# A New Approach to Teaching and Learning Statics

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## Introduction

As engineers need to be increasingly flexible in their careers and adjust to an ever-widening range of technologies, a firm command of basic engineering subjects, such as mechanics, is increasingly important. Such a command must include the ability to apply mechanics. Along with many instructors, we are often disappointed with the extent to which students are able to use mechanics in the analysis and design of real systems and structures which they confront in their subsequent education [1].

Our approach to helping students use mechanics is consistent with the ideas put forth by Diana Laurillard [2] who argues that in higher education we ask students to learn a way of *viewing and representing the world*. In mechanics this way of viewing the world involves mathematical symbols that represent interactions between parts of mechanical systems and their motions and deformations. We take failure to relate the symbol to that which it represents (relating the "sign" to the "signified" in Laurillard's parlance) as underlying much of the difficulty that students have in applying mechanics.

Our instructional approach is also strongly rooted in the idea that students learn new things by building upon what they already know [3]. New ideas should be presented so that students can build upon their existing ideas. Finally, different students favor different learning styles [4]. Instruction typically shortchanges students who are visual and sensing learners, as compared to those who are verbal or intuitive learners. Moreover, students can learn by interacting with each other and with instructors. Activities that facilitate learning in a variety of modes enable more students to succeed.

For these reasons, we contend that the initial study of Statics needs to be refocused *away* from machines and structures. Students often have trouble envisioning the forces between inanimate bodies, e.g., between relatively rigid contacting parts of a machine. When the forces are not real to students, Statics is an exercise in mathematics for them: manipulating variables that have no physical counterparts. Instead, students should first work with forces (and couples) that they can, indeed, perceive. This includes forces and couples that students exert with their own hands, as

well as forces and couples that are evident by virtue of the perceivable deformations and motions associated with them. Indeed, there are a number of aspects of Statics, such as interactions between connected members, which are better understood with reference to motion. Besides, forces and couples are measured in engineering practice by the motions or deformations they cause, and Statics is to be used ultimately in circumstances in which the motion of objects, or their deformation and failure, is in question. While it may be intellectually satisfying to professors, we think that limiting Statics to rigid, unmoving bodies, hampers student learning.

Students will also be able to build on what they know if the individual concepts of Statics are decoupled from one another and presented sequentially. This is preferable to confronting students immediately with problems that draw on many concepts, although eventually they must be able to solve such problems as well. While we follow the traditional approach to Statics insofar as moving from two- to three-dimensional problems and from single to multiple bodies, we do so in a much more staged fashion.

Accordingly, we have revised our teaching of Statics so that all major concepts - forces, moments, couples, static equivalency, free body diagrams and equilibrium – are introduced separately in sequence, using simple objects on which students can exert forces and couples or considering the balance of the human body. We even take students to the point where they are considering problems featuring multiple bodies that must be separated, while still focusing on forces that students can readily appreciate. Only later do we address the more complex subject of how bodies interact in different ways through connections. This decoupled approach focusing on readily comprehensible forces has been followed in lectures, homeworks, and exams.

More specifically, with this approach we address a wide range of topic in Statics, for example: the necessity for the presence of a force and a couple at a fixed support, the moment due to a force in three-dimensions, various types of connections, when two hinges provide support each needs only to exert a force but no couple, two-force bodies, tipping versus sliding in friction, and distinct internal forces or torques in a body with multiple loads along its length. In this paper we describe how students can explore some basic concepts of Statics in ways that make the forces readily evident. In the companion paper [5], we describe approaches for implementing this approach, augmented with PowerPoint Presentations and Concept Questions, in ways that promotes significant class interaction and discussion.

## **Couples and Static Equivalency**

Statics is a significant step beyond mechanics as typically taught in physics courses for a number of reasons. A primary one is the importance of rotational equilibrium, and the inclusion of moments of forces and couples. We view a couple as a convenient representation for a collection of two or more forces which has a net tendency to rotate a body, but not to translate it. We use the term couple for such a collection of forces between bodies, with the goal of distinguishing it from the moment about a point due to a force. The notion of a couple is tied, in turn, to the concept of static equivalency. Our approach is to enable students to encounter these concepts with physical objects in way that makes these quantities completely real and allows their

understanding to grow. Part of this growth involves seeing the same concept in apparently distinct physical circumstances.

After students already have experience with the concept of the moment due to a force about a point, we offer students a first direct contact with the concept of the couple. As seen in Figure 1, students confront an elongated body lying on a table in the orientation shown. The rubber bands are hooked to the body, and their remote ends are fixed. The goal for the students is to cause the body to rotate about its center (as evidenced by both rubber bands stretching equally). A series of questions guides them to consider the various means of doing this. For example, the first question is: Where would you apply a single force to produce this motion of the body?

After discussion and attempting to do so, it becomes clear that no single force will do. Then they are asked whether there is more than one way of producing the motion and what is the minimum number of forces. They are asked whether forces parallel to the main member will work (they can use the two short pegs oriented perpendicular to the bar), and how the magnitude of the forces changes as the positions change. Finally, they are told to take the nut driver and produce the motion. It becomes clearer that the effect of this single interaction between the nut driver and the nut is equivalent to various different combinations of two or more forces. Notice this approach allows students to deal with forces that are perceived by the senses of touch (forces they apply) and vision (forces stretching the rubber bands).



Figure 1a

Figure 1b

Figure 1c

Figure 1. Concept of a couple as a combination of forces with a net tendency to rotate.

As an aside, we point out that the member displayed in Figure 1, is part of a L-shaped object with various attachments which can be disassembled into two parts. We have designed this object to facilitate a number of other activities that are part of our instructional approach.

Static equivalency revolves around the interplay between forces and couples. To complement the mathematical formulation of this concept, we seek physical embodiments. We try to convey the idea that statically equivalent combinations of loads can produce similar motions, deformations, or reactions. A first approach relies on comparing the effects of changes of load on the motion of a rigid object (Figure 2a). The bar shown lies on in the horizontal plane and has a short peg running through the center. First, a load is applied at the peg perpendicularly to the bar causing it to translate. (The frictional resistance is nearly constant along the length of the bar.) Next, the force is applied near the right end, and the bar is seen to pivot counterclockwise (and to translate). This reinforces the idea that effect of a force changes when it is moved perpendicularly to its line of

action. However, the original, purely translational, motion can be restored if a couple is added to the moved force. This couple can be applied by fingers at the pegs or by the nutdriver and at different locations (Figure 2b and 2c). This reinforced the notion that the couple can be applied in different ways and at different points, with the same effect on the motion of a relatively rigid object.



Figure 2a

Figure 2b



Figures 2. Statically equivalent loads produce equivalent motions.

One can also show that statically equivalent loads are different ways of maintaining equilibrium under the same applied load. In Figure 3, the rod is balanced against gravity in the vertical plane with fingers at the center. Then the rod is balanced by hanging from a rubber band which is off center, and hence requires a couple. Again, the independence of the method of couple application (pair of fingers on the pegs, versus nutdriver on the nut) is made evident, in this instance by the unchanging stretch of the rubber band.



Figure 3a

Figure 3b

Figure 3c

Figures 3. Statically equivalent loads balance same applied load (gravity).

As another example, in a configuration familiar to all students, a spoon is held horizontally by the hand (Figures 4a and 4b). The loads outside the hand are clearly unchanging, but the spoon can still be supported by the hand in ways that differ as to the details. With free body diagrams, we are able to argue that the hand in both cases is supplying a force and a couple.

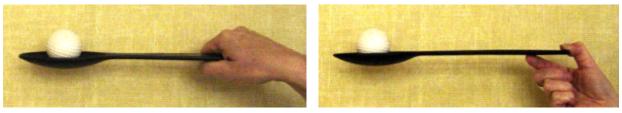
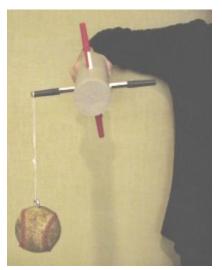


Figure 4a

Figure 4b

Figures 4. Distinct, statically equivalent loads to balance the spoon.

That statically equivalent loads can cause similar deformations is made evident in Figure 5. A deformable bar, approximately 60 cm long and 10 cm in diameter, is pierced by two stiff thin rods perpendicularly to the axis to form a cross. The deformable bar is held fixed at one end, and a weight is hung from end of one of thin rods, causing the bar to bend and to twist. The same weight is then hung from the deformable bar itself, although now a couple also needs to be applied to the cross to produce the twisting of the bar.





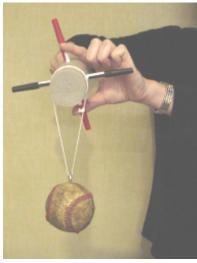
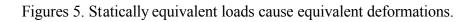


Figure 5b



## Incorporation of Friction Conditions into Statics Problems

The modeling of friction in static situations appears to be highly challenging to many students. Put succinctly, students feel obliged to set the friction force *equal* to the normal force times the friction coefficient ( $\mu$ ), even if it is inconsistent with equilibrium. Several exercises, which exploit the sense of touch and balance, have been helpful in allowing students to sort out this apparent conflict.

A student is asked to quantify the friction force between a book and the table on which it rests. It is not uncommon for a student to respond that the friction force is  $\mu$  times the normal force or *Proceedings of the 2003 American Society for Engineering Education Annual Conference & Exposition* 

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weight. The book is then placed at the student's hands which are positioned next to each other facing upward. When asked about the friction force between the book and the hands, the student now correctly responds zero. Then, as the instructor presses tangentially with increasing force on the book, the student recognizes that friction force (which her hands are exerting) is increasing. Our sense is that students have trouble distinguishing between the concept of the maximum possible frictional resistance and the actual prevailing friction force. But it is clear that the interaction between inanimate objects (book and table) is more difficult for students to characterize than an interaction in which the student is one of the participants.

A second exercise involves the question of how large a force a person can apply to a wall (or to push a car). This problem is an extremely rich one, and it draws nicely on the necessity of *simultaneously* satisfying the equilibrium and the friction conditions. After some discussion, students deduce that the maximum force that can be applied is affected by friction, by the orientation of the body and by the strength of the individual. The goal of the exercise is for students to understand how to assess these different effects quantitatively.

Students are told to plant their feet in one position, and to exert different amounts of force on the wall. Students come to appreciate that a larger force can be exerted when they move their body center closer to the wall (Figures 6a and 6b). Provided their feet are sufficiently close to the wall, students come to see that the force they can exert is limited by the tendency for them to tip backward. Then, students are asked to step farther away from the wall. If the feet are planted far enough away, then the student sees that the force can now be limited by the other effects: by slippage at the feet or by inadequate strength to maintain the body in position (Figure 6c).



Figure 6a

Figure 6b

Figure 6c

Figures 6. Force applied to wall is affected by positions of center of gravity and feet.

The class then considers the free body diagrams of the situation and the conditions imposed by equilibrium and by the friction law between the feet and floor. From these considerations,

students are lead to rationalize the ranges of parameters in which the three effects of friction, body position and body strength play a decisive role.

### Summary

In this paper, we have argued that fluency in using Statics to understand and quantify real machines and structures requires students to have made connections between the quantities and variables of Statics and their physical counterparts. These connections cannot be made when the forces of interest are not real to students. Accordingly, we have revised our instructional approach to focus on forces which are readily perceivable, including those exerted directly by students and those which cause motions or deformations that are visually perceivable. Several examples of this approach have been explained in this paper. In a companion paper [5], we describe how this approach is augmented with PowerPoint Presentations and Concept Questions in ways that promote interaction and discussion in the classroom.

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