

2006-1937: HOW DOES HIGH SCHOOL MATHEMATICS PREPARE FUTURE ENGINEERS?

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How Does High School Mathematics Prepare Future Engineers?

Mathematics instruction in high schools varies widely across the country, but movement toward high stakes testing has had undoubted influence on curricular content. In addition, the availability of high tech calculators to students has influenced activities in the classroom and on the test. Research holds that mathematics can act as a strong predictor of future college success. What mathematical concepts are future freshman engineering students taught in high school and are they sufficient preparation for the rigors of an engineering curriculum? This paper will take an introductory look at these questions by examining the North Carolina approach to high school math, through the Standard Course of Study, by selected classroom snapshots and by assessment of selected mathematics skills of college freshmen.

1.0 Introduction

Experience working with teachers¹ has show that it is common for teachers at one level to not be aware of what math is taught at the previous and next levels, including the transition from high school to college. In addition, high stakes testing is having a definite impact on high school classroom teaching, and the results are as yet unknown. The increasing use of technology, in the form of graphing calculators, in the classroom, is also having an impact. Research shows that technology can help in conceptual understanding, but does it shift the focus away from basic skills? In addition, other issues such as block scheduling, variability across the state of course offerings and textbooks play a role in what high school mathematics students are exposed to. Less emphasis in the classroom is placed on mathematical algorithms and procedures and how math ideas connect to one another than on solving specific types of problems and learning mathematical concepts².

2.0 Construction of experiment

The question of mathematical preparation is very broad and not easily answered without complex examination of goals, objectives and methods. A snapshot, however, is easier to obtain and can lend insight into this difficult question. The researchers in this experiment chose to set it up as follows. One high school class was chosen that contains a large number of learning objectives that are important in a wide variety of engineering courses. Of the material in that course, four objectives were chosen from the standard course of study for closer examination. Teachers with varying degrees of experience in teaching in Wake County, North Carolina public schools were chose for interview. In addition, freshman engineering students were asked to assess themselves on the skill encompassed in the chosen objectives through answering test questions and then self-analyzing their results.

2.1 Selection of class for examination

According to survey data from the year 2000², most high schools in the United States offer Algebra II, but a fair number do not offer calculus. Although this data is now five years old, informal student contact verifies that some gap still exists. The standard course of study for Algebra II also contains several skills that are directly linked to introductory engineering classes, including matrix manipulation, conic sections, series, and linear regression. Therefore, Algebra II was deemed a good course for closer examination. Most high school students who are on an engineering-bound track in the state of North Carolina take this course in their sophomore year, so any skill learned in the class would have had to be retained or applied in some subsequent course for students to feel comfortable with them.

2.2 Selection of four objectives from the standard course of study

Of the many objectives in the North Carolina Standard Course of Study³, four were selected for closer examination in this experiment in order to assess specific teaching techniques and specific skills acquired by students. The objectives were specifically chosen to relate to skills needed in introductory engineering problem solving classes. They are:

- *2.02 Use quadratic functions and inequalities to model and solve problems; justify results. Solve using tables, graphs, and algebraic properties. Interpret the constants and coefficients in the context of the problem.*
- *2.04 Create and use best-fit mathematical models of linear, exponential, and quadratic functions to solve problems involving sets of data. Interpret the constants, coefficients, and bases in the context of the data. Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.*
- *2.05 Use rational equations to model and solve problems; justify results. Solve using tables, graphs, and algebraic properties. Interpret the constants and coefficients in the context of the problem. Identify the asymptotes and intercepts graphically and algebraically.*
- *2.10 Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.*

2.3 Interviews of teachers

Current teachers of algebra II were interviewed to assess how teachers address the specific chosen objectives and what external influences affect that teaching. The questions asked were:

- Describe the resources that you use when planning units and lessons in Algebra II
- To what extent is the content and methods for teaching Algebra II guided by your textbook (school planning team, end of course exam)?
- Describe the types of activities or projects that you use with students.

- Describe the ways in which you and your students use technology to teach/learn Algebra II.
- If you have taught Algebra II for several years, what changes have you made in Algebra II? Why?
- In what way(s) do you think using technology in the classroom enhances (or not) the teaching and learning of algebra II concepts?
- Describe the characteristics of different students to whom you teach Algebra II (what courses have they taken and what do they take after Algebra II).
- Please take a look at the following objectives related to Goal 2. Describe a lesson or two that you might teach related to the goal. Specifically tell how you use technology in these lessons.

The teachers interviewed varied in experience level and preparation level. Their characteristics are summarized in the following table.

Number of years teaching math	Number of years teaching Algebra II	Undergraduate major	Graduate major	Have you pursued National Board Certification?
7	7	Secondary math ed.	Secondary math ed.	Yes
17	1.5	Math education	Education	Yes
4	0.5	Accounting and math	Secondary ed.	No
6	5.5	Math and math ed.	N/A	Yes
2	2	Secondary math ed.	N/A	No

Teachers were chosen from RAMP-UP partnership schools and non-partnership schools, but the teachers who were approached had some previous acquaintance with one or more of the researchers. Extensive face-to-face or electronic interviews were conducted with five teachers from five schools, one an academic gifted magnet, one a science and technology magnet and three non-magnet high schools. Their participation was completely voluntary. Interviews lasted approximately two hours.

2.4 Examination of textbook

The textbook currently used in Wake County, North Carolina for Algebra II is Key Curriculum Press, Discovering Advanced Algebra, an Investigative Approach⁴. The textbook is highly inquiry and technology based, using calculator investigations to introduce each concept before students are taught calculations and applications. The publisher’s web site advertises the book as strongly linked to the desired outcomes expressed by the National Council of Teachers of Mathematics⁵, “We want our students to gain in factual knowledge, become proficient at procedures, improve at learning on their own, and become better at applying their learning in this

changing, technological world. Research indicates that these four outcomes will follow if we achieve one underlying goal: conceptual understanding. Moreover, if by emphasizing conceptual understanding, you can motivate a large part of that diverse group of students who are not able to remember facts and procedures unless they understand them.” The North Carolina Department of Public Instruction specifies three other textbooks, which are not as inquiry oriented.

2.5 Questions to college students

Questions were developed that align with each of the objectives under examination and distributed to engineering freshmen. The students were asked to solve the problems and then comment on whether they felt prepared by their high school mathematics courses to answer questions like those asked. The questions asked were:

Objective 2.02: Use quadratic functions and inequalities to model and solve problems; justify results.

1. The shape and thickness of a contact lens depends on the vision and correction that is needed. The intersection of the following inequalities describes a typical contact lens cross section. (x and y are measured in millimeters.) Sketch the region. How wide is the contact lens? How thick is it?

$$y \leq -0.065x^2 + 2.34$$

$$y \geq -0.055x^2 + 1.98$$

(Problem from Larson, R. Kanold, T. & Stiff, L. (1997). *Algebra 2: An Integrated Approach*. Lexington, MA: D.C. Heath Publications.)

Objective 2.04 Create and use best-fit mathematical models of linear, exponential, and quadratic functions to solve problems involving sets of data. Interpret the constants, coefficients, and bases in the context of the data.

Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

1. The following data were collected from an experiment testing how many times a piece of metal could be bent before breaking, as a function of temperature. Find an equation to represent the data and predict how many bends would lead to breaking at 0 degrees C. How good is your prediction (justify)?

Temperature	Number of bends to breaking
10 degrees C	25
15 degrees C	35
20 degrees C	45
25 degrees C	55

Objective 2.05: Use rational equations to model and solve problems; justify results.

1. The weight of an object above the earth's surface is given by the function

$w(h) = \left(\frac{r}{r+h}\right)^2 w_0$ where r is the radius of the earth (approximately 3950 miles), w_0 is the weight of the object at sea level, and h is the height of the object above the earth's surface. How much will an object whose weight at sea level is 200 pounds weigh when it is 400 miles above the earth's surface?

(Problem from *Contemporary Precalculus through applications*. Janson Publications)

2.10 Use systems of two or more equations or inequalities to model and solve problems; justify results. Solve using tables, graphs, matrix operations, and algebraic properties.

1. The currents in three points of an electric circuit are given by the following equations. Solve.

$$3 i_1 + 2 i_2 + 5 i_3 = 10$$

$$-i_1 + 4 i_2 + i_3 = 5$$

$$2 i_1 + i_2 + 3 i_3 = 0$$

3.0 Teacher interview results

Some of the teachers were apparently comfortable with the use of the inquiry –oriented textbook, and some were not. The North Carolina End-of-Course test for Algebra II is highly skills based, as opposed to application based. In some instances tests in individual classes aligned more with skills (i.e. finding the determinant of a matrix) versus applications (i.e. solving simultaneous equations using matrices). In some instances teachers taught material using calculators exclusively, and some taught their students to solve problems by hand first, then using the calculator. The following paragraphs summarize the teacher interview responses.

Describe the resources that you use when planning units and lessons in Algebra II.

Resources used to prepare units and lessons included multiple textbooks, previous course material from teaching Introduction to College Math, Wake County curriculum guides, the Internet, and materials collected from a workshop. The more experienced teachers tended to rely on previous experience, while the less experienced teachers were more likely to seek resources on the Internet and from other teachers.

To what extent is the content and methods for teaching Algebra II guided by your textbook (school planning team, end of course exam)?

Two of the teachers stated that they used the textbook to guide their coursework. Both of these teachers said that they, “rarely let the final exam dictate what I’m teaching or how I teach.” The other three teachers said that they felt the inquiry-oriented nature of the book made it unsuitable for teaching Algebra II so that students could perform well on the end of course exam. One teacher stated that, “most teachers don’t use the book.” Another noted that the curriculum is so “jam packed” that she cannot teach any topic with any depth, now that the county has moved to the block schedule. These teachers stated that the end of course exam completely dictates what and how they teach, to the extent that they do not cover some of the standard course of study, because it is not on the exam (sequences and series were given as an example). These results are consistent with other studies that show that high stakes testing does not necessarily produce desired learning outcomes^{6,7,8}.

Describe the types of activities or projects that you use with students.

Describe the ways in which you and your students use technology to teach/learn Algebra II.

All of the teachers made extensive use of graphing calculators in their classes. Some stated that removing the need for tedious calculation made it easier for students to grasp the meaning of what they were learning. Most, however, stated that they use the calculators because they are required on the end of course exam. Each of the teachers described projects involving modeling using functions that required out of school work.

In what way(s) do you think using technology in the classroom enhances (or not) the teaching and learning of algebra II concepts?

All of the teachers stated that technology allowed them to take students into greater depth in the exploration of functional modeling. One concern was that the perception of the graphs on the calculator could be incomplete due to changing scales and the students’ lack of the, relatively, sophisticated ability to choose scale correctly to interpret results.

Describe the characteristics of different students to whom you teach Algebra II (what courses have they taken and what do they take after Algebra II).

Most honors students came from honors geometry. Non-honors students came through a diversity of paths, including algebra I and geometry, Algebra 1A and 1B and tech math and geometry.

Please take a look at the following objectives related to Goal 2. Describe a lesson or two that you might teach related to the goal. Specifically tell how you use technology in these lessons.

Each of the teachers described lessons consistent with their own characterization of whether they followed the inquiry-oriented textbook or prior experience. Lessons varied accordingly from inquiry-oriented, technology-based lessons to simple computational activities. Several teachers stated that computational skills such as reducing rational functions and simplifying radicals are “dead skills” due to students’ dependence on calculators.

4.0 College student question results

The students who responded to the questionnaire were above average in terms of their performance in first semester freshman math classes. This is not surprising, as students who do not have confidence in math are unlikely to respond to a voluntary survey that involves computation. The students who performed the best in their math classes, and correspondingly on the questions on this survey, were from large counties like Wake and Mecklenburg with high test scores. Half of these students stated that they felt that their high school algebra courses prepared them well for college. The students who performed less well on the “quiz” were, without exception, from smaller counties in rural portions of North Carolina. They also stated that they did not feel prepared coming from high school. The students who felt partially prepared stated lack in areas that surprised the researchers, as both linear fits and simultaneous equations are strongly represented in the North Carolina Standard Course of Study.

Fall semester math grade	Claims prepared	Correct answers	County test performance	Solved with technology or by hand
B+	Yes	Partially	Medium	Hand
C-	No	No	Low	Computer
B-	Strong no	No	Low	Hand
A+	All but one question (linear fit)	Yes	High	Computer
B+	Yes	Yes	High	Hand
A-	Yes	Yes	Medium	Calculator
B+	All but one (simultaneous equations)	Partially	High	Computer
B	Yes	Yes	High	Hand

These results were correlated with experience in the freshman engineering class where students were asked to take experimental data, find a linear fit to the data and discuss the quality of the fit (taken directly from the Algebra II standard course of study). The students were able to complete the project with no difficulty, but could not discuss the quality of the linear fit until the professor pointed out the “R” button on their calculator. The students then remembered that “R” had something to do with linear regression, but could not explain what.

Interestingly, half of the students chose to solve the problems by hand. This might indicate that these students, at least, do not depend on technology to be able to solve mathematics problems.

5.0 Comments on overall results and questions for future study

The sample sizes in this study are small, primarily because the study is intended to be exploratory in nature to investigate potential research questions for a more in-depth, statistical

study. The results of this study have produced many questions that beg further study. Technology appears to be helpful in some ways in teaching classes such as Algebra II, but may inhibit learning and retention of basic skills. Of more concern, however, is the effects that high stakes tests appear to have on what is being taught in the classroom. Of the teachers in this study, it appears that the more experience teaching, the more a teacher is likely to “teach to the test” rather than to the mathematical future of his/her students. Most students in the study appeared to retain specific skills related to Algebra II as they enter engineering school, but their ability to apply those skills in the solution of new problems remains to be seen. The researchers expected all of the students to retain the skills examined, but to rely on technology for problem solution. Clearly, the students in the study did not retain as many skills as expected, and did not appear to rely on technology to the degree expected either. Further study might use a more comprehensive preparation test of a large sample of freshman engineers.

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