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Paper 910

TEACHING PROBLEM SOLVING IN AN INTEGRATED MATHEMATICS-WRITING CURRICULUM

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ABSTRACT: It is crucial that students realize that solving equations and writing papers are not exercises done to please teachers, that equations represent real-world events, that the process of writing a paper reflects the process of reporting information, that problem-solving is what adults do on the job. Consequently, we teach mathematics and English, not as separate subjects, but as two grammars that must be used to formulate solutions to problems. Whenever possible, we apply the two grammars to problems from the six engineering technology disciplines pursued at Ward College of Technology (at the University of Hartford). In this paper, we discuss our integrated math-English curriculum (developed with support from a University-wide FIPSE grant), the subject matter of which is problem-solving. We demonstrate the comparison and contrast of the two languages in problem solving and how the solutions of problems extend into the students' technical courses through a perspective of multiple intelligences.

Introduction

Under a grant from FIPSE (Fund for the Improvement of Postsecondary Education), The University of Hartford offers courses called First-Year Interest Groups, or FIGS. Each FIG comprises two courses, generally a writing course and a content course with a common theme, and students must enroll in both courses to receive credit. For example, the Barney School of Business at the University offers a FIG that links BAR 110, Introduction to Business, with RLC 110 Reading and Writing I. Among the many FIGs offered by the College of Arts and Sciences is one that links ENG 140, Introduction to Literature with PHI 110, Introduction to Philosophy. Although the two courses share a theme, the curriculum of each course is separate; occasional assignments in each course relate to the curriculum of the other course.

When Ward College of Technology was invited to participate in the FIPSE grant, the original plan was to have a traditional FIG with a writing course and a content course in which the writing course assignments would relate to one of the technology courses. However, in the course of developing the FIG, the writing instructor and one of the mathematics instructors at Ward College (the authors of this paper) began to talk about the problems they encountered in teaching the support courses for technology students.

Those problems centered on what the authors thought was the inability of the freshmen in our classes to solve problems in math and in writing. Discussion led to a decision to solve the problem of that inability by forming the Ward College FIG, not with a traditional writing course and a content course, but with the freshman writing course, EN 111, Expository Writing, and the freshman math course, MTH 112, Algebra and Trigonometry. The theme of the FIG would be problem solving.

Further, instead of simply sharing the theme, the authors decided to structure the courses so that the entire curriculum in both courses would be problem solving. Both English and mathematics would be presented as grammars for use in describing the world and solving problems in the world.

In this paper, we report on the first semester of our FIG and what we learned teaching it.

Expected Outcomes

In planning the FIG, the authors completed OATAs (Outcome/Activity/ Technique/Assessment) in a grid format. Figure 1 shows the OATAs the authors created as they prepared to teach the FIG for the first time in the fall 2001 semester.

Figure 1: First-Year Interest Groups (FIGs) ILB-IOTA Grid

Integrative Learning Block (**ILB**)	Student Learning **Outcome**	Learning **Activity**	Instructional **Technology**	Learning **Assessment**
English and Math are the fundamental "languages" of the technology curriculum	Students will solve basic math problems.	Solve problem sets both individually and in groups.	Students work in groups to discuss and solve problems. Instructor articulates and identifies problem-solving steps	
	Students identify and differentiate between descriptive and embedded word problems	Discuss problem types	Instructor discusses types of problems; walks through hierarchy to solve problems; presents worksheet with steps	
	Students solve descriptive and embedded word problems by translating English into math. Students differentiate significant and nonsignificant information	Solve problem sets	Groups discuss and solve problems. Instructor discusses drawing problems.	
	Students recognize importance of math and English as they relate to knowledge and research in technology	Review tech. journals and other literature and present findings orally or in writing	Individuals conduct research in library and other hardcopy sources and online sources	Oral presentations and papers
Students learn basic problem-solving skills that translate from application to application	Students apply problem-solving techniques in math and English	Discuss Bloom's Taxonomy, other problem-solving paradigms	Instructor introduces terminology	
		Discuss components of analysis, synthesis, evaluation	Groups discuss brain teasers; writing problems; math problems	Groups report results
		Apply analysis, synthesis, evaluation methods.	Individuals solve problems, write papers	Papers and problem sets reviewed

Writing our OATAs, we fully expected to teach our freshmen students to solve problems with little difficulty.

The Class

We implemented our plans in the fall of 2001 with the writing class and the math class meeting one immediately after the other in the same classroom and with an enrollment of 19 students. Each instructor attended the other's class not only so that we could reference the other's content in our own classes but it also provided symbolic team teaching of the two classes thought of as two sections of the "same" course.

Our first step was to understand our students, to which end we administered both a multiple intelligences (MI) inventory (using Martin Gardner's formulation of that theory)[1] and a Myers-Briggs Type Inventory (MBTI)[2]. In the MI inventory, as summarized in Figure 2 below, we found that our Audio Engineering Technology students were strongest in musical intelligence, our Architectural Engineering Technology students were strongest in visual-spatial and bodily-kinesthetic intelligence as well as musical, and our Mechanical and Computer Engineering Technology students were strongest in bodily-kinesthetic intelligence. None was strong in linguistics, and the logical-mathematical intelligence was stronger in the two majors strongly connected to the arts (architecture and audio).

Please note that the information presented is based on only 18 students. We're reporting on trends until we have more data, at which time we may be able to perform quantitative analysis and draw significant conclusions. We also have to note that every class may differ, meaning that the trends reported here apply only to the class taught in the fall 2001 semester.

Figure 2. Results summary of MI and MBTI tests.

MI	AET	AuET	MET	CET
Musical	5.8	7.3	6	5
Logical-Mathematical	5.7	6.1	2	2
Linguistic	3	2.9	1.5	3
Bodily-Kinesthetic	6.3	5.3	7.5	7
Visual-Spatial	5.8	4.9	5	3
Interpersonal	4.7	4.6	3.5	5
Intrapersonal	4	5.6	4.5	2
Naturalist	3.2	3	2	1

MBTI	AET	AuET	MET	CET
E	6	6.6	3	8
I	4	3.4	6.5	2
S	12.3	11.6	17	17
N	7.5	8.4	3	3
T	7.5	10.5	11	10
F	12.5	9.6	9	10
J	11.7	9.1	10.5	12
P	8.3	10.6	9.5	8

Where the majors' abbreviations and populations are

Abbreviation	Major	# of students
AET	Architectural Engineering Technology	6
AuET	Audio Engineering Technology	9
MET	Mechical Engineering Technology	2
CET	Computer Engineering Technology	1

Based on the "average" student, our students have the following MBTI profiles:

Major	Ave
AET	ESFJ
AuET	ESTP
MET	ISTJ
CET	ES?J

Seeing those results, we immediately realized that one of our tasks would be to strengthen the students' linguistic and mathematical-logical intelligences, not simply to teach them writing and math. In other words, our task would be to teach them the patterns of thought represented in writing and math, in addition to the skills embodied in writing and mathematical problem solving.

The MBTI revealed that our students are concrete thinkers; all are Sensers. These profiles provide a particular challenge for teaching the FIG in that both professors are Ins, whereas the students were all ESs. "Observers (SPs and SJs) seem more at home when … attending to concrete things … and to practical matters …. In turn, Introspectors (NTs and NFs) tend to be more content when these concrete concerns are handled by someone else and they are left free to consider the more abstract world of ideas."[3]

Armed with this information, we approached the classes, first, by being explicit about our objectives. We explained the results of both the MI and Myers-Briggs inventories and the fact that one of our goals was to strengthen their linguistic and mathematical-logical intelligences. We also pointed out how our profiles, as faculty, affected the way we teach. We invited their suggestions for how we might modify our teaching styles to match their learning profiles. We wanted to be clear that the students had strengths that were not traditionally valued in high school, but that would be highly valued in their chosen professions. However, in order to prosper in their chosen professions, they would have to learn certain tools, namely, writing, presentation, and mathematics.

We also presented our MI profiles and Myers-Briggs results to the students in order to demonstrate, first, that individuals with different strengths can work together to solve problems. We also demonstrated that our different strengths (Professor Segal is extremely strong in musical and linguistic intelligences, whereas Dr. Townsend is extremely strong in mathematical-logical and visual-spatial intelligences) lead us to prefer different media for problem solving, but that both of us have mastered the skills supported by other intelligences in order to perform various problem-solving tasks at work.

In addition, we explicitly stated that the true content of both courses was problem solving, that we would be talking about mathematics in the writing class and writing in the mathematics class, because mathematics and writing are the tools used to describe problems and solutions at work. Even when the problems involved can be visualized, even if the problems involve music or space, words and numbers will be involved in the solution. Having been explicit about our program, we proceeded to carry out our OATAs.

The Writing Class

The writing class began with the following statement: "The central problem we are trying to solve in this class is, 'How can I write (speak, show) information so that my audience will understand what I say and do what I want them to do?'" As the class proceeded, we related various reading and writing assignments to that initial problem statement. One frequent question about various readings was, "what problem is this author trying to solve with this piece of writing?" Another set of questions was, "How did *this* author solve the problem of audience understanding and response? *Did* the author solve that problem?" The intent of such questions was to help students understand that writing is a response to a problem and often an attempt to solve a particular problem. It also demonstrates potential solutions to the problem of writing for an audience, as does the organization and discussion of the readings and writing assignments into rhetorical strategies such as description, narration, definition, comparison and contrast, classification and division, and the like.

We also discussed mathematics in the writing classroom. One exercise involved a comparison of descriptions in English and in mathematical equations. Figure 3 shows one such comparison, a discussion of velocity in words and in an equation.

Figure 3: Comparison of Two Grammars, English and Mathematical Equation[4]

Comparison/Contrast—Two Grammars

Plain English:
. . .consider the ball rolling 2 feet in one second. . . .In one second it covers 2 feet for an average velocity of 2 ft/sec. In two seconds, it covers 8 feet, for an average velocity . . . of 4 ft/sec. . . .we are dealing with average velocities. What is the velocity of a rolling ball at a particular moment? Consider the first second of time. During that second the ball has been rolling at an average velocity of 2 ft/sec. It began that first second of time at a slower velocity. In fact, since it started at rest, the velocity at the beginning (after 0 seconds, in other words) was 0 ft/sec. To get the average up to 2 ft/sec, the ball must reach correspondingly higher velocities in the second half of the time interval. If we assume that the velocity is rising smoothly with time, it follows that if the velocity at the beginning of the time interval was 2 ft/sec less than average, then at the end of the time interval (after one second), it should be 2 ft/sec m ore than average, or 4 ft/sec.

If we follow the same line of reasoning for the average velocities in the first two seconds, in the first three seconds, and so one, we come to the following conclusions: at 9 seconds the velocity is 0 ft/sec; at two seconds, the velocity is 8 ft/sec; at three seconds, the velocity is 12 ft/sec; at four seconds, the velocity is 16 ft sec, and so on.

Notice that after each second, the velocity has increased by exactly 4 ft/sec. Such a change in velocity with time is called an *acceleration* . . . To determine the value of the acceleration, we must divide the gain in velocity during a particular time interval by that time interval. For instance at one second, the velocity was 4 ft/sec, while at four seconds it was 16 ft/sec. Over a three-second interval the velocity increased by 12 ft/sec. The acceleration then is 12 ft/sec divided by three seconds. (Notice particularly that it is *not* 12 ft/sec divided by 3. Where units are involved, they *must* be included in any mathematical manipulation. Here you are dividing by three seconds and not by 3.)

In dividing 12 ft/sec by three seconds, we get an answer in which the units as well as the numbers are subjected to the division—in other words 4 ft/sec/sec. This can be written 4 ft/sec/sec (and read four feet per second per second). Then again, in algebraic manipulations a/b divided by b is equal to a/b multiplied by $1/b$, and the final result is a/b^2. Treating unit-fractions in the same manner, 4 ft/sec/sec can be written 4 ft/sec^2 (and read four feet per second squared.

You can see that in the case just given, for whatever time interval you work out the acceleration, the answer is always the same: 4 ft/sec^2. For inclined planes tipped to a greater or lesser extent, the acceleration would be different, but it would remain constant for any one given inclined plane through all time intervals.

This makes it possible for us to express Galileo's discovery about falling bodies in simpler and neater fashion. To say that all bodies cover equal distances in equal times is true; however, it is not saying enough, for it doesn't tell us whether bodies fall at uniform velocities, at steadily increasing velocities, or at velocities that change erratically. Again, if we say that all bodies fall at equal velocities, we are not saying anything about how those velocities may change with time.

What we can say now is that all bodies, regardless of weight (neglecting air resistance), roll down inclined planes, or fall freely, at equal and constant accelerations. When this is true, it follows quite inevitably that two falling bodies cover the same distance in the same time, and that at any given moment they are falling with the same velocity (assuming both started falling at the same time). It also tells us that velocity increases with time and ata constant rate.

OR

Mathematical Equation

$$v=at$$

The plain English formulation seems highly wordy to the N (intuitive) mathematics professor but provides the S (senser) student with logical development of an idea and concrete examples of the idea. The plain English formulation is also significantly more interesting to the writing professor and the students than just the short equation. The terse mathematical formulation makes sense only after digestion of the explanation that precedes it—except, of course, to the math professor. For everyone else in the class, presented without the words, $v=at$ is meaningless. However, it does provide a clear method for investigating other cases besides those presented in the explanation.

Another exercise involved using brainteasers of various sorts to abstract the problem-solving process from the students' majors. One interesting finding in the brainteaser exercises was that different intelligences led students down different paths in their attempted solutions. For instance, one alphabet game had the instructor ask questions that required a letter of the alphabet in the answer. "What letter of the alphabet is a bird?" The expected answer is *J*; the Audio Engineering Technology students, whose musical intelligence includes a high degree of aural involvement, responded very quickly with that answer. Other students, whose strengths lie in visual or kinesthetic intelligence, couldn't answer the alphabet riddles as quickly. One Architecture student said, "I was looking for something that *looked* like a bird." On the other hand, when we moved to problems that could be diagramed for solution, for example, logic puzzles involving names that must be matched to professions on the basis of four or five clues and that are commonly solved by drawing a grid, the visually and kinesthetically stronger students were quicker to grasp solutions or methods of finding solutions. Hence, any problem-solving class must take individual intelligences into account and provide rubrics for the different intelligences to use.

The Mathematics Class

Prominent features of the math class were (1) comparison of math and English as rule-based disciplines, (2) in-class practice in solving the problems and (3) emphasis on the real-world application of the tools students were learning,

Since the math class followed immediately after the English class, Dr. Townsend had the opportunity to compare and contrast the two disciplines in real time. In English, ambiguity of meaning is often encouraged; not so in math. At this level, math is a small collection of rules, much like those of English grammar. Anyone performing a symbolic manipulation must use one of the rules.

The job of the students is to then practice using these rules, i.e., do homework and solve in-class problems, until they build an intuition on which rule to use. It is a question of pattern matching: "What rule matches the task at hand?" We found that our freshmen did not typically have the mental discipline to train their minds by concentrating on homework to the point of internalizing their new intuition. They tended to try to use misremembered rules from their past.

It is often said that the students want to see how they are going to use the information we teach them in their professions. They also want to know "What formula do I use?" without the complication of an accompanying real-world description. These statements provide an interesting contradiction that must be addressed before the next incarnation of this FIG. Students consider word problems (i.e., real world applications) to be different from math—in other words they see math as a computational machine and word problems as an entirely different order of task. To put our problem in terms of the Myers-Briggs profiles, solution of word problems requires a strong N component in the approach to problem solving, but the students are all S's. Word problems do not necessarily present the question in a nice, orderly, concrete way.

Our Assessment of the FIG

When we examine our actual outcomes against the OATAs we wrote before the class, we appear to have achieved what we wanted to. We carried out the activities listed on the grid, for example, having students solve problem sets, discussing problem types, and so on.

However, we feel that the accomplishments are superficial, that we did not, in fact, accomplish the overarching goal of teaching the students that mathematics and writing are tools they will use on the job and in their lives to solve problems. Part of the problem is, of course, that we are teaching freshmen. They appear to be novices on the Wankat and Oreovicz scale of novice-to-expert problem solvers. The table below (figure 4) is a reproduction of that scale, which shows the characteristics of novices—and, not coincidentally, freshmen technology students. We assumed that our task would be to introduce freshmen to the vocabulary and the ways of thinking that they will use later on, but that we would not turn them into expert problem solvers in one semester.

Figure 4: Comparison of Novice and Expert Problem Solvers[5]

Characteristic	**Novices**	**Experts**
Memory	Small pieces Few items	“Chunks” or pattern - 50,000 items
Attitude	Try once and then give up Anxious	Can-do if persist Confident
Categorize	Superficial detail	Fundamentals
Problem statement	Difficulty redescribing Slow and inaccurate Jump to conclusion	Many techniques to redescribe Fast and accurate Take time defining tentative problem

Characteristic	Novices	Experts
		May redefine several times
Simple well-defined problems	Slow Work backward	~ 4 times faster Work forwards with known procedures
Strategy	Trial and error	Use a strategy
Information	Don't know what is relevant Cannot draw inferences from incomplete data	Recognize relevant information Can draw inferences
Parts (harder problems)	Do NOT analyze into parts	Analyze parts Proceed in steps Look for patterns
First step done (harder problems)	Try to calculate (Do It step)	Define and draw Sketch Explore
Sketching	Often not done	Considerable time Abstract principles Slow motion
Limits	Do not calculate	May calculate to get quick fix on solution
Equations	Memorize or look up detailed equations for each circumstance	Use fundamental relations to derive needed result
Solutions procedures	"Uncompiled" Decide how to solve after writing equation	"Compiled" procedures Equation and solution method are single procedure
Monitoring solution progress	Do not do	Keep track Check off versus strategy
If stuck	Guess Quit	Use Heuristics Persevere Brainstorm
Accuracy	Not concerned DO NOT check	Very accurate Check and recheck
Evaluation of result	Do not do	Do from broad experience
Mistakes or failure to solve problems	Ignore it	Learn what should have done Develop new problem solving methods
Actions	Sit and think Inactive Quiet	Use paper and pencil Very active Sketch, write questions, flow paths. Subvocalize (talk to selves)
Decisions	Do NOT understand process No clear criterion	Clear criterion.

Our assumption that we would be teaching novices was incorrect. One major issue is that we are not addressing problem solving at the first step in the process. To clarify that statement, we make reference to an earlier paper, "Word Problems or Problems with Words: A Possible Solution."[6] In that paper, we argue that it is not sufficient to tell students to analyze a mathematical word problem without first teaching them what "analyze" means and demonstrating various methods of analysis. Likewise, to teach students to solve problems in whatever field those problems might arise, we must step back and define what we mean by "problem" and "solving" and how to apply those definitions.

We must also establish a connection between our classroom and the world of work. We are not referring here to making material "relevant to the real world." We refer instead to the actuality that what we teach is, in fact, part of the real world, that writing and mathematics are tools used in developing and reporting information. Despite the fact that we explicitly state that fact, that in the math class, the problems they solve are taken from their majors, that in the writing class, they are taught rhetorical strategies for technical writing, the one question we hear repeatedly is "Why do I need to know this?"

It is now apparent to us that our students are *not* at the novice level. They often do not recognize any strategies to apply, cannot handle even well-defined problems. We must actually teach our students to think like novices, and that in order to teach our students to think like novices, we must understand where they are starting from in their problem solving. Transition from high school to college is a paradigm shift that we must address explicitly in order for them to feel more comfortable about our novel approach to teaching the subjects they "know" so well. So we are taking a step back and interviewing high-school teachers about what our students have been learning and, more importantly, how they learn in the high-school setting. What are our students expecting when they walk into class as new freshmen?

In doing so, we are addressing our revised hypothesis, which is that because our students' linguistic and logical-mathematical intelligences are not strong, their basic problem-solving skills, taught in high school as logical-mathematical and linguistic skills, are likewise not strong. Hence, in addition to teaching them the writing and mathematical skills they need, we must find methods that draw on *their* strengths. It is interesting to note in conjunction with our hypothesis that the literature on problem solving centers on the traditional intelligences (that is, linguistic and logical-mathematical) and methods and that the literature on multiple intelligences is heavily concentrated on elementary education with some information on secondary schools. So we find an investigation of high-school methods necessary for our teaching. Armed with knowledge of our students' starting point, we can work to build a transition for our students from high school to the novice column on Wancat and Oreowicz's scale.

ENDNOTES

[1] Adapted from *7 Kinds of Smart: Identifying and Developing Your Multiple Intelligences, revised and Updated with Information on 2 New Kinds of Smart.* Thomas Armstrong (New York: Penguin Putnam, Inc., 1999).
[2] Adapted from *Please Understand Me II: Temperament, Character, Intelligence.* David Keirsey (Del Mar, Calif.: Prometheus Nemesis Book Company, 1998), pp 346–350.
[3] http://keirsey.com/pumII/ns.html
[4] Adapted from *Understanding Physics,* Isaac Asimov (New York: Barnes & Noble Books, 1993). pp 14–17.
[5] Adapted from *Teaching Engineering*, Phillip C. Wankat and Frank S. Oreovicz (New York: McGraw-Hill, Inc., 1993). pp 69–70.
[6] "Word Problems and Problems with Words", presented at the 2001 ASEE Annual Conference, Albuquerque, NM, 24-27 June, 2001, session 2248, paper number 902.

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