2006-197: COST EFFECTIVE MECHANICAL DESIGN IN RELATION TO MATERIAL AND STRUCTURAL RIGIDITY AND DESIGN ALTERNATIVES

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Cost Effective Mechanical Design in Relation to Material and Structural Rigidity and Design Alternatives

Abstract

The integration of cost effective design techniques into Engineering and Engineering Technology programs is necessary in order to provide graduating Engineers the necessary skills to become more immediate contributors to the goals and profits of their chosen companies. Example teaching and analysis techniques are discussed which will allow faculty to introduce and reinforce cost effective design into Mechanical Engineering courses. These techniques can be applied to other courses as well.

Introduction

In over 30 years of work with engineers, designers and architects it has been observed that many have difficulty determining the proper combination of material and shape to meet design and cost criteria. There are a number of recognized methods available to evaluate the structural rigidity or integrity of design components. However many design professionals lack the ability to incorporate cost effectiveness into their design. How do you get the most rigidity for the least cost and in many cases at the lowest weight? That is, to say, "the most bang for your buck". Graduating Engineering and Engineering Technology students do not have a good grasp of this concept and it is suggested that faculty have the responsibility to introduce and nurture cost effective design. It is the purpose of this paper to demonstrate one method of introducing this concept to Mechanical Engineering students in typical Strength of Materials courses. Rigidity will be defined considering both the material and the shape of the cross section. Different combinations of material and shape will be evaluated. A simple decision matrix will be shown as one method of comparison and the entire concept will be pulled together. This concept should be incorporated into a variety of other Engineering and Engineering Technology courses in order to demonstrate and reinforce its application.

Definitions

The rigidity (or stiffness) of a material is simply a measure of the amount of deflection, δ, that occurs when a simple cantilevered beam is exposed to some applied load as shown in Figure 1.

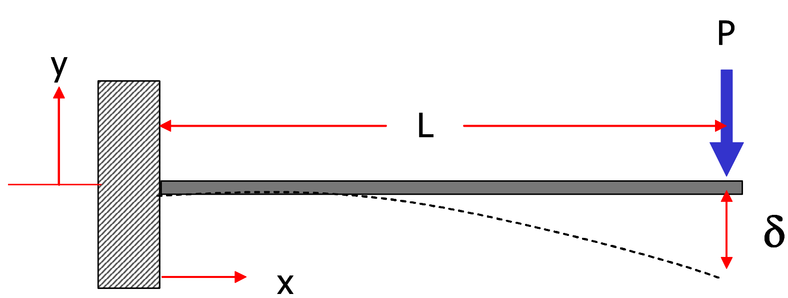


Figure 1 A simple cantilevered beam showing an applied load at the end of the beam and depicting the amount of deflection.

The amount of deflection, δ, is a function of both a material property and the cross sectional shape of the beam. The material property is the Modulus of Elasticity, E, of the material being used and can be determined by a simple tension test or found in published literature. Normally the Modulus of Elasticity is a constant for each specific metal but can vary by molecular weight in polymers. The shape property is the Moment of Inertia, I, of the cross sectional shape which can be determined using a number of mathematical and graphical methods or found in published literature.

The Modulus of Elasticity is simply the slope of the elastic portion of the stress strain curve from a tension test as shown in Figure 2.

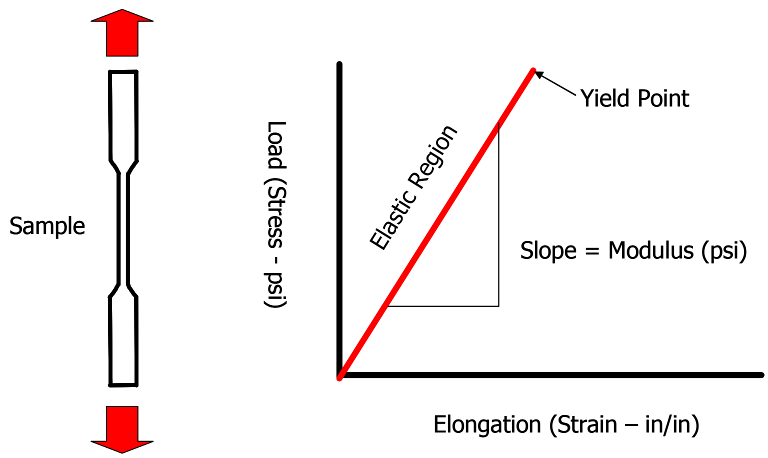


Figure 2 Determination of the modulus of elasticity from a simple tension test.

The modulus of various materials is different but is normally constant for alloys of the same material. Examples are shown in Figure 3.

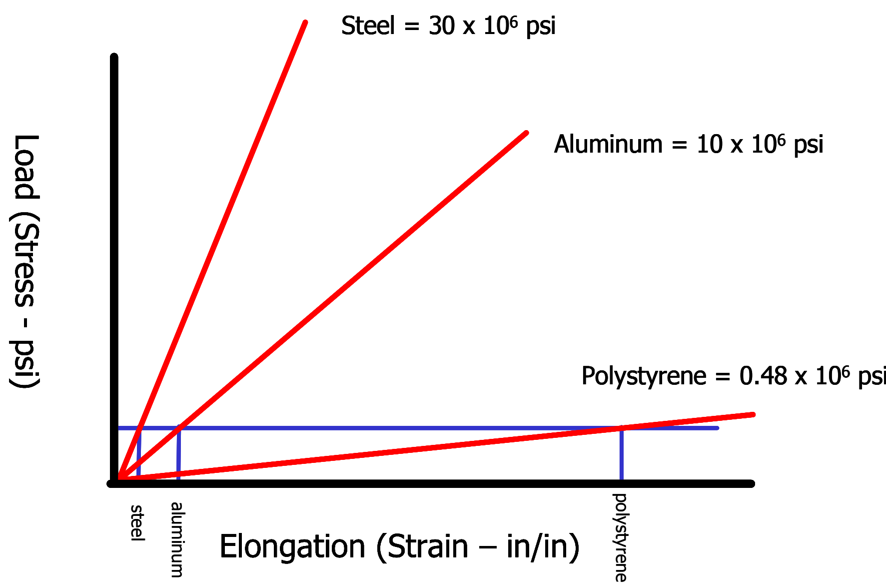
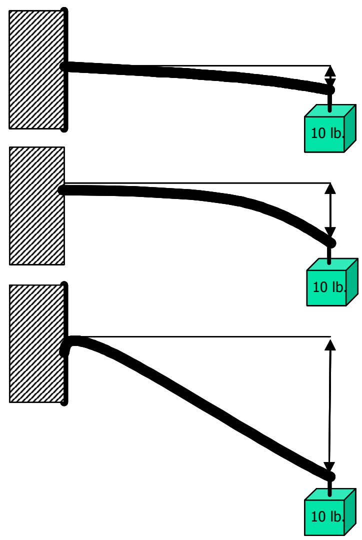


Figure 3 A comparison of the slope of the elastic portion of the stress strain curve for steel, aluminum and polystyrene.

Intuitively, looking at the slopes in Figure 3 it can be seen that for a given stress level each material will deflect differently simply based on the Modulus of Elasticity which is a material property. In looking at the deflection of cantilevered beams made from the three different materials the following can be observed.



Deflection	***Material***	***Modulus of Elasticity***
1"	***Steel***	***30 x 10⁶ psi***
3"	***Aluminum***	***10 x 10⁶ psi***
60"	***Polystyrene***	***0.48 x 10⁶ psi***

Figure 4 A comparison of modulus materials with the deflection of a simple cantilevered beam. Assuming all pieces are equivalent size and shape and neglecting the weight of each member.

The material rigidity and density can be compared using the specific stiffness ratio which is the ratio of the Modulus of Elasticity to the density.

$$\text{Specific Stiffness} = S_p = E/\rho$$

For example in comparing steel to aluminum the following is observed:

Steel = 30×10^6 psi / 0.28 lbs/in^3
Aluminum = 10×10^6 psi / 0.10 lbs/in^3

This gives an equivalent ratio for each material. Thus both steel and aluminum are very similar if looking only at the amount of stiffness per pound of weight, not considering the shape of the cross section. The ratio for polystyrene is significantly lower demonstrating that polymers are much less rigid than most metals.

As will be discussed, the shape effect must always be considered and can be expressed as the Moment of Inertia, I. This is the capacity of a cross-section to resist bending. It is always considered with respect to a reference axis such x-x or y-y. It is a mathematical property of a section concerned with the cross-sectional area and how that area is distributed about the reference axis. This reference axis is usually a centroidal axis. This Moment of Inertia is an important value which is used to determine the state of stress in a section, to calculate the resistance to buckling, and to determine the amount of deflection of a member.

Here is an example to consider. Consider the two 1”x 4” solid bars shown in Figure 5 and determine which will deflect more and why.

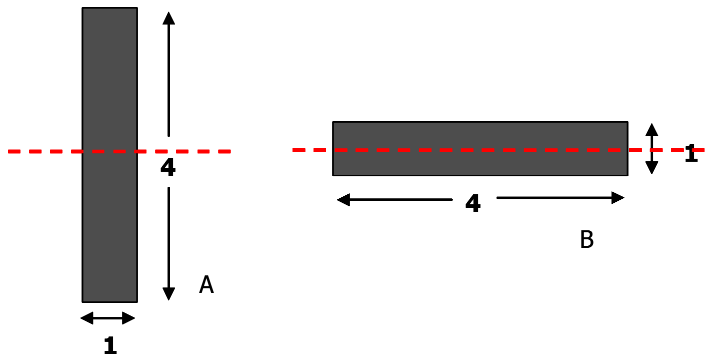


Figure 5 Example depicting the variation of the moment of inertia of the same cross section oriented relative the horizontal axis.

Bar A, has its 1” dimension parallel to the horizontal axis, while bar B has its 4” dimension parallel to the horizontal axis. The Moment of Inertia for a rectangular cross-section in relation to the horizontal centroidal axis can be calculated using the following equation:

$$I_x = \frac{bh^3}{12}$$

Equation 1 Moment of Inertia for a rectangular cross section

In Equation 1, b is the length of the base and h is the height of the cross section. Other shaped cross sections require different equations to calculate their moments.

Using this equation and substituting values for the respective base and height dimensions it is seen that bar A has a moment value of 5.33 in^4, while bar B has a value of 0.33 in^4. Both bars are the same size and shape however they are oriented differently. Bar A is significantly more rigid (16 times !) than bar B. Although the cross sectional area of both bars is the same it is distributed differently above and below the horizontal axis which results in a greater stiffness for bar A. Intuitively, envision a 2” x 8” piece of dimension lumber. It is clear that its rigidity when oriented with the 2” dimension oriented parallel to the horizontal axis (like a floor joist) is significantly greater than with 8” dimension parallel to the horizontal axis.

Analysis

Combining the material property, E, and the shape property, I, into one equation gives the total deflection, δ, of the cantilevered beam as shown in Figure 6 and Equation 2.

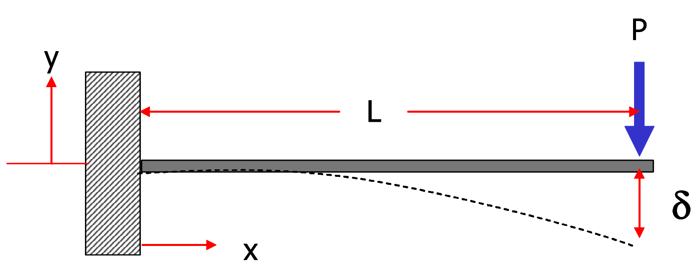


Figure 6 A cantilevered beam used in the determination of the deflection of the end relative to the applied load.

$$\delta = \frac{-PL^3}{3EI}$$

δ = Deflection at the end
P = Load
L = Length
E = Modulus of Elasticity (material property)
I = Moment of Inertia (shape factor)

Equation 2 Equation to predict the deflection of the end of a cantilevered beam related to the modulus of elasticity, the moment of inertia and the applied load.

δ is the total deflection for the cantilevered structure pictured in Figure 6. Look again at the 1" x 4" steel, aluminum and polystyrene bars to see that the total deflection of each can be calculated. Assume that the applied load, P, is 500 lbs., the length of the cantilevered bar is 36", and that the bar has the 1" dimension parallel to the horizontal axis. Substituting these values into the equation above gives the following results for each beam:

δ_{steel} = - (500 lbs)(36 in.)^3 / 3(30 x 10^6 psi)(5.3 in^4) = - 0.0489 in

$\delta_{aluminum}$ = - (500 lbs)(36 in.)^3 / 3(10 x 10^6 psi)(5.3 in^4) = - 0.1467 in

$\delta_{polystyrene}$ = - (500 lbs)(36 in.)^3 / 3(0.48 x 10^6 psi)(5.3 in^4) = - 3.0560 in

This reveals that the aluminum bar deflects 3 times as much as the steel bar of the same shape. This also demonstrates that polystyrene has a huge deflection (60 times greater!) and is probably not a consideration in most designs.

Consider only the steel and aluminum bars. How can the deflection of the aluminum bar be made the same as, or similar to, the steel bar? The answer can be determined by rearranging Equation 2 and solving for I to obtain Equation 3.

$$I = - PL^3 / \delta(3)(E)$$

Equation 3 Rearrangement of Equation 2

Substitute in the value of E for aluminum, 10 x 10^6 psi. Use a load of 500 lbs and a length of 36" (same as before) and set the deflection, δ, of the aluminum bar to be - 0.0498 in. The result is 15.61 in^4.

Thus, to get a deflection of the aluminum bar equal to the deflection of the steel bar

(- 0.0498”), an aluminum bar must have a Moment of Inertia, I, equal to 15.61 in^4. Look at the various cross sectional shapes available for aluminum and determine which shape has a Moment of Inertia equal to or greater than 15.61 in^4. One example of a shape that meets this criteria is a 4” x 6” aluminum I-beam which has an I value of 21.99 in^4. This would give a total deflection of - 0.0354 in which is 0.0139 in less than the steel bar.

Using this method, equation, and E and I values, we can also look at other combinations of material and cross section shape in order to arrive at the lowest defection characteristics for the lowest density.
In the above comparison, the 1”x 4”x 36” steel bar would weigh 40.32 lbs and the 4”x 6”x 36” aluminum I-beam would weigh only12.09 lbs. Thus an aluminum I-beam has greater rigidity than the steel bar and weighs 28.23 lbs. less! This evaluation practice is very common in the aerospace and transportation industries but can be used in just about any situation. Obviously, for differently supported beams and parts the deflection equations are different and can be found in any Strength of Materials text or reference book.

Cost will be the next consideration. This must be introduced in almost all comparative design processes. This is the part that is missed in many typical courses. There are many methods that can be used to evaluate the cost factor. One simple process it to look at the material cost per pound in the above example. This gives the following:

The cost of bulk steel is approximately \$0.25 per pound and bulk aluminum is approximately \$1.00 per pound.

1”x 4” x 36” x 0.28 lbs /in^3 = 40.32 lbs
\$0.25 / lb x 40.32 lbs = \$10.08 for the steel bar

4” x 6” x 36” aluminum I-beam weighs 4.03 lbs / ft = 12.09 lbs
\$1.00 / lb x 12.09 lbs = \$12.09 for the aluminum I-beam

Therefore, the aluminum I-beam gives less deflection and costs only \$2.01 more than the steel bar base on bulk prices.

Another process is to consider the actual costs per foot of the above bar and I-beam. Quoted price for 1”x 4” 1020 cold rolled steel bar is \$16.60 per foot and the 4”x 6” aluminum I-beam is \$16.14 per foot. The cost for a 36” sections of each is \$49.80 for the steel bar and \$48.42 for the aluminum I-beam. So on an actual cost basis, the aluminum I-beam is less expensive, significantly lighter weight and more rigid. This demonstrates a example of cost effective design. In an aircraft, automobile or boat this weight difference is significant because the weight factor is one of the most important design criteria. Ultimately, cost is almost always the major consideration in the real world and should be well understood by students.

To evaluate the above factors, a simple selection model such as a decision matrix (Table 2) can be used. To create a decision matrix the following steps should be followed:

- Establish the Design Criteria. In our example the Design Criteria might include deflection, weight, cost, size, and safety. Many other criteria can also be included.
- Assign a weighting factor to each of the Design Criteria based upon the relative importance of each. Since there are five design criteria in this example, a five point

scale (1 thru 5) could be used. A weighting factor of 1 would be the least important and 5 would be the most important.

- Develop a list of Design Alternatives or, in our case, material and shape combination options: 1"x 4" steel bar, 1"x 4" aluminum bar, 1"x 4" polystyrene bar and 2"x 6" aluminum I-beam.
- Establish a Rating Factor which indicates the performance of the Design Alternative with respect to each Design Criteria. These could be as follows:

Rating Factor	Definition
1	Failure
2	Low Performance
3	Average Performance
4	High Performance
5	Outstanding Performance

Table 1 Rating factors

- Use the above five Rating Factors to rate each Design Criteria for each of the Design Alternatives.
- Multiply each Rating Factor by each of the Weighting Factors and obtain a Value for each Design Alternative
- Finally the best design alternative is determined by summing the respective Value Columns within the decision matrix. The column with the highest sum is the best choice.

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		1" x 4" Steel Bar		1" x 4" Aluminum Bar		1" x 4" Polystyrene Bar		2" x 6" Aluminum I-beam	
Design Criteria	Weight Factor	Rating	Value	Rating	Value	Rating	Value	Rating	Value
Deflection	**5**	5	25	4	20	1	5	5	25
Weight	**4**	1	4	5	8	5	20	5	20
Cost	**5**	4	20	3	25	2	10	5	25
Size	**2**	4	8	4	8	3	6	3	8
Safety	**3**	4	12	4	12	1	3	4	12
Totals			**69**		**73**		**44**		**90**

Table 2 Decision Matrix

Clearly the aluminum I-beam is the best option. The aluminum bar has the second highest total so it might be an alternative if the deflection meets the design standard.

Summary

To summarize, we as educators have the responsibility to teach Engineering and Engineering Technology students all aspects of design. Based on my years of experience in industrial design and manufacturing, we fall short of the goal in many courses because we do not introduce or emphasize the economics and design alternative factors. Many students graduate, begin a job and are shocked by the fact that the design they must create for their company is not always the best mechanical design. It is the acceptable design that is the lowest cost or the lightest weight. In several situations it is possible to do both as shown in the example above.

These are just a few examples of how these factors can be introduced in typical mechanical design courses. There are other ways to give students the complete picture and I urge you to seek them out and be sure that your students understand these concepts. They will be more effective Engineers and will be immediate contributors to the company of choice.

References

Mott, R.L.; *Applied Strength of Materials* 4th edition; Prentice-Hall: Upper Saddle River, NJ Columbus, OH 2002