Design Project for Advanced Mechanics of Materials

C.J. Lissenden, G.S. Wagle, and N.J. Salamon

Department of Engineering Science and Mechanics, Penn State University

Abstract
Advanced mechanics of materials is a broad subject encompassing many topics. However, often there is only room in the curriculum for a single course. Thus, there is a tendency to pack the course full of topics, in which case sufficient depth of coverage can be lost. Furthermore, design is at the heart of engineering and demands attention. Advanced mechanics of materials is a major part of many design problems. In this paper we describe the implementation of a team design project into an advanced mechanics of materials course. The project has been developed such that it can be initiated at the beginning of the course, but it builds on itself and students progressively apply principles learned in class. The design of a crank arm for a bicycle was chosen as the project because of the familiarity that students have with it, its simple function, it poses interesting and common design dilemmas, and because the analysis can range from being very simple to being very complicated. The project contains many parts: development of design specifications, material selection, analysis of a straight crank having a circular cross-section, design of a straight crank, validation of analysis with experimental results, design of an elliptical cross-section and a rectangular cross-section, and comparison of results from simple circular, elliptical, and rectangular cross-sections with finite element results from actual crank arms. The primary topics that this project covers are: design, combined stresses, prediction of yielding, fatigue, torsion of noncircular cross-sections, and finite element analysis. Student teams from a separate finite element class conducted the actual finite element analyses. A website is under development to assist students performing design in this and other mechanics courses. It was clear to the instructor that the project increased the students' level of interest during the course.

Background
The Engineering Science and Mechanics department at Penn State offers a technical elective entitled, "Advanced Strength of Materials and Design." This course follows the elementary strength of materials course and introduces the field equations of mechanics, covering such topics as: principal stresses and maximum shear stress, failure criteria, energy methods, torsion of noncircular sections, shear flow, unsymmetrical bending, thick-walled cylinders and disks, plates, and shells. Other topics that could be included, but are not due to a lack of time are: beams on elastic foundations, curved beams, details of finite element analysis, plastic structural analysis, and buckling. In addition to learning these topics, engineering students need to learn how to design[1, 2]. We have introduced design into a course traditionally laden with analysis by implementing a semester-long team design project.

This paper is subdivided into the following sections: objectives, project definition, basis for components of the project, evaluation, and student feedback.

Objectives

The overall objectives of the design project that we have implemented are as follows.

- Apply the principles learned during a course to a practical problem.
- Provide an opportunity to apply basic knowledge in addition to what is learned in the course to solve an engineering problem.
- Learn to design and conduct experiments, as well as analyze and interpret data.
- Provide an opportunity to function on multi-disciplinary teams, which requires communication with team members and to learn professional and ethical responsibility.

In order to select a design project for a course, the course objectives must be clearly defined. Advanced mechanics of materials students will be able to:

- develop models of mechanical components by making reasonable assumptions and writing appropriate equations,
- apply appropriate failure criteria,
- formulate a design methodology.

For this course, a project involving the design of a left crank arm for a bicycle was selected because of the familiarity that students have with it, its simple function, it poses interesting and common design dilemmas, and because the analysis can range from being very simple to being very complicated. This particular project helps achieve the course objectives. The technical objectives of the project are to:

- select a simplified model of an actual crank arm by considering a straight and prismatic cross-section,
- analyze the simplified model to determine stresses,
- design a suitable cross-section by applying appropriate failure criteria,
- develop interaction skills in students to enable them to work efficiently in team projects.

Project Definition

"Design a crank arm for a bicycle." A bicycle crank arm is a critical component of the drive mechanism as it transmits the force applied by the rider to the crankshaft. It also supports the rider weight. The failure of a crank arm can cause injury to the rider's leg by contact with the broken crank or the rider can lose control of the bike, fall and be injured. In 1997, Shimano American Corporation of Irvine, California received more than 630 reports of crank arm failures in North America resulting in 22 injuries, including cuts and fractures **leading to a recall of more than 1 million crank arms** installed on bicycles in North America[3, 4]. Replacement of these cranks was a significant cost burden to the company (potentially millions of dollars) and highlights the importance of a good design. Therefore, the design of the left crank arm of a bicycle has been assigned since it poses an interesting design problem from a number of viewpoints.

For logistical reasons, the project is subdivided into a number of tasks. The division is such that tasks involving application of fundamentals of mechanics of materials can be initiated at the beginning of the course, while the subsequent ones build on these as students progressively apply principles learned in class such as failure criteria, fatigue, torsion analysis and finite element analysis. This also facilitates assigning different tasks to students with different backgrounds.

Tasks A-D, some of which are divided into subtasks are reproduced below. These are as assigned to the students including due dates and percentages of project grade.

A1. [5%] *Team record*. Each team shall keep a written record of all work done on the design project. This record will be turned in periodically and graded. It should be organized in a notebook (a 3-ring binder will work well). As a minimum, this record shall contain:

- notes from meetings,
- lists from brainstorming sessions,
- work from individual team members,
- analysis calculations,
- reference material,
- a clear record of who did what.

Each group should consider assigning tasks to individuals on a rotating basis. Possible non-technical tasks are:

- discussion facilitator, administrator;
- recorder, secretary;
- organizer, scheduler;
- coordinator.

A2. [10%] *Design specifications* including for example: loads, size, material and manufacturing, failure modes. Each student will list ten items that should be addressed in the design and five suggested specifications. Students will bring these lists to a team meeting and the team will then develop a detailed set of specifications. Due 6 September.

A3. [20%] *Crank analysis*. Must include the following:

- clear and complete free body diagram;
- internal force calculations and diagrams;
- stress calculations for any point in the cross-section due to: axial force, bending about both axes, shear from transverse load, torsion, and effective stress;
- plot equivalent stress as a function of crank orientation angle for 8 points on the circumference of the cross-section;
- determine the critical condition, i.e., if effective stress is the critical quantity, for what crank orientation angle and where in the cross-section does the maximum effective stress occur;

Calculations can be done using a spreadsheet having many columns (e.g., force, moment, P/A stress, Mc/I stresses, VQ/It stress, Tr/J stress, effective stress). The formulae for these stress components should be in terms of the coordinates in the cross-sectional plane. The rows of the spreadsheet are the crank orientation angles. Increment the crank orientation angle by no more than 5 degrees to get a smooth plot of stress as a function of angle. This format should be easy to modify (later) for an elliptical or rectangular section. Due 20 September.

Field data has been obtained from the bicycle. A pedal dynamometer has been used to measure the normal strain on the surface of the pedal spindle due to bending about two orthogonal axes. These strain signals were calibrated to applied force components normal to the top of the pedal (F_z) and tangent to the top of the pedal (F_x). These force components were then combined to find the resultant force (F_r) acting on the pedal. Plots of the resultant force as a function of time for various 'rides' are provided. The data for each ride is available in spreadsheet format upon request.

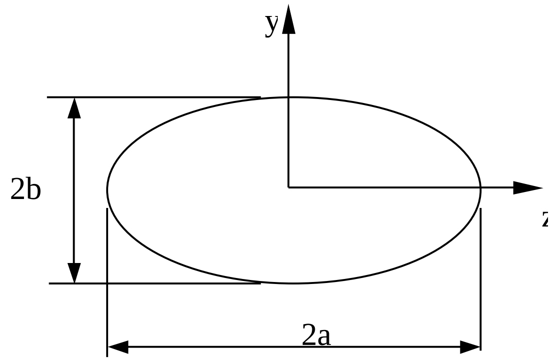
A4. [25%] *Crank design*. While you are free to use your own judgment, you are encouraged to consider designing the crank to remain elastic during occasional overloads and have a pre-defined service life when subjected to normal cyclic loading. The report must include all final specifications and design criteria, calculations and analysis, as well as dimensions. EMch 400 students should lead the design for overload and EMch 500 students should lead the design to prevent fatigue failure. Due 11 October.

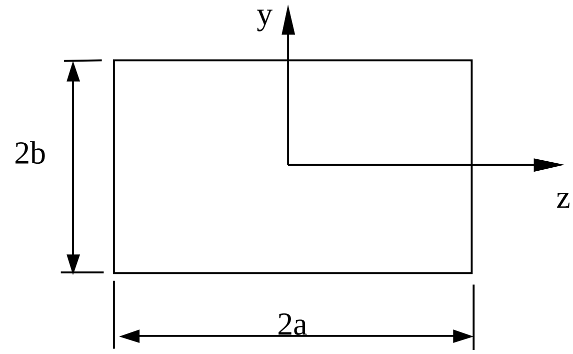
B. [10%] *Analysis Validation*. EMch 400 ONLY.

Part I: Suggest a laboratory experiment or set of experiments that will enable you to validate your crank arm analysis. Note that this is not intended to be field testing of a prototype. The solid model of the crank arm is shown in Figure 1 and a dimensioned drawing in Figure 2. (Figures not included in this paper.) Due 25 October.

Part II: Validate your team's analysis with the experimental results. At points A & B compare measured strain with predicted strain from your analysis for: L= 125 mm, D = 25 mm, e = 95 mm, F_{vert} = -445 N, E = 72 GPa, and ν = 0.33. Point A is on the outside face and Point B is on the top face of the crank when in the horizontal position. Consider crank orientation angles of 0, 45, 90, 135, and 180 degrees as measured from the vertical. Due 27 November.

C1. [10%] *Torsion analysis*. EMch 500 ONLY. Develop equations and an algorithm for shear stress at any point due to torsion in elliptical and rectangular cross-sections. Determine the rate of twist, β, and the shear stress components τ_{xy} and τ_{xz} at 8 points around the perimeter of elliptical and rectangular cross-sections having an aspect ratio of 2:1 (take *b*=0.5 inch) for a constant internal torque of 1000 lb-in. Due 1 November.





C2. [15%] *Design*. Combine torsion analysis with bending and axial force to design first an elliptical cross-section crank arm and second a rectangular cross-section crank arm. This portion is for the full team to work on. EMch 500 students should provide the analysis tools and EMch 400 students should use the tools to do the design. Due 29 November.

D. [15%] *Finite element comparison*. In your efforts to design a better crank arm you have supplied a solid model to the finite element geeks in your company (actually, the EMch/ME 461 students). They, in turn, have discretized the model and performed a 3D stress analysis. In order to check whether their results can be trusted, you have also asked them to analyze the straight circular crank arm. They have analyzed the following conditions:

Material: linear elastic aluminum E=72.4 GPa, ν=0.33
Constraints: crank fixed at connection to crank shaft
Loading: 445 N applied in the direction of gravity and 95 mm from the centerline of the crank along the pedal spindle
Crank Orientations: 90, 45, and 0 degrees from vertical
Straight Circular Crank (A): 25 mm diameter and 175 mm long
Better Crank (B): 175 mm long with complex geometry including curvature, tapered cross-section, and cutout

A report containing the results of the finite element analysis has been submitted for your use. Examine it and answer the following.

1. Crank A. Describe why the strains shown at the section z=125 mm (which is where strain gages were bonded to the prototype) are or are not qualitatively correct for each of the three crank orientations.
2. Crank A. Explain why the von Mises stress distributions are as shown at the 25%, 50%, 75%, and longitudinal sections.
3. Crank A and Crank B. Based on the von Mises stress distributions shown at the 25%, 50%, 75%, and longitudinal sections, which crank orientation results in the critical condition for each crank?
4. Crank B. Based on the von Mises stress distributions shown at the 25%, 50%, 75%, and longitudinal sections, what geometric feature is the most detrimental to the service life of the crank (i.e., causes the worst stress concentration)?

Notes: Be sure that you understand the orientation of the sections and where they are cut. Colors correspond to different magnitudes on different contour plots so be sure you look at the legends. Due 11 December. (Lissenden et al[5] discuss the projects implemented in the finite element analysis class.)

Basis for Components of Project
The basis for assigning these tasks is given below.

A1. *Team Record*. This notebook is intended to help keep the team's work organized and shared equally amongst the members as well as provide a safe means of transport for design documents.

A2. *Define specifications*. The first step in a design problem is to develop design criteria based on the target market, function, materials, mechanics, and cost of the process. It helps define clear objectives for the design and enables the selection of a simplified model for analysis based on the requirements of design.

The specifications need to include the design load. One method of determining the design is via field testing. A pedal dynamometer has been constructed to measure the force applied to the pedal by the rider. The dynamometer is a protective aluminum box that encloses a potentiometer to measure the pedal angle with respect to the crank arm, and a pedal spindle that is instrumented with eight strain gages (Measurements Group EA-06-125BZ-350) connected in two full Wheatstone bridge circuits that measure force components normal and tangent to the pedal. Potentiometer and strain gage signals are acquired by a field computer system (Somat 2100). A three-axis accelerometer is also available.

A3. *Crank analysis*. A simplified model of the crank arm having a straight circular cross-section is considered for analysis. Since EMch 13 (Strength of Materials) is a prerequisite to the EMch 400 & EMch 500 courses, the students have the tools necessary to analyze this simplified model at the start of the semester. Each subtask listed above helps define the steps involved in the analysis of a straight circular crank arm. Analysis of the circular cross-section enables students to understand kinetics of the crank arm.

A4. *Crank design*. Use the analysis tools developed in A3 to converge on the optimal diameter given the loading conditions specified and the material chosen.

B. *Analysis validation*. Pro/Engineer solid modeling software is used to create an engineering design of the cross-section achieved from the analysis (although this is not done by the students). This is converted into a machining drawing and a prototype is built using CNC machining. Tests are done to measure strains at certain points on the circumference of the crank arm for different loading conditions to compare the theoretical strains achieved from the analysis. The cost of the prototype was $500, which highlights the need for accurate modeling to reduce the number of prototypes that need to be made and tested.

C1. *Torsion analysis*. In order to model more accurately the cross-section of an actual bicycle crank arm, a straight crank arm with rectangular or elliptical cross-section is considered. This task follows discussion on torsion analysis for non-circular cross-sections in the class. Since the analysis of these sections under torsion is a complex topic, this task is assigned to the EMch 500 students.

C2. *Design*. Combining the results of the torsion analysis with the rest of the stress analysis involves the entire team and requires EMch 500 students to instruct EMch 400 students on how to use their analysis tools for design.

D. *Finite element comparison*. The stresses from the simplified analytical models were compared to the stresses in an actual crank arm using finite element analysis. Solid models of an actual Shimano bicycle crank arm and a crank arm with straight circular cross-section made using Pro/Engineer solid modeling software were discretized and analyzed by the EMch/ME 461 (Finite Element Analysis) students using Pro/Mechanica. The results from the finite element analysis were provided to the EMch 400 and EMch 500 students for comparison with the analytical and experimental results of the analytical models (circular, elliptical and rectangular) to give an idea of which section closely matched the actual cross-section. Students enrolled in both EMch/ME 461 and EMch 400 (or EMch 500) shared their analysis with the EMch 400 and EMch 500 classes during class.

Note that while many engineering failures are associated with connections, essentially no emphasis was placed on the connections of the crank arm to the pedal and the crank shaft. That just was not part of the project due to a lack of time and the focus being on topics covered in class.

Evaluation

Team grades are given based on quality of the work, thoroughness, and creativity. The percentages of the individual tasks are given in a previous section. Also, peer evaluations of teammates are done twice during the semester. Salamon and Engel[6] address grading mechanics of materials design projects.

Student Survey

An anonymous survey was given to the students at the end of the semester. 83% of the students participated in the survey. One question on the survey asked, "Using the scale below, rank each of the following teaching methods according to how useful they are in helping you learn the material for the course. 1 = Not at all useful, 2 = Somewhat useful, 3 = Useful, 4 = Very useful." Table 1 shows the student responses in terms of the mean as well as percentage of respondents.

Table 1 – Usefulness of teaching methods

	Mean	**1**	**2**	**3**	**4**
Listening to the instructor's lecture	2.92	8	24	36	32
Participating in class discussion	2.24	16	52	24	8
Working in groups on design projects	2.92	8	16	52	24
Working with others to complete homework assignments	2.83	9	35	22	35
Working alone to complete homework assignments	2.92	12	12	48	28
Listening to students answer questions asked by the instructor	2.16	12	68	12	8
Reading the textbook	2.88	8	32	24	36

Clearly, the students felt that the design project was useful. Students indicated that they spent an average of 0-6 hours/week on the design, with 36% of the students saying that they spent an average of 2 hours/week. This is comparable to the amount of time that they spent on homework and on reviewing notes. It was apparent to the instructor that the project increased the students' interest level in the course; see Wankat and Oreovicz[7] for more on retention. Some students provided additional comments on the survey that related to the design project.

- "The project work for the class was very good. The project taught us some things, which wouldn't have occurred if we did not have a project. Unfortunately the project was limited to torsion and bending problems. Therefore, I suggest to include several small projects in other areas like shear centre, FEA, unsymmetrical bending etc."
- "The group project was a little too long. This should be a project that lasts only about ½ of the semester. The project should be more open-ended in the fact that each group should pick something they want to design/analyze. There is no need to add torsion of elliptical/rectangular cross-sections because the project was structured such that the methodology and ideas about how to approach the design was formulated early on. Experimental validation part was interesting and necessary part of the project."
- "I thought when I enrolled in this class I would be learning material that is applicable to the real world. But apparently I was wrong most of the material is geared toward grad work or upper level research. The only practical thing was the project."

Bibliography

1. J.D. Lang, S. Cruse, F.D. McVey, J. McMasters, 1999, "Industry expectations of new engineers: a survey to assist curriculum designers," J. Engng. Education 88:43-51.
2. T. Mase, 1997, "Integrating design in advanced mechanics of materials through industry collaboration," 1997 ASEE Annual Conference Proceedings, Session 2266, pp. 1-11.
3. http://www.af.mil/news/Jul1997/n19970724_970900.html , as of 10 January 2002
4. http://www.chainreaction.com/huge.htm , as of 10 January 2002
5. C.J. Lissenden, G.S. Wagle, N.J. Salamon, 2002, "Applications of finite element analysis for undergraduates," 2002 ASEE Annual Conference Proceedings, Session 3568.
6. N.J. Salamon and R.S. Engel, 2000, "A management/grading system for teaching design in mechanics of materials," Int. J. Engng. Education 16:423-429.
7. P. Wankat and F. Oreovicz, 2000, "Memories are made of this: projects can make your classes unforgettable," ASEE Prism, February 2000, p. 33.

Biographical Information

CLIFF J. LISSENDEN, Ph.D. (University of Virginia, 1993) is an associate professor of Engineering Science and Mechanics at Penn State. In addition to teaching engineering mechanics courses ranging from statics to plasticity theory, he performs experimental and modeling studies of material response in the presence of multiaxial stress states. He is a member of ASEE, SES, ASME, ASCE, and Sigma Xi.

GAUTAM S. WAGLE is a Research Assistant at Penn State. He received a B.E. degree in Mechanical Engineering from the University of Bombay, India in 1997. He worked for a year as a Pre-planning engineer in the Switchgear manufacturing division of M/s Larsen and Toubro Ltd, Bombay, India. Gautam received his M.S. in Engineering Mechanics from Penn State in 2000.

N.J. SALAMON, Ph.D. (Northwestern University, USA) has been a professor at Penn State since 1985. Prior to that he was associate professor at West Virginia University and assistant professor at the University of Wisconsin Milwaukee. He has taught mechanics at the undergraduate and graduate level since 1975 and is a proponent of project work in engineering classes, in particular design. He does research in stress analysis of materials and structures with the emphasis on computer simulation. He is a fellow of the ASME and member of ASEE. His hobby is hiding away in forests.