The Draining of a Tank: A Lab Experiment in Fluid Mechanics

Dominique Savage, Trent Porterfield, W. Roy Penney and Edgar C. Clausen
University of Arkansas, Ralph E. Martin Department of Chemical Engineering

Abstract

An improved apparatus has been constructed and employed in the fluids laboratory for the draining of a tank through a sharp-edged orifice and, with minor modifications, other orifices of interest. The experiment was operated at steady state to accurately determine the discharge coefficient, C_D, with only a 2-5% deviation from literature values. In addition, transient data were collected and compared to model data generated from a combined Bernoulli balance and mass balance, again with minimal deviations between the experimental and model data. The experiment meets all requirements for a well-designed lab experiment.

Keywords

laboratory, fluid mechanics, experimentation, modeling, tank draining

Introduction

Laboratory is an essential part of the undergraduate engineering experience. The undergraduate laboratory is extensively used in engineering curricula to expose students to engineering equipment, allow students to work with their hands, demonstrate principles and correlations that were previously presented in the classroom, develop teamwork and leadership skills and serve as a vehicle for strengthening oral and written communication skills. Feisel and Rosa [1] present a history of the development of educational laboratories and how changes have been incorporated throughout the years. The use of laboratory as part of the engineering curriculum can trace its roots to the training of military engineers, combining theory with lots of practice. Following World War II, the ASEE Grinter report noted that the engineers being produced were too practically oriented and were not sufficiently trained to seek solutions by referring to first principles. While engineering programs became more theoretical after World War II, a balance was struck in engineering curricula to include laboratories in an effort to ensure that the graduating engineers were prepared for their industrial careers. More recently, there have been significant developments in the use of virtual teaching labs, which has led to arguments on the pros and cons of using virtual labs in place of physical labs. Korestky *et al*. [2] note that virtual labs are better for experimental design, critical thinking and dealing with ambiguity, while physical labs are better for understanding lab protocols and specific content.

Penney and Clausen [3] developed several inexpensive fluids and heat transfer exercises that could be used in the classroom as physical demonstrations or as laboratory exercises. Many of these experiments illustrated correlations from the literature using equipment that could easily be constructed at low cost in a departmental machine shop. One of these experiments was a simple sharp-edged orifice demonstration for the fluid mechanics classroom, shown in Figure 1 [4]. The apparatus consisted of a 4 in (10.2 cm) inside diameter, 24.25 in (61.6 cm) long PVC pipe (0.25 in walls), containing the sharp-edge orifice at the bottom of the pipe, and attached upright to a metal support tripod. The PVC pipe had a sight glass tube (0.25 in clear PVC) attached to its side to observe liquid level in the pipe. A 17 gal (64 L) utility tub was used to collect water flowing from the pipe, and Erlenmeyer flasks and graduated cylinders were used to hold, feed and collect water flowing in and out of the system. A stopwatch was used for timing the flow of water. The apparatus was portable for use in the classroom and could be used in finding the discharge coefficient, C_D, from either a steady state or transient experiment. Although the demonstration unit was an effective learning tool, the demonstration worked best in a steady state overflow mode, where water from a hose was continually fed to the pipe and allowed to overflow from the top and into the overflow container. This often created a bit of a mess in the classroom.

Figure 1. First Iteration of the Sharp-edged Orifice Demonstration [4]

The purpose of this paper is to describe and present data from an improved tank draining system which can be operated very effectively in the laboratory in either the steady state or transient mode. The discharge coefficient, C_D, is obtained from steady state operation and good height vs. time data are also obtained from a transient experiment that agree very well with data generated from a Bernoulli balance model. The apparatus also has the flexibility, with a simple modification, of being able to use several different orifices in addition to the sharp-edged orifice.

Experimental

Apparatus

Figure 2 shows a photograph of the tank for draining as mounted to the end of metal scaffolding in the lab and Figure 3 shows a schematic of the experimental apparatus. The “tank” was actually a clear acrylic tube that had a height of 10 ft (3.1 m) and an inside diameter of 3 in (7.6 cm). The use of a long and slender tube in the place of a typical tank enabled much more accurate height measurements than in a tank with a diameter essentially equal to its height. A simple plastic valve with a garden hose connection was mounted approximately 3 ft (0.9 m) from the bottom of the tank to permit the introduction of water and regulate the inlet flow of water. A rubber sleeve containing the orifice was connected to the bottom of the tank. The tank had a measuring tape attached to its side to monitor liquid height, but also had a Magnetrol® Eclipse® 705 guided wave radar transmitter [5] mounted inside the tank to monitor the liquid level. The transmitter, which is optional for this lab experiment, was connected to a Measurement Computing data acquisition device, USB-TC-AI, driven by a 2 amp/12 volt source and connected to the USB port of a Dell Latitude E 5510 laptop computer. Omega TracerDAQPro software was used to convert the signal to liquid height, and then to display and analyze the data. Water flowing from the tank flowed into a 5 gal (18.9 L) receptacle and then into a floor drain. A stopwatch was used for timing the flow of water and a 1 L Erlenmeyer flask was used to collect water from the exit of the tank to determine the flow rate. A simple $\frac{1}{2}$ in (1.3 cm) garden hose was used to bring water to the tank.

Figure 2. Tank (Acrylic Tube) Used for Draining (left) and
the Bottom of Tank with Orifice (right)

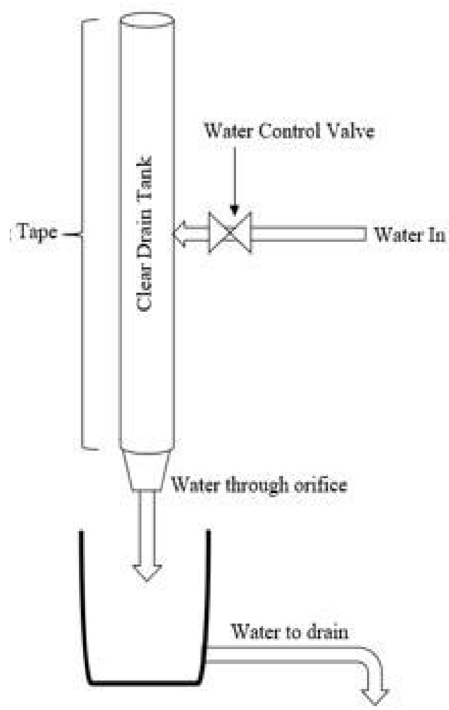

Figure 3. Schematic of the Experimental Apparatus

Although many different orifices may be used in this experiment by changing out the sleeve and orifice at the bottom of the tank, a sharp-edged orifice was used in this experiment because this orifice has a very well-defined discharge coefficient of 0.61 [6]. The orifice must be properly designed and constructed with standard dimensions to minimize the error in the discharge coefficient. In this case, the $\frac{3}{16}$ in (4.76 mm) orifice was machined with a 30° downstream relief angle and a 0.5 mm (0.020 in) land, the minimum orifice wall thickness. Other orifices that could be employed in the experiment and their estimated discharge coefficients are shown in Figure 4 [6].

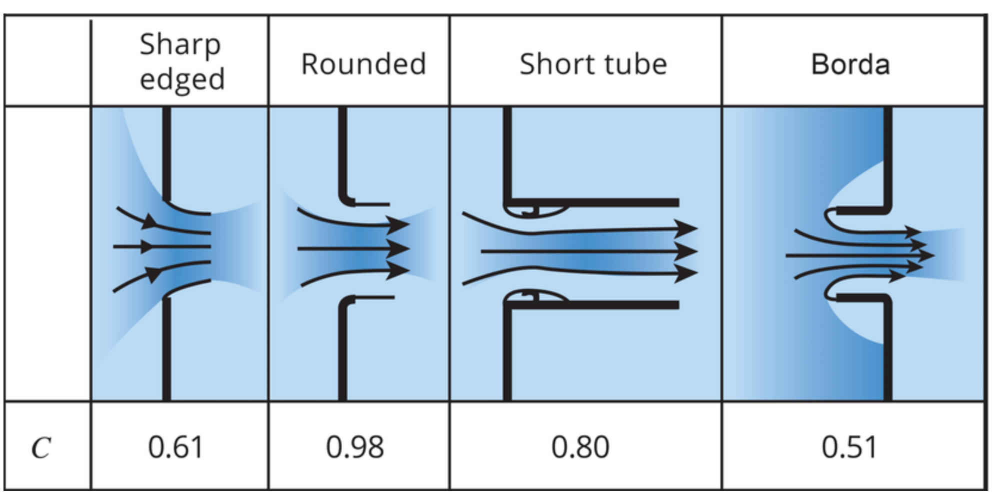

Figure 4. Orifices and Their Discharge Coefficients [6]

Experimental Procedure

The experiment consisted of three parts:

1. Calibration of the guided wave radar transmitter
2. Collecting steady state data to determine C_D
3. Monitoring tank level with time to compare experimental data with model data

Procedures for each of these tasks are briefly described below.

To calibrate the guided wave transmitter, the 5 gal (18.9 L) receptacle is placed under the cylindrical tank, as is shown in Figure 2, and the rubber tube is extended from the receptacle to the floor drain. A Tygon® plug is secured in the orifice to prevent liquid from draining from the tank during filling. The tank is filled to a height of about 9 ft (2.7 m) using the garden hose. After turning on the radar transmitter and readying the computer and software, the Tygon® plug is removed and both the output from the computer and the physical height of the water in the tank are recorded with time. A plot of transmitter height vs. physically measured height can then be used as a calibration curve.

To operate the system at steady state, the tank is filled to a convenient eye-level height of about 3 ft (0.9 m). With the Tygon® plug removed, the flow through the orifice is matched with the flow from the water supply to maintain a steady state height. Once steady state is achieved, a 1 L Erlenmeyer flask is used to measure the volume of water exiting the orifice for a measured time period to calculate the steady state flow rate. The flow rate is then used in calculating the discharge coefficient, C_D.

Finally, to monitor the tank level with time, the tank is filled to a height of about 9 ft (2.7 m). The Tygon® plug is then removed and the wave radar transmitter is used to record water height in the tank with time. This information is used in comparing experimental data with data generated from a Bernoulli and mass balance model.

Experimental Data

Tables 1-3 show the experimental data collected by one student group during the Spring 2021 semester. Table 1 shows the calibration of the guided wave radar transmitter. As the tank drained, the students physically measured the height using the tape measure mounted on the outside of the tank in $\frac{1}{2}$ ft increments for the entire height of the tank. The wave transmitter radar also measured the height in inches but could not measure height below 5 in. Table 2 shows the steady state data from the experiment where the time to collect a given volume of water from the system was measured as the height in the tank was held at 3 ft (0.9 m). Finally, Table 3 shows transient data showing the height of the water in the tank, measured as a function of time using the wave radar transmitter. The tank drained completely from a height of 108 in (2.7 m) in just over 4 min.

Table 1. Calibration of the Guided Wave Radar Transmitter

Physically Measured Height, in	Wave Transmitter Height, in
108	107.9

102	102.7
96	96.5
90	91.05
84	84.8
78	78.7
72	72.05
66	66.08
60	60.5
54	54.39
48	47.8
42	41.5
36	36.05
30	30.08
24	23.7
18	18.12
12	12.27
6	6.17
0	5

Table 2. Steady State Experimental Data*

Run	Collection Time, s	Volume Collected, ml
1	8.84	425
2	8.47	400
3	9.93	500

*tank held at a height of 3 ft (0.9 m) in collecting the data

Table 3. Transient Experimental Data

Time, s	Tank Height*, in
0	107.683
10	102.394
20	96.1793
30	90.229
40	84.147
50	77.8
60	72.379
70	66.958
80	61.933
90	56.909
100	52.149
110	47.257
120	42.629
130	38.662
140	34.299
150	30.596
160	26.63

170	23.192
180	20.151
190	16.977
200	14.201
210	11.292
220	8.912
230	6.4
240	5.077
250	5.077

*as measured by a guided wave radar transmitter

Model Development

Model development for this experiment was previously shown by Penney *et al.* [3] and is repeated here to aid the reader. The basic Bernoulli balance, with no work in the system and negligible friction losses, is described by Wilkes *et al.* [7] as

$$\frac{v_1^2}{2g} + z_1 + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + z_2 + \frac{p_2}{\rho g} \tag{1}$$

For application in this experiment, point 1 was selected as the fluid level in the tank, and point 2 was selected as the location of the *vena contracta*, labeled with the subscript *vc*, which is located one-half of an orifice diameter from the orifice entrance [8]. Since both ends of the tank were open to the atmosphere, $p_1 = p_2$. The velocity at the top of the liquid in the pipe, v_1, may be neglected, and the *vena contracta* is at zero height, so that $z_{vc} = 0$. With these simplifications, Equation (1) may be rearranged to solve for the velocity at the *vena contracta*, v_{vc}:

$$v_{vc} = \sqrt{2gz_1} \tag{2}$$

The flow through the orifice may be described by the equation

$$Q = A_{vc}\, v_{vc} \tag{3}$$

However, the area of the vena contracta is difficult to measure. Thus, we introduce the discharge coefficient, $C_D = \frac{A_{vc}}{A_o}$, so that Equation (3) may be rewritten as

$$Q = C_D A_0\, v_{vc} \tag{4}$$

where A_0 is the area of the orifice, equal to $\frac{\pi d_o^2}{4}$. Thus, C_D may be calculated as

$$C_D = \frac{Q}{A_0\sqrt{2gz_1}} \tag{5}$$

for the steady state system, where the volumetric flow rate is calculated as the volume of water collected, divided by the time of collection ($Q = \frac{V}{t}$) at steady state.

In considering the time-dependent system where height changes with time, the simplified Bernoulli balance of Equation (2) must be combined with the mass balance,

$$\frac{dm}{dt} = m_I - m_{vc} \tag{6}$$

For a draining tank, $m_I = 0$, since there is no water flowing into the tank. Furthermore, $\frac{dm}{dt}$ may be written as $\rho A \frac{dh}{dt}$, and m may be written as $\rho v A$. Thus, Equation (6) becomes

$$\rho A_I \frac{dh}{dt} = -\rho v_{vc} A_{vc} \tag{7}$$

Once again, we do not know A_{vc}. Reintroducing C_D yields

$$\rho A_I \frac{dh}{dt} = -\rho v_{vc} A_o C_D \tag{8}$$

Combining Equations (2) and (8) yields

$$\frac{dh}{dt} = -\frac{A_O C_D}{A_1}\sqrt{2gh} \tag{9}$$

Separating variables and integrating Equation (9) from $h = h_0$ at $t = 0$, and $h = h$ at $t = t$ yields, with rearrangement

$$h = \left(\frac{C_D t A_O \sqrt{2g}}{-2A_1} + \sqrt{h_0}\right)^2 \tag{10}$$

Finally, taking the square root of each side yields

$$\sqrt{h} = \frac{C_D t A_O \sqrt{2g}}{-2A_1} + \sqrt{h_0} \tag{11}$$

Thus, a plot of $\sqrt{h}$ vs. t will yield a straight line, the usual method of presenting this type of data.

Reduced Results and Discussion

Figure 5 illustrates the results from the calibration of the guided wave radar transmitter, which showed nearly perfect agreement between the transmitter tank height and the physically measured height. Table 4 displays the steady state results and calculated values for C_D, the discharge coefficient, using Equation 5. The average value of C_D was 0.64, which is 5% higher than the value suggested by Vennard and Street [6] and less than 2% higher than the value suggested by Wilkes *et al*. [7].

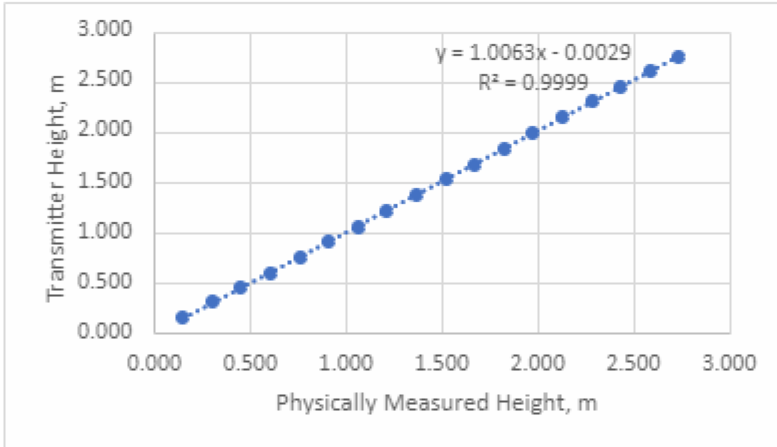

Figure 5. Calibration of the Guided Wave Radar Transmitter

Table 4. Calculated Discharge Coefficients from Steady State Data

Run	*t, s*	*V, ml*	$Q, \frac{m^3}{s}$	C_D
1	8.84	425	0.000425	0.64
2	8.47	400	0.0004	0.63
3	9.93	500	0.0005	0.67
			Average C_D:	**0.64**

Figure 6 shows a plot of the square root of the tank height with time according to Equation 11. The experimental plot used $C_D = 0.64$ and the ideal plot used $C_D = 0.61$. In comparing the plots there was essentially no deviation between the two plots at $t \leq 100$ s, a 3.6% deviation at $t = 120$ s and a maximum deviation of 19% at $t = 240$ s. Finally, Figure 7 shows a plot of height vs. time for $C_D = 0.64$ (experimental) and $C_D = 0.61$ (ideal) in solving Equation 9 using MATLAB. As expected, these plots were also nearly identical.

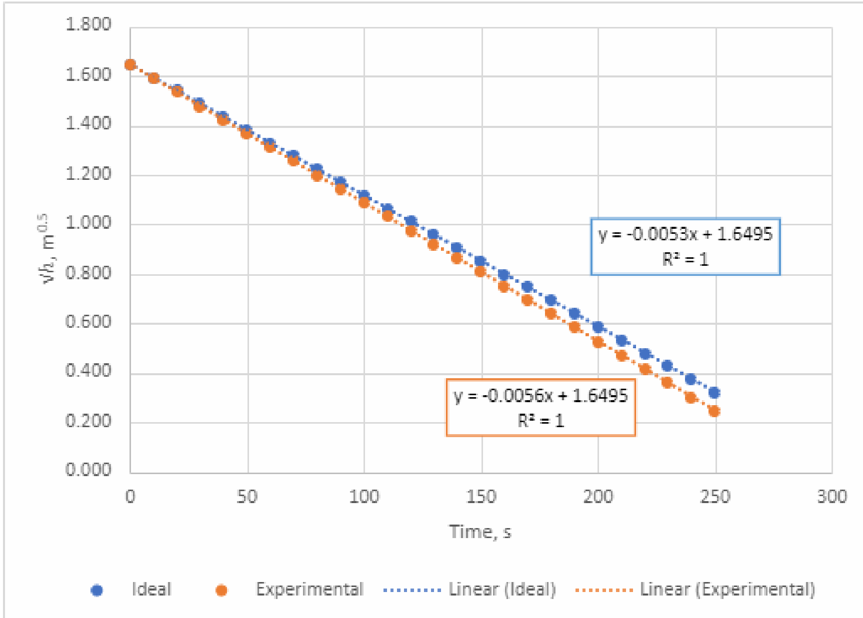

Figure 6. Comparison of the Square Root of Height vs. Time for $C_D = 0.64$ (experimental) and $C_D = 0.61$ (theoretical)

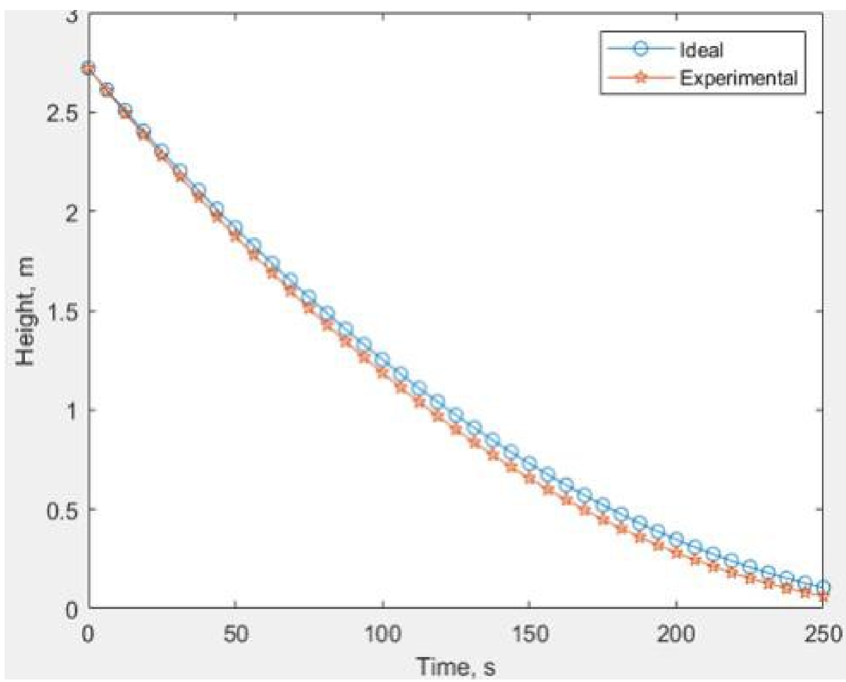

Figure 7. Comparison of Height vs. Time for $C_D = 0.64$ (experimental) and $C_D = 0.61$ (theoretical)

Conclusions

1. This experiment is an excellent teaching tool because it shows students how a sharp-edged orifice must be machined and applies the Bernoulli and mass balances to reduce experimental data and develop a tank draining model.
2. The well-designed orifice yielded discharge coefficients which were almost identical to those described in the literature, with errors of only 2-5%.
3. This experiment meets all the requirements of a well-designed laboratory experiment:
    - The apparatus is relatively inexpensive and can be constructed in an engineering machine shop. The major cost is the acrylic tube, which costs about $600 for a 10 ft section. The radar detector was available from a previous experiment (free) and is optional in constructing the experiment.
    - The apparatus can be easily modified to use other orifices
    - Fundamental principles can be applied to model the experiment
    - The experimental results agree with literature data
    - The experimental data and model predictions are easily compared using linear plots with excellent agreement. If desired, MATLAB can also be used to generate h vs. t data, and these plots show excellent agreement as well.

Nomenclature (SI units shown)

Latin Letters

A_0	Area of the orifice, m^2
A_1	Area of the tank pipe, m^2
A_{vc}	Area of the vena contracta, m^2
C_D	Discharge coefficient
$C_{D\ ideal}$	Discharge coefficient using the ideal value of 0.61
$C_{D\ experimental}$	Discharge coefficient that is experimentally found
d_0	Inside diameter of shape-edged orifice, m
d_1	Inside diameter of the cylindrical tank, m
g	Gravitational constant, m/s^2
h	Height of water in the tank, m
h_0	Initial height of water in the tank, m
m_1	Mass at fluid level in the tank, kg
m_{vc}	Mass at the *vena contracta*, kg
p_1	Pressure at point 1, fluid level in the tank, Pa
p_2	Pressure at point 2, selected as the location of the *vena contracta*, *vc*, Pa
Q	Volumetric flow rate out of the orifice
t	Time, s
v_l	Velocity of the fluid level tank, m/s
v_t	Velocity of the water in the drain tube, m/s
v_{vc}	Velocity at the vena contracta, m/s
z_1	Height of the free surface in the tank as marked by the tape measure, m
z_2	Height of the free surface in the tank where the water exits, m

Greek Letters

ρ Density of water, kg/m^3

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Dominique Savage

Ms. Savage is a chemical engineering undergraduate with minors in Mandarin and Sustainability at the University of Arkansas. She has had engineering internships with Eastman Chemical and Becton, Dickinson and Company. Ms. Savage has received the College of Engineering Presidential Scholar Award, the Tau Beti Pi Scholarship, and the Thomas O. Spicer Endowed Scholarship In Chemical Engineering for the 2021-22 academic year. She plans to work in industry upon graduation in May 2022.

Trent Porterfield

Mr. Porterfield is a chemical engineering undergraduate at the University of Arkansas and is from Houston, Texas. He has been named to the Chancellor's List in each semester from Spring 2020 to Spring 2021. He plans to work in industry upon graduation in May 2022.

W. Roy Penney

Dr. Penney is a Professor Emeritus of Chemical Engineering at the University of Arkansas. His research interests include fluid mixing and process design, and he has been instrumental in introducing hands-on concepts into the undergraduate classroom. Professor Penney is a registered professional engineer in the state of Arkansas.

Edgar C. Clausen

Dr. Clausen is a University Professor in the Ralph E. Martin Department of Chemical Engineering at the University of Arkansas. His research interests include engineering education, teaching improvement through hands-on experiences and enhancement of the K-12 educational experience. Professor Clausen is a registered professional engineer in the state of Arkansas.