“Introducing Special Interest Topics into an Engineering Analysis Course”

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Abstract

The Masters Degree program at Union College is primarily a part time program that serves engineers who work in area industries. Until recently no math course was required. The justification was the sufficient math content is embedded in the required engineering courses. Recent adjustments in the curriculum have added an Engineering Analysis Course which focuses on applied engineering mathematics.

Some special interest topics have been added along with the traditional differential and integral calculus, vectors, matrices and transform methods. These special interest methods topics place emphasis on the mathematical modeling of various processes and secondary emphasis on solution methods. The philosophy is that if a process can be mathematically modeled it can be solved by computer based numerical techniques. The fact that any process that can be modeled can be solved by numerical methods also means that questionable assumptions such as linearity do not have to be made during the process of developing the equations. Thus more realistic results can be obtained.

1. Introduction

Special interest topics that have been introduced in an engineering analysis course. These topics are of engineering importance but fall outside the boundaries of a traditional classical Applied Engineering Mathematics Course. They include a) Markov processes to model the transitional forecasting process that can be defined as fact to fuzz, b) the Global Positioning System problem of determining position by finding the intersection point of spheres, c) 2nd law analysis of thermodynamic cycles to develop the ideal fuel burning engine based on the varying temperature combustion products which are the source of heat, d) development of a finite element model of a cooking hotdog with constant and with temperature dependent conductivity, e) statistical analysis of the risk associated by launching a rocket outside the experience envelope as defined by the 1986 space shuttle Challenger accident, and f) development of simplified differential equation to represent nuclear reactor steady state and transient processes and then solving this differential equation for a variety of operating conditions.

2. Description

This section will further describe the special topics defined in the introduction.

a. Markov Model for Transitional Statistics

Forecasting can be defined as stepping from the present fact to far future fuzz. There is an intermediate time that is a combination of fact and fuzz. The outside temperature now is a fact that is obtained by looking at the thermometer and seeing 20 F. The estimate of the temperature on this day and at this time a year from now would be obtained from processing historic temperatures into a statistical distribution and finding the average. However, the best forecast for an hour from now should be the temperature now as a reference and modified by the upwind conditions. Thus, the temperature now is a fact, the long time future is a stationary statistical distribution or fuzz, but the intermediate temperature forecast should be a combination of the present measured fact and the distant fuzz.

Another example is some troublesome equipment. It has a 10% probability of failing any day it is operating and a 30 % probability of repair any day it is broken. On the average this equipment will be available 75 % of the time. However, the forecasting the availability for tomorrow and the next day depends upon whether it is working or broken today.

If the equipment is working today there is a 90 % probability it will be working tomorrow. It will be available two days from now if it goes two days with out failing, which has a probability of .81 or if it fails the first day and is fixed the second which has a probability .3 . Thus, the probability of availability two days from now is 84%.

Alternatively if the equipment broken today there is a 30% probability it will be working tomorrow. The probability for two days is the sum of being fixed on the first day and not failing on the second day which is $.3 \times .75 = .225$ plus the probability of not being fixed the first day and being fixed the second day which is $.7 \times .3 = .21$ for a total probability of second day availability of 43.5 %.

A random number generator can be used to simulate the process and thus show a transitional available probability distribution for each future day as a function of whether the machine is working or broken today.

b. The Global Positioning System (GPS) Problem

The navigation problem requires developing a coordinate system and determining your position. The coordinate system may be either two or three dimensional. It can be flat or spherical to correspond to the equator and pole defined latitude, arbitrary reference for longitude, and with sea level providing an the elevation reference.

The navigation problem is solved by determining your position relative to known reference points or beacons. The two methods are triangulation and ranging. The former is performed by measuring angles to the beacons. Your position will be the intersection point on lines. Ranging is based on determining distance from beacons. Your position will thus be the intersection point of circles in two dimensions or spheres in three dimensions. The distance between the radio beacon and the receiver is the time of signal in transit times the speed of light. Thus, precise time is required for ranging.

The amazing Global Positioning System problem is based upon ranging (reference 1). An artificial constellation of twenty four satellites 12,000 miles above the earth in six planes that cross the equator at 53 degrees and that circle the earth each twelve hours. A ground based atomic clock provides the universal time standard, which sets the atomic clocks on the satellites. Since only a less precise quartz clock is practical for the hand held receiver an additional beacon is required to determine the quartz clock error.

Each satellite is continually determining its position by ranging on ground beacons. In turn each satellite is transmitting a signal containing its identification, position and time of transmission.

The hand held receiver must process signals from four satellites. This provides the basis for solving a problem with an equation for each satellite and your four unknowns. Your unknown position is defined as x_s , y_s , z_s and your unknown time error is t_e . Define one satellite "a" with known x_a , y_a , z_a and measured time of signal to travel from the satellite to receiver is t_m . The following ranging equation using the law of the right triangle is developed from the law of the right triangle and c is the speed of light.

$$(x_a - x_s)^2 + (y_a - y_s)^2 + (z_a - z_s)^2 = (c(t_m + t_e))^2$$

Signals from three other beacons defined as "b", "c" and "d" provide the four equations to solve for the three dimensions of your unknown position and your quartz clock error. The GPS receiver uses your last known position as a trial solution in an algorithm that solves for the unknowns. Students are asked to develop and program their own algorithm to solve this problem of intersecting spheres.

c. Efficiency of the Ideal Fuel Burning Engine

Engineering students for more than a century have learned that the Carnot cycle represents the ideal or most efficient heat engine. This Carnot efficiency is a function of the absolute temperatures of the hot reservoir T_{hot} and cold reservoir T_{cold} .

$$\text{Carnot Efficiency} = (T_{hot} - T_{cold}) / T_{hot}$$

The author recognized that the basis for this efficiency is not understood by students. He starts by confirming the Carnot equation by combining the 1st and 2nd law with the definition of efficiency which is the ratio of net work to input heat.

He then notes that the Carnot engine is based on constant temperature heat source and heat sink. Nature provides a virtually infinite heat sink so T_{cold} is a constant. However, the hot reservoir are the combustion products that release heat over the entire temperature from T_{hotmax} down to T_{cold} . The variable hot side temperature means that calculus rather than just algebra is required to determine the efficiency of an ideal fuel burning engine.

The resulting efficiency of the ideal fuel burning engine which is called the Wicks cycle is a function of T_{hotmax} and T_{cold} and is called the Wicks efficiency (reference 2).

$$\text{Wicks efficiency} = 1 - T_{\text{cold}} \ln(T_{\text{hotmax}}/T_{\text{cold}}) / (T_{\text{hotmax}} - T_{\text{cold}})$$

A typical text book problem is to find the efficiency for an engine with $T_{\text{hot}}=2500$ R and $T_{\text{cold}}=500$ R. The Carnot efficiency is 80 %. However, a fuel burning engine with a maximum combustion product temperature of T_{hotmax} would have an ideal or Wicks efficiency of 59.76%.

d. Cooking a Hotdog

A hotdog typically starts at a uniform temperature of 10 C and is placed in boiling water at 100 C. During the cooking process the temperature T is a function of both position r and time t and is thus defined as $T(r,t)$. The parameters that determine the temperature response are convection coefficient at the skin, conductivity, heat capacity, density and radius.

The problem can be modeled by a partial differential equation (PDE) in space and time. This PDE is called the diffusion equation . It can be solved with much difficulty by separation into a time function and a space function and a resulting infinite series.

It can also be solved graphically by means of a Heisler chart which plots normalized dimensionless temperature vs a dimensionless time and family of curves for different Biot numbers.

Alternatively, it can be modeled as a finite element problem. This reduces the advanced classical mathematics to arithmetic. This allows nonlinear effects such as the temperature dependent conductivity to be represented. Students are assigned to solve the linear problem by each method. They are then shown the additional power of the finite element model to solve the nonlinear problem which can yield a more accurate result.

e. Rocket Failure Probability

The launch of the space shuttle Challenger on a cold morning in February, 1986 was a disastrous failure. A field joint that used a teflon o-ring for sealing failed. The resulting flame ignited the liquid hydrogen tank. All on board perished.

The investigation attributed the disaster to the low temperature and the corresponding transition of the o-ring from flexible to brittle at the lower temperature, which was also outside the experience envelope. The renowned physicist Richard Feynman dramatically demonstrated the embrittlement of teflon at low temperatures at a congressional hearing. He placed a sample in ice water. A bending action broke the teflon. The subsequent William Rogers Commission report suggested that failure was certain because of the cold weather.

The author teaches that the cold weather increased the risk, but failure was not certain. One field joint failed, but five others worked. Thus, the failure was statistical. Assume there was an identical launch on a parallel pad under the same conditions. What would be the probability of success?

Using the statistics from the failed mission, the probability of failure for each joint was $1/6$, and the probability of success for each joint was $5/6$. Since all joints must work the probability of success is given by:

$$\text{Probability of success} = (5/6)^6 = .334897976$$

Now assume that each vehicle had eight rather than six field joints and one failed on the original launch. The probability of launch on the second is given by:

$$\text{Probability of success} = (7/8)^8 = .343608919$$

The probability of success is seen to have increased slightly. A general success equation can be developed by induction as a function of the number of joints and for a single failure on the original vehicle:

$$\text{Probability of success} = ((n-1)/n)^n$$

Probability of success was shown to approach a constant number of .367879441 as n approached infinity. It was then recognized that this number is the inverse of e . Thus the rocket risk assessment led to the unexpected result of a new method (at least new to the author) of determining the base number for the natural logarithm.

$$(n/(n-1))^n = 2.718281828 = e \quad (\text{as } n \text{ goes to infinity})$$

This discovery is an example of how certain natural constants such as π and e are pervasive in nature. They can occur in unexpected ways and situations.

f. Nuclear Reactor Equation Differential Equation and Solutions

Nuclear reactors are based upon a chain reaction in which the uranium or plutonium atom is split by a neutron. The result is two fission product atoms and two or three more neutrons. The generation time for these neutrons are about .00001 seconds. Thus it would

at first seem that a controllable chain reaction would be impossible. A slight unbalance of a unity multiplication factor would result in an instant explosion or instant shutdown.

The controllable chain reaction is possible because 7 out of 1000 neutrons come from the decay of fission products with half lives ranging from 1 to 80 seconds. Thus, a critical reactor is subcritical on prompt neutrons. The additional 7 out of 1000 delayed neutrons are necessary for the chain reaction. Thus, the rate of power increase or decrease is retarded and controlled by the delayed neutrons.

The author had the opportunity to ask the pioneering nuclear physicist Hans Bethe who discovered delayed neutrons. Dr Bethe answered that it was Enrico Fermi who also proclaimed that “If god did not want us to have a controlled nuclear chain reaction he would not have given us delayed neutrons”. Fermi and Leo Szilard subsequently demonstrated a controllable neutron based chain reaction under Chicago Stadium in December, 1942.

Many of the students in the Engineering analysis course are employed as engineers in local nuclear related industries. The author developed a simplified model of the nuclear chain reaction (reference 3). One ordinary differential equation (ODE) is for neutrons and the coupled ODE is for delayed neutrons. The equations are combined and solved for a variety of conditions such as supercritical period and scram response. The results quantify the effect of delayed neutrons on reactor dynamics.

3. Conclusion

The author had never had the opportunity to teach an engineering mathematics course prior to the subject engineering analysis course. The course provided some flexibility in the topics to be covered. This has allowed for the introduction of the previously described special interest topics along with the traditional engineering mathematics. Students have shown enthusiasm for the variety of material as demonstrated by favorable evaluations and comments.

References

1. Frank Wicks, “Where on Earth?”, Mechanical Engineering, cover and pp 34-37, July, 2002.
2. Frank Wicks, “The Thermodynamic Theory and Design of an Ideal Fuel Burning Engine”, Proceedings of the Intersociety Energy Conversion Engineering Conference, pp 474-481, 1991, Boston.
3. Frank Wicks, “Formulation and Application of an Equation for Low Power Reactor Operations and Experiments”, Transactions of the American Nuclear Society, volume 45, page 32, 1983, San Francisco.

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