdependency are not included, and that differences of opinion are probable regarding the shapes and values assumed for the variable distributions.

Example 1: Factor of Safety

Consider the simple beam shown in Figure 1. The conventional approach to the design of this beam would involve calculating a single value for the Factor of Safety with respect to failure by yielding. The Factor of Safety would be equal to the material yield strength divided by the calculated maximum fiber stress in the beam. For this example, the following values were selected for the parameters:

P = **18** kips Beam Section = WI 6x31 L = 120 in. F_{yld} = **36** ksi

The maximum bending moment is determined to be PL/4, and the maximum fiber stress is defined by the equation Me/l. Based upon these equations, a maximum fiber stress of 22.9 ksi (T or C) is calculated. Thus, the Factor of Safety = 36.0/22.9 = 1.57.

Suppose the client asked the designer if there is any chance of failure (that the Factor of Safety could fall below 1.0). The designer would probably answer: “Yes, there is a chance, but failure is very unlikely.” Knowledge of the ‘real world’ tells us that there is variability in all of the values that were used in the previous calculations. Because conventional Factor of Safety calculation methods do not consider such variabilities, estimates of the percentage of times the Factor of Safety would fall below 1.0 are not possible.

Now consider a solution of the same problem using a Monte Carlo Simulation in which all values influencing the Factor of Safety are treated as random variables. The software package PC: Solve® was used for this analysis. The values assumed for the deterministic solution above will be taken as the mean values for the respective random variables. The dispersion of each random variable will be taken as three standard deviations. The means and dispersions of each random variable in the Monte Carlo Simulation were:

P = 18 ± 6 kips	(actual variation would depend on load type)
L = 120 ± 3/8 in.	(based on value specified in the standard mill practice section of the AISC LRFD Manual of Steel Construction)
W16x31	(variations in width and depth were taken from the standard mill section of the AISC LRFD Manual of Steel Construction)
F_{yld} = 36 ± 9 ksi	(actual variation would depend on results of material tests or on allowable variations permitted by ASTM)

As input to the PC: Solve® analysis, one must specify both a mean and variance for each distribution created. Once these distributions (all assumed to be normal distributions in this example) are created, they can be manipulated just as one would manipulate any “variable.” Thus, repeating the above problem many times using the means and variances assumed, the distribution of Factor of Safety obtained is shown in Figure 2.

Examination of this distribution indicates that some Factor of Safety values fall below 1.0. The next logical question is: “What percentage of the time will the Factor of Safety fall below 1.0?” This can be estimated by asking the software to count the number, and calculate the percentage, of values which fall below 1.0. For this example, the Factor of Safety fell below 1.0 approximately 0.102% of the time. One would therefore estimate that approximately 1 out of every 1000 such beams will fail.

Example 2: Cost Estimate

As another illustration of Monte Carlo Simulation, the cost of excavating a simple storm sewer pipe trench was estimated using a user-friendly software package called Crystal Ball®. Traditionally, the storm sewer trench excavation volume is taken off by multiplying the average depth between catch basins and manholes by the width and length of the trench taken from engineering drawings. Different width trenches are taken off separately. Even though the final trench width depends on many variables (such as the tendency of the soil to slough and the experience of the operator), no attempt is usually made in the traditional take-off process to account for such variations. Consider for example the cost of excavating a 300 ft long x 3 ft-6 in wide x 5 ft deep trench. Traditionally, the excavation volume (LxWxD) would be calculated as 5250 ft^3 (195 CY). If the cost of excavation is assumed to be $4.50/CY, the total cost will be $877.50.

Suppose that an excavation contractor, considering submitting a bid of $1,250 for this work, wonders what his/her chances of suffering a loss would be. Only a vague guess could be made based on the conventional estimating procedures described above. A far more useful estimate of the answer to the contractor's question can be obtained through a Monte Carlo Simulation. The software Crystal Ball® will be used in this example.

In this example, the variables are the trench length, trench width, trench depth, and unit cost of excavation. The trench width and depth were assumed to be dependent and positively correlated, a condition which Crystal Ball® manages with ease. A "forecast cell" was defined in which the many values of calculated total cost were saved. To illustrate the flexibility and simplicity of using Crystal Ball®, a different type probability distribution was defined for each of the above variables.

Figure 3 shows the "forecast" of trench excavation cost under the specified assumptions. Of the 9,928 cost calculations, a click of the mouse caused Crystal Ball® to count approximately 8.5°A of these values exceeding $1,250.

Conclusions

Civil engineers (and all engineers and scientists) must conceive of, and manage, physical quantities and properties as the random variables they are. This will permit a far more realistic modeling of natural phenomena, and it will provide the priceless opportunity to estimate quantitatively the chances of failure. Undergraduate civil engineering curricula founded on, and saturated at all levels with, Monte Carlo Simulation will help to achieve these important objectives. The simple examples presented above show the "power" and simplicity of such computer-assisted applications in civil engineering education.

JAMES E. SCHUELER, Prof., Dept. of Civ. Engrg., Valparaiso Univ., Valparaiso, IN 46383.

ZUHDI AL-JOBEH, Asst. Prof., Dept. of Civ. Engrg., Valparaiso Univ., Valparaiso, IN 46383.

GERALD R. SEELEY, Prof., Dept. of Civ. Engrg., Valparaiso Univ., Valparaiso, IN 46383.

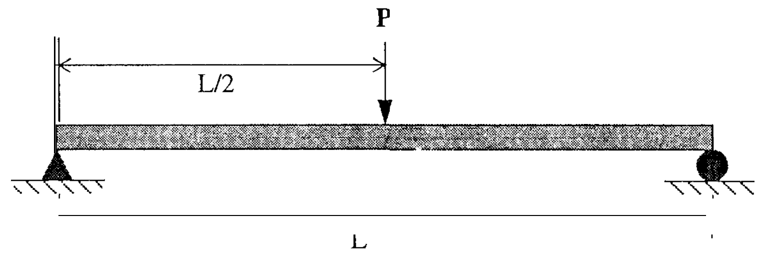

Figure 1: Simple Beam

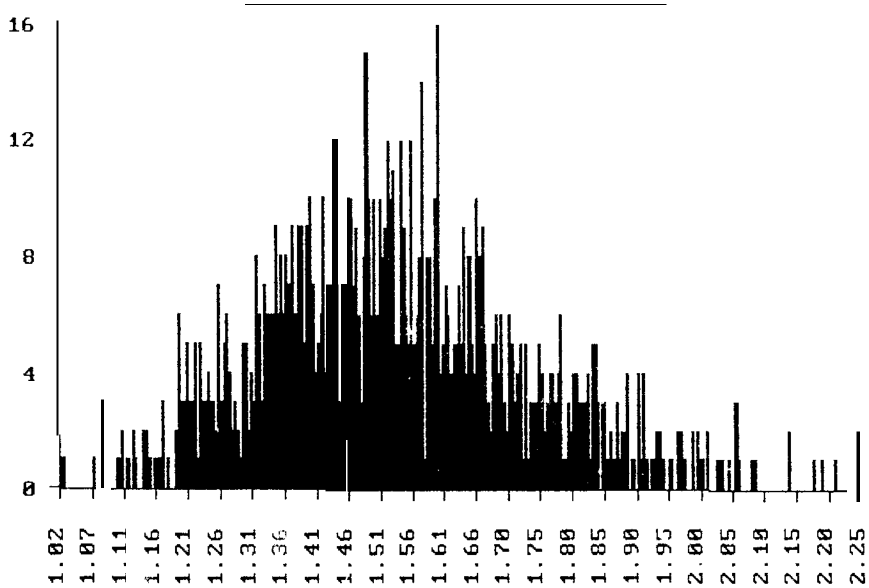

Figure 2: Safety Factor Distribution

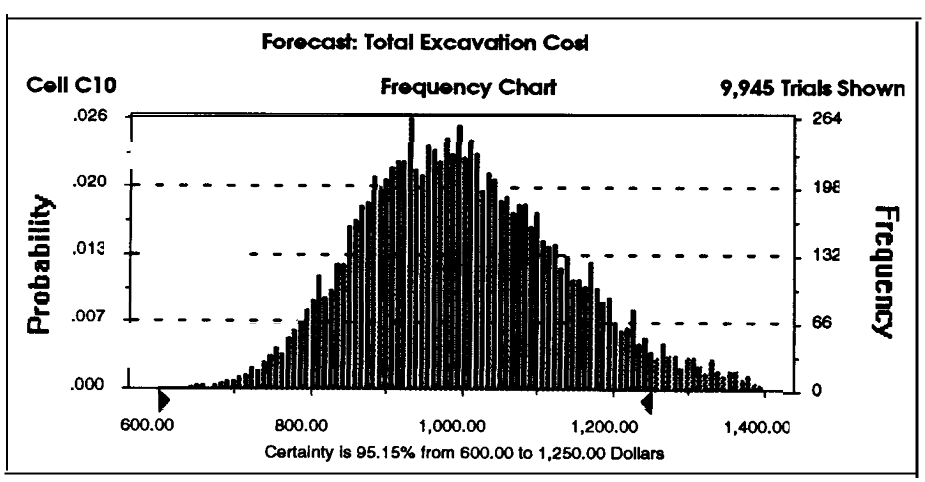

Figure 3: The Probability Distribution For the Forecasted Total Excavation Cost