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Using a Finite Element Stress Analysis Program to Enhance Learning In a Machine Design Course

Dr. Robert G. Ryan
California State University, Northridge

Abstract

Engineering educators generally agree that students should learn modern computational tools related to their engineering discipline. The recent trend has been to integrate the use of finite element analysis tools throughout the curriculum, instead of delaying their introduction to the senior year for use in senior design or elective courses. However, time constraints dictate that computational analysis tools must be used efficiently in engineering core courses so that teaching of fundamentals is not compromised. This paper describes a shaft design project assigned in a junior level machine design course, and compares the teaching effectiveness of a traditional analysis by "hand" versus a computational approach using COSMOS/Works. Both approaches are found to be beneficial for student learning, and the experience suggests that a finite element analysis tool complements, but does not replace, traditional analysis techniques in the classroom.

Introduction

The use of modern computational tools in an undergraduate curriculum is a necessary component of today's engineering education. Introduction of new topics or techniques inevitably creates a tension between an engineering program's need to stay current versus the traditional coverage of the fundamentals of engineering science and design. The use of solid modeling (CAD) and finite element analysis (FEA) tools can require the introduction of new courses and/or a significant allotment of time within existing courses. Integration of the use of FEA software in core mechanical engineering courses can potentially absorb valuable time to accommodate the "learning curve" associated with CAD/FEA tools.

In 2001, concerns about this learning curve led the Mechanical Engineering department at California State University, Northridge to adopt the SolidWorks family of design and analysis tools as the standard for our curriculum. The most important reasons for this choice was the relative ease of use of the solid modeling CAD package (SolidWorks), and the close integration among the analysis tools (COSMOSWorks, FloWorks, etc.) and the SolidWorks interface. Recent changes to our lower division curriculum have been designed to introduce this family of tools in freshman and sophomore classes, in order to support the ultimate goal of integrating CAD and FEA tools throughout the curriculum. This is a significant change from our past practice which tended to delay the application of CAD and FEA tools until the senior design capstone course.

Engineering educators tend to worry (rightly!) that students will tend to become enamored with impressive and colorful outputs of FEA tools and ignore the underlying physical principles which govern a design problem. One approach for resolving this dilemma is to require students to solve a problem using two or more approaches, thus providing experience in validating the results produced by an FEA package and an understanding of what an analysis tool can and cannot do. The importance of understanding classical analysis techniques for becoming a "well educated" – as opposed to "well trained" – user of FEA tools was clearly stated by Jolley [1] et al. This paper presents a specific example of an assignment used in a junior level machine design course which is designed to illustrate the advantages and disadvantages of a finite element analysis versus a classical analysis of a simple shaft and gear system. Ultimately, a library of similar assignments will be created to enhance other core mechanical engineering courses such as heat transfer, fluid mechanics, and kinematics.

Problem Definition

A shaft design problem was selected to introduce the use of COSMOSWorks for stress and deformation calculations and for frequency analysis. This problem, with slight variations, has been used as a "design project" in CSUN's junior level machine design course for several years, with the students' calculations being done by "hand" or using a spreadsheet. Requiring students to approach this problem using COSMOSWorks in addition to the classical calculations allows students to compare the accuracy and efficiency of the two methods for arriving at an appropriate shaft design. A schematic of the shaft and gear system is shown in Figure 1. Students are given values of the gear radii, widths, axial locations, and pressure angle, shaft length, operating rotational speed, and the amount of transmitted power. Students are required to specify the size and material for the shaft.

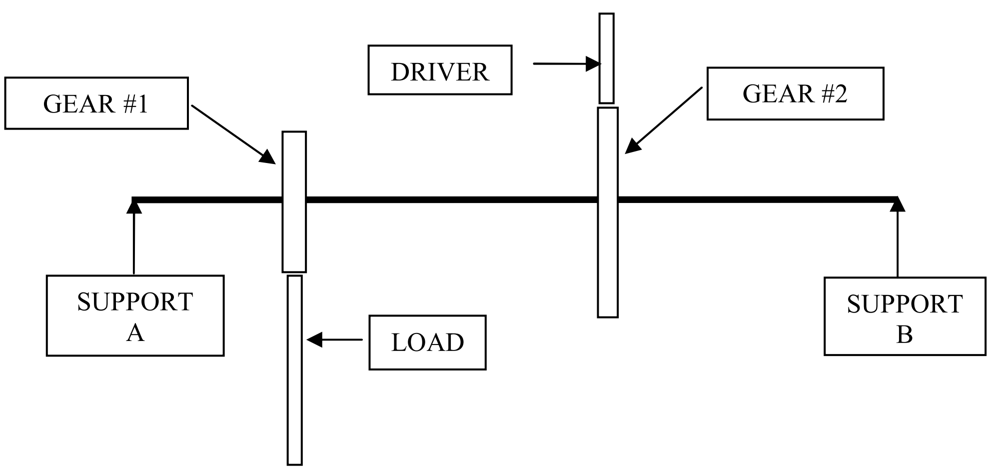

Figure 1 Schematic of Shaft/Gear System

To simplify the problem, students are asked to use a shaft with a uniform diameter along its length. This diameter must be specified to satisfy three criteria:

- The shaft should have an infinite fatigue life with a safety factor of at least two.
- The shaft should have deflections which are smaller than recommended limits.
- The critical speed of the shaft should be at least two times the operating speed.

Spreadsheet Analysis Procedure and Results

The analysis procedures for calculating fatigue safety factor, shaft deflections, and critical speed are relatively straightforward for a shaft of uniform diameter. Automating these calculations on an Excel spreadsheet allows the effect of diameter change on each of the design criteria to be quickly evaluated. The following is a summary of the required calculations:

- The transmitted power, pressure angle, and gear radii are used to find the radial and tangential components of the forces acting on the gears.
- Treating the shaft as a beam with simple supports at the shaft ends with concentrated loads acting at the midplanes of the gears, the bending moment acting along the length of the shaft is calculated.
- The location of the maximum stress on the shaft must be identified. In this case it will be adjacent to the gear with the smallest radius. Knowing the bending moment and torque acting on the shaft at this point, and a desired fatigue safety factor, the ASME shaft design equation [2] can be used to calculate a shaft diameter.
- Simply supported beam equations are used for calculating shaft deflections. Deflections at the ends (i.e. at the bearing supports) and at the gears can be checked versus recommended limits [2].
- The Rayleigh method [2] is used to estimate the critical speed of the shaft. To implement this technique, the static loads due to the weights of the gears are applied to the shaft, and the resulting deflection is calculated. The natural frequency of the shaft is then found from the gear weights and the deflections at the gear locations. If the shaft weight is not negligible compared to the gear weights, it should be included as well.

Values cited in Table 1 were calculated using an Excel spreadsheet, augmented by functions written in Visual Basic for Applications (VBA). The VBA functions were used to evaluate the beam equations for bending moments and deflections. Results are shown for diameters of 1.25 inches and 2.00 inches, using AISI 1020 steel for the shaft material. The smaller diameter is sufficient to provide infinite fatigue life for the shaft but has excessive deflections at the bearing and gear locations. The larger diameter satisfies all design criteria.

Criterion	Recommended Min Or Max [2]	D = 1.25 inches	D = 2.00 inches
Fatigue Safety Factor (ASME Eqn.)	2 (depends on design)	2.34	9.15
Maximum Von-Mises Stress, psi	< Sy to prevent yield	9,219	2,251
Deflection @ Gear #1 (inch)	0.005	0.0256	0.00390
Deflection @ Gear #2 (inch)	0.005	0.0183	0.00278
Angular Deflection @ Gear #1 (degrees)	0.03	0.153	0.0229
Angular Deflection @ Gear #2 (degrees)	0.03	0.180	0.0275
Angular Deflection @ Support A (degrees)	0.04	0.239	0.0363
Angular Deflection @ Support B (degrees)	0.04	0.223	0.0339
Critical Speed to Operating Speed Ratio	2	2.81	7.12

Table 1 Results of Spreadsheet Analysis

Several assumptions made for this analysis should be mentioned. As noted previously, reactions at the bearings and gear forces are applied as point loads on the shaft, and the bearings act as simple supports. The effect of stress concentration (e.g. for keyways for gear attachment, etc.) was not considered, although this is not a difficult addition for common geometric discontinuities for which published data are available. The effect of shaft weight on the critical speed was neglected for simplicity. It's also worth noting that analyzing more complex shaft geometries, such as a stepped shaft with different diameters, makes the evaluation of shaft deflections somewhat more difficult.

COSMOSWorks Analysis and Results

Analysis of this problem using COSMOSWorks involves a significantly different thought process. First, a solid model of the shaft/gear system must be created in SolidWorks. A number of approaches were considered for the model, including a simpler model including only the shaft, and an assembly including the shaft, gears, and bearing supports. After some experimentation, the model shown in Figure 2 was chosen for the analysis. There was no attempt to make a realistic model of the gear teeth since the focus here is on the shaft design. The teeth in the model were created to provide an edge for the application of the gear forces at the pitch radii of the gears, and the overall gear geometry is adequate for modeling the gear weights. Generally it is best to make the SolidWorks assembly file available to the students for this assignment so that they can concentrate on the use of COSMOSWorks.

Figure 2 SolidWorks model of shaft/gear system

Once the solid model is created in SolidWorks, the COSMOSWorks environment is entered by simply clicking a tab. To set up the analysis, first the loads and restraints on the assembly must be defined. For this problem, fixed restraints were established at the bottom face of the bearing supports, and tangential and radial gear loads were applied at the tip of the appropriate tooth on each gear to match the loading arrangement shown in Figure 1. Gravity forces are also imposed in the appropriate direction. Next, relationships among the parts in the assembly must be established to specify how forces are transferred between each "mate" in the assembly. A "bonded faces" condition is appropriate at the gear/shaft interface, but the best condition to use at the bearing supports is less clear. The effect of this boundary condition on the results is discussed below. The next step is mesh the assembly. A fairly standard mesh was used, although

COSMOSWorks does provide a number of ways to customize the mesh to enhance accuracy in areas of interest.

Two types of COSMOSWorks analysis "studies"are relevant to this design problem: a static analysis produces values for stresses, displacements, and related quantities; and a frequency analysis evaluates resonant frequencies, which can be related to the critical speed.

The static analysis produces a variety of color output plots of the distribution of parameters such as Von-Mises stress, strain, deflection, and deformation. These plots can be customized by changing aspects of the plot format or the specific parameter that is displayed; for example, different stress components can be displayed on separate plots. A "design check" plot allows the display of static safety factors based on different values of strength and static failure criteria. These plots and additional information related to maximum and minimum values of key parameters, mesh parameters, material properties, and the numerical solvers used to generate the solutions are compiled in a convenient report file for archiving analysis results.

Assuming that no hand calculations are used up front to estimate a shaft diameter (e.g. the ASME design equation), analysis studies for a range of diameters would have to be run in order to iterate to the desired solution. Thus a complete reliance on a FE analysis could be a somewhat inefficient approach for achieving an optimal design – a good lesson for students to learn! Two design criteria need to be evaluated from the FE analysis – shaft displacement and fatigue safety factor. Shaft deflection is easily viewed on its output plot, and fairly precise numerical results can be read from the plot by customizing the range of values associated with the color bands. Fatigue safety factor, though, is not easily determined since the bending and torsional loads are applied statically in the FEA solution, while the fatigue safety factor is based on a failure envelope which involves an alternating bending stress (due to the shaft's rotation) and a constant torsional stress [2]. Producing separate plots of the bending stress and shear stress would allow the extraction of stress values to perform a hand calculation of fatigue safety factor, but this is clearly awkward at best. One could argue that this is a moot point since the shaft deflection criteria is the governing constraint for this design, and in fact this is typical for this type of shaft design problem. However, it is important to emphasize to students that the definition of the safety factor used in a design must be consistent with the applicable failure theory.

The results of the spreadsheet analysis shown in Table 1 indicate that a shaft diameter of 2.00 inches will satisfy the shaft design criteria. For comparison purposes, FEA results for a diameter of 2.00 inches were generated for a number of different boundary conditions defined at the bearing supports. Definition of these conditions and their effect on the results may be summarized as follows:

- A "bonded faces" condition [3] allows no movement between the shaft and bearing support. This condition resulted in large bending loads near the shaft ends, reduced stresses in the middle of the shaft (by a factor of 2), and lower deflections in the middle of the shaft (by a factor of 3). (Comparisons are relative to the values from the spreadsheet analysis.)

- A “surface contact pair” condition [3] allows for a small amount of relative motion between the shaft and bearing support, but does not allow these surfaces to penetrate each other. This condition produced a good shaft stress distribution, but tended to over-estimate deflections in the middle of the shaft by about 50% because of the non-zero shaft deflections adjacent to the bearing supports. It also appeared that there was a discontinuity in the deflections at the bearing support-shaft interface, despite the prohibition against surface penetration. This may be dependent on the fineness of the mesh, and requires further investigation outside of the scope of this paper.

- The value of Young’s modulus was artificially reduced for the bearing supports for some runs so that they would deflect much easier than the shaft. These conditions were designed to minimize the bending moment exerted at the bearing supports, which should be consistent with physical reality when using a proper bearing design. The best results were obtained when the Young’s modulus of the supports was reduced by a factor of 100, and the bonded faces condition was used. Key values agreed with the spreadsheet analysis, and these boundary conditions are also consistent with physical expectations (minimal bending moment at the bearings, minimal deflection at shaft ends).

Deflection and stress plots for the run with a 2 inch shaft with bonded faces and flexible bearing supports are shown in Figures 3 and 4. The values for shaft displacement at the gear locations and the maximum Von-Mises stress on the shaft (adjacent to Gear #1) are in excellent agreement with the spreadsheet analysis values (within the precision that the plot values can be read, much easier from a color reproduction of the plot). Equally important from a pedagogical viewpoint is the strong effect that the end boundary conditions have on the stress and deflection results – another important lesson for students!

Additional “frequency studies” were run to evaluate the fundamental natural frequency of the shaft. For these studies, the gear loads are not imposed, but the rest of the boundary conditions are the same. Two studies were run – one which accounted for shaft weight, while the other assumed a “weightless” shaft (done by modifying the shaft density to near zero) to allow for direct comparison with the spreadsheet analysis. COSMOSWorks produces the first five (as a default) fundamental frequencies in Hertz. Multiplying the lowest (fundamental) frequency by 60 gives the critical speed in rpm. Table 2 summarizes the results.

Calculation Method	**Critical Speed (rpm)**
Rayleigh Method (spreadsheet), weightless shaft	8540
COSMOSWorks, weightless shaft	7800
COSMOSWorks, with shaft weight	7240

Table 2 Comparison of Critical Speed Values

The critical speed found from Rayleigh’s method is 10% higher than the corresponding value produced by COSMOSWorks. This is consistent with the expectation that Rayleigh’s method over-estimates the critical speed by a few percent [2]. While the inclusion of shaft weight in the

COSMOSWorks calculation is simple (easier than excluding it), the Rayleigh method becomes somewhat ponderous because the shaft weight must be accounted for by modeling it as several discrete lumps along the shaft length.

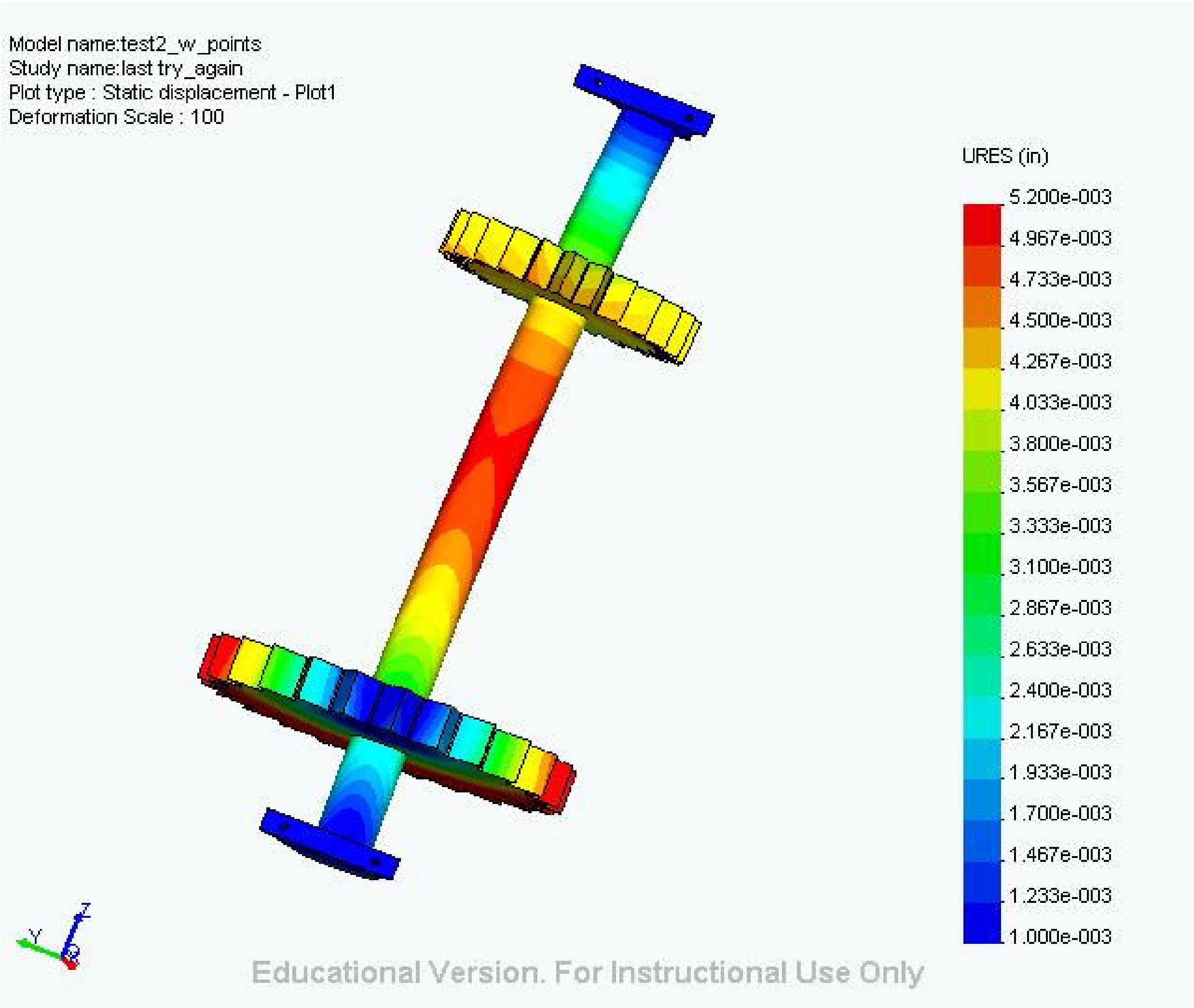

Figure 3 Deflection Plot for 2 Inch Diameter

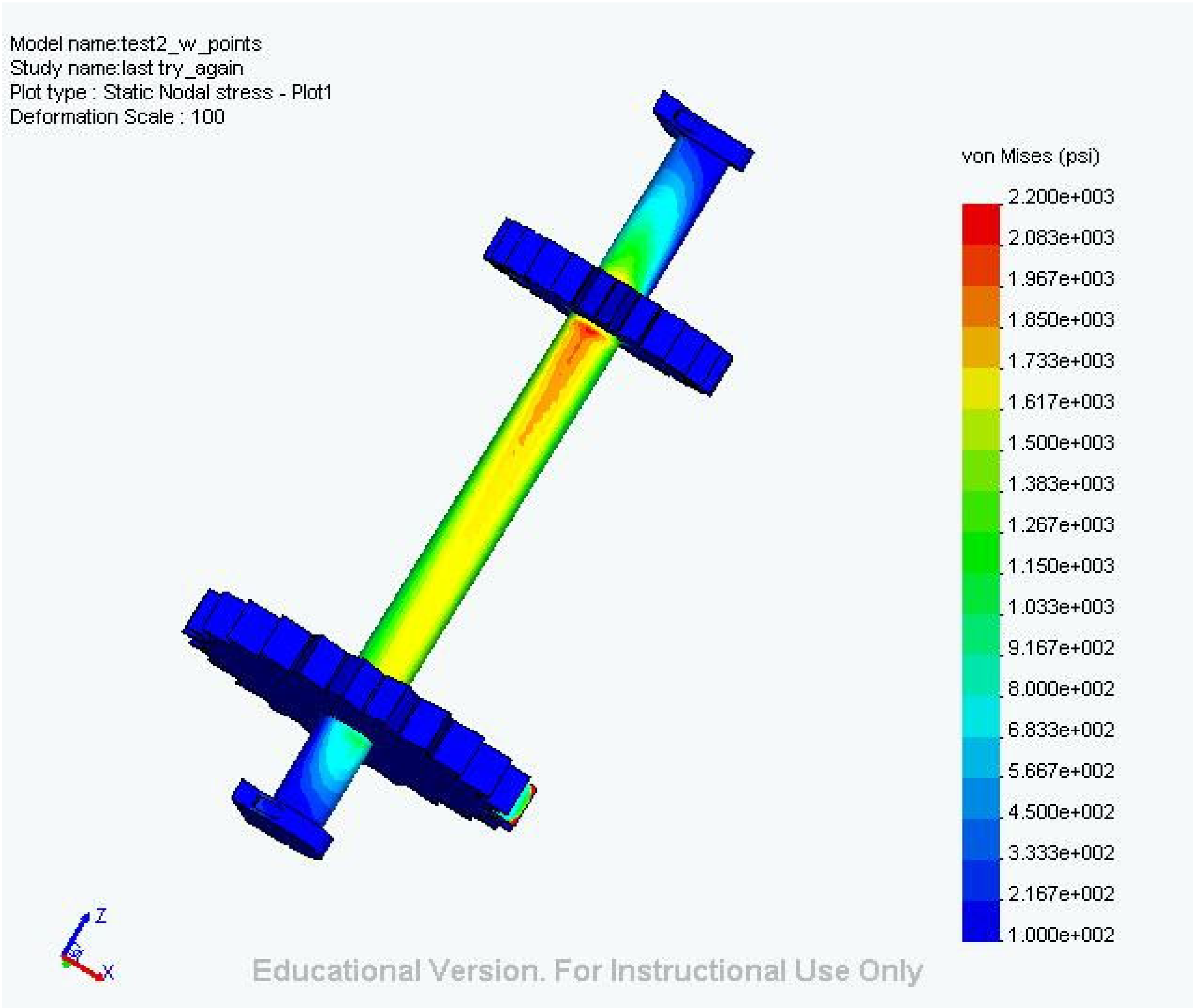

Figure 4 Von-Mises Stress Plot for 2 Inch Diameter

Summary and Conclusions

A shaft design problem has been created to enhance students' understanding of the use of a CAD/FEA software package for machine design applications. Of particular importance is the proper use of a FEA tool for achieving design solutions consistent with classical techniques. Calculations for a typical case have been presented to demonstrate the comparison of the FEA results with values found using a spreadsheet analysis. The expected learning outcomes of this design problem are the following:

- Using a CAD/FEA tool is not a replacement for a fundamental understanding of the physics governing a design. CAD/FEA tools are extremely valuable for performing complex calculations and providing useful graphical representations of these results. Every engineer must have an understanding of how these tools are used in modern engineering practice.

- Shaft design calculations using classical techniques are reasonably straightforward for a shaft of uniform diameter. Shafts with steps or shoulders for axial placement of gears, pulley, etc. make the calculation of shaft deflections significantly more difficult. CAD/FEA tools can handle complex geometries with little difficulty.

- Using a FEA tool to find the optimum shaft diameter can be somewhat inefficient without an initial hand calculation to get at least a reasonable estimate of the diameter required to provide infinite fatigue life (this generally gives the lower limit of shaft diameter which can be used at the point of maximum bending moment). The static analysis performed in COSMOSWorks does not allow the direct evaluation of fatigue safety factors, and thus must be supplemented by hand calculations. This will generally be true for any type of part design involving fatigue life. It is important for students to understand the definitions of different safety factors so that they are not misused.

- Boundary conditions have a significant effect on calculated stress and deflections. Classical techniques are usually limited to fairly simple boundary conditions. The effect of different boundary conditions can be tested fairly easily with an FEA tool. However, an understanding of the physical problem is required to apply the correct boundary conditions for a given design problem.

- The frequency analysis performed by a FEA tool is an efficient way to find fundamental resonant frequencies of a mechanical assembly. The results obtained for a uniform shaft are consistent with Rayleigh's method.

Complete implementation of this assignment in the junior level machine design class is scheduled for the Spring 2004 semester. It is expected that the use of CAD/FEA for design projects of this nature throughout the undergraduate curriculum will not only enhance learning, but will generate students' interest and curiosity in exploring additional design modifications. Students' perception of the value of this assignment will be measured via a self-assessment questionnaire which will be completed by all students at the end of the semester.

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Robert G. Ryan

Robert Ryan received his PhD degree in mechanical engineering from the University of California at Los Angeles. He is currently an Assistant Professor in the Mechanical Engineering Department at California State University, Northridge. He is the coordinator for the department's Measurements Laboratory and Thermo-fluids Laboratory.