# 2006-2368: AN INNOVATIVE APPROACH TO A CLASSIC DESIGN PROJECT

**James Penrod, University of Dayton**

James P. Penrod, P.E., is Assistant Professor of Mechanical Engineering Technology at the University of Dayton. He holds a B.S. and M.S. in Mechanical Engineering from the University of Cincinnati and a M.S. in Aerospace Engineering from the University of Dayton. He is a member of the ASME, SAE, and ASEE.

# An Innovative Approach to a Classic Design Project


## Abstract

Each year many gear reducers are designed by students of mechanical engineering and mechanical engineering technology in their machine design courses. In many instances, these design projects offer little challenge other than perhaps the volume of work that must be completed. This paper outlines a gear reducer design project that was created to be intentionally challenging. This was accomplished by requiring the gear reducer to have concentric shafts, double reduction, standard diametral pitches, *and* an exact train value that just might be a prime number. The project is structured around American Gear Manufacturers Association (AGMA) design procedures for spur gears, and is patterned after a line of commercially available gear reducers having a similar configuration and performance. The design specifications for the project, a methodology to satisfy the technical requirements, and a spreadsheet tailored to analyze the student's gear designs are presented and discussed.


## Introduction

At the University of Dayton, junior mechanical engineering technology students are required to take a course in machine design titled *Design of Mechanical Elements*. The content is typical of many introductory courses in machine design with the content divided into three groupings. The first grouping contains topics that are a review and extension of those covered in an introductory strength of materials course. These topics typically include transverse shear stresses in beams, combined stress, symmetrical bending of beams in 2-planes, transformation of stress, principal stresses, and Mohr's circle. The second grouping covers failure theories and fatigue. The third grouping contains topics from machine design like the design of shafts, spur gears, springs, fasteners, shear pins, keys, couplings, seals, roller bearings, and plain bearings. The text for the course is *Machine Elements in Mechanical Design* by Mott[1]. As an integrating capstone-type experience, the students are required to complete a design project.

## Project Description

The subject and scope of the project given in the Design of Machine Elements course varies somewhat depending on the instructor of the course. However, the project is typically the design of either a power transmission or a power transmission component. Either approach provides an excellent vehicle for the student to integrate many of the course topics into a single design project.

This paper focuses on a portion of a design project where the students are required to design a gear reducer. In this project the student must design two stages of spur gearing, the input, intermediate, and output shafts, select appropriate bearings and (where applicable) seals for each shaft, and configure the assembly. The project has now been successfully used three times and has been refined after each use. The current design specifications for the project include:

1. The gear reducer shall have a specified exact train value.[a]
2. The gear reducer shall have concentric (i.e. in-line) input and output shafts.
3. The input and output shafts shall turn in the same sense.
4. The gear reducer shall include two stages of reduction (i.e. double reduction).
5. The gear reducer shall have a specified input power rating for an 1150 rpm input.[a]
6. The gear reducer shall provide 20,000 hours of life at 99% reliability for the rated input power and speed.
7. The gear reducer should be considered a commercial enclosed unit.
8. All gears shall be spur.
9. All gears shall be of standard diametral pitch.
10. All gears shall have a pressure angle of 20°.
11. All gears shall be AGMA Q10.
12. All gears should be made of Grade 1 carburized and case-hardened steel.
13. The face width of all gears shall be the AGMA nominal recommended value of $12/P_d$.
14. The rim of all gears should be sufficiently thick that the rim thickness factor is unity.
15. The contact ratio of each stage shall exceed 1.2.
16. Each pair of mating gears shall run free of interference without undercutting the teeth.
17. Backlash should be provided in accordance with AGMA standards.
18. All shafts shall be preferred/standard U.S. customary sizes.
19. All shafts shall be fabricated from cold-drawn AISI 1144.
20. All shafts shall be designed to a minimum safety factor of two.
21. All keyways shall be the appropriate standard U.S. customary sizes.
22. The input and output shafts shall include appropriately sized keyways.
23. All bearings shall be standard U.S. customary sizes and **not** selected from the tables provided in the text.
24. Only rolling element bearings should be used.
25. All ancillary components such as seals, fasteners, etc., shall be standard U.S. customary sizes.
26. Assume that the prime mover and load impart uniform loadings on the gear reducer.
27. Assume that the gear reducer is used in an application where no overhung load exists.

There are a variety of origins from which the specifications were derived. Many of them simply reflect established standards. In some cases, the specification is intended to duplicate the conditions that exist in a line of commercially available gear reducers that the design was loosely patterned after. In other cases, specifications were added to promote some uniformity in the student's designs and, in turn, simplify grading. In yet other cases, constraints on the design were added to elevate the technical complexity and difficulty of the project. Clearly, the extensive list of specifications limits the open-endedness of the design. The creative aspect of the project is embodied in the ingenuity that is required to optimally satisfy the design requirements.

As is apparent from the design specifications, the project normally involves designing not only gearing, but also the shafts and layout of the gear reducer assembly. Additionally, bearings and

---

[a] The values for train value and input power are different for each student.

other ancillary hardware like seals are selected as part of the project requirements. This paper focuses on the gear design since it is the most interesting and challenging aspect of the project.

**Individualized Train Value and Input Power Specifications**

One feature of the project worthy of note is that unique train values and input power specifications can be assigned for each student and the resulting designs should be relatively similar in size (i.e. the gearing of each student's design should fit a common housing size). This is the case because the train value and corresponding input power requirement were patterned after a line of commercially available gear reducers which are offered in a variety of ratios and power ratings, but share a common housing size.

The line of gear reducers used to develop the individualized specifications was the Link-Belt® Type D (i.e. double reduction), size DDI (i.e. the housing size and style), in-line, helical gear reducers.[b] Technical data for these units are contained on pages F-23 through F-30, F-35, and F-47 of the Link-Belt® catalog[2]. Table 1 provides a summary of the pertinent performance data for models having ratios up to 25.6 and Figure 1 contains a plot of the data with a smooth curve fitted through the data points.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Nominal Train Value | 6.2 | 7.6 | 9.3 | 11.4 | 13.9 | 17.1 | 20.9 | 25.6 |
| Input Power (hp) | 33.5 | 27.4 | 22.1 | 18.3 | 14.8 | 12.4 | 10.1 | 7.91 |

**Table 1:** Performance ratings for Link-Belt® Type D, size DDI, in-line, helical gear reducers for an 1150 rpm input speed.

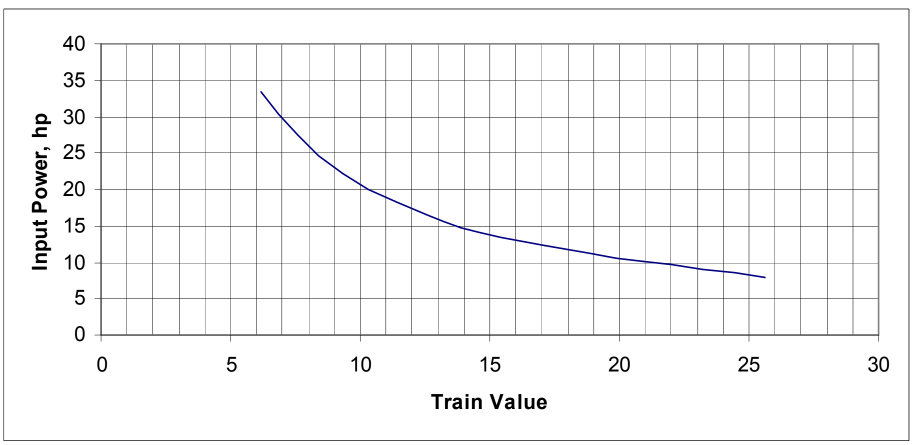


**Figure 1:** Plot of rated input power versus train value for Link-Belt® Type D, size DDI, in-line, helical gear reducers for an 1150 rpm input speed.

From Figure 1, it is possible to extract any number of train value/input power combinations that should produce similarly sized gear trains. For example, it is apparent from Figure 1 that for a train value of 13, the corresponding input power requirement would be 16 hp.

[b] Link-Belt® was acquired by Rexnord® in 1988. In 2005, Rexnord® acquired the Falk® line of gear reducers and as a result, the line of gear reducers that the specifications were patterned after was discontinued.

## Examining the Kinematical Challenge of the Project

The primary difficulty is to satisfy the requirement that the input and output shafts be concentric for an exact train value while using only standard diametral pitch gears. Concentric shafts for a double reduction gear reducer suggests that

$$C_1 = C_2 \tag{1}$$

where $C_1$ denotes the center distance of the first stage of reduction and $C_2$ denotes the center distance for the second stage of reduction. The center distance of each stage is given by the familiar expression

$$C = \frac{1}{2P_d}(N_P + N_G) \tag{2}$$

where $P_d$ is the diametral pitch, $N_P$ is the number of teeth on the pinion, and $N_G$ is the number of teeth on the gear. Using (2), equation (1) can be rewritten as

$$\frac{1}{2P_{d_1}}(N_{P_1} + N_{G_1}) = \frac{1}{2P_{d_2}}(N_{P_2} + N_{G_2}) \tag{3}$$

where additional subscripts are included to distinguish between the two stages. However, (3) can be further rewritten in terms of the velocity ratio for each stage. After some algebraic manipulation, doing so yields

$$\left(\frac{N_{P_1}}{N_{P_2}}\right)\left(\frac{P_{d_2}}{P_{d_1}}\right) = \left(\frac{1+VR_2}{1+VR_1}\right) \tag{4}$$

where $VR_1$ and $VR_2$ denote the velocity ratio of the first and second stage, respectively.[c] Finding a solution to equation (4) is best demonstrated via an example.

Suppose that a student were assigned a train value of 13. A good starting point would be to find the square root of 13 which is approximately 3.61. This value would represent the velocity ratio of each stage if it was split equally. Doing so is likely a mistake though even if the square root of the train value were an integer value (e.g. 4 for a train value of 16). The second stage of reduction must transmit much higher torque. As such, larger gear teeth are desirable in the second stage as compared to those used in the first stage. Because of the in-line constraint, this can only be achieved by placing a disproportionate amount of the total ratio (i.e. train value) in the first stage. Thus, the velocity ratio of the first stage should be somewhat greater than 3.61 and the velocity ratio of the second stage should be correspondingly smaller than 3.61. Please note that without the in-line requirement, it is a trivial matter to satisfy the train value requirement and have arbitrarily large gear teeth in the second stage.

---

[c] In the past, students were not provided with equation (4) and some struggled mightily to produce an acceptable design. The equation is now given to the students, but only after they attempt to develop their own methodology.

Generating combinations of the velocity ratio for each stage to satisfy the train value specification is a simple matter. For instance, suppose the velocity ratio of the second stage is picked to be 3. The corresponding velocity ratio for the first stage would then be

$$VR_1 = \frac{TV}{VR_2} = \frac{13}{3}$$

Inserting this result into equation (4) and simplifying yields

$$\left(\frac{N_{P_1}}{N_{P_2}}\right)\left(\frac{P_{d_2}}{P_{d_1}}\right) = \frac{3}{4} \qquad (6)$$

At this point, one might select the pinions in each stage to have identical numbers of teeth. If this is the case, then (6) becomes

$$\left(\frac{P_{d_2}}{P_{d_1}}\right) = \frac{3}{4} \qquad (7)$$

Thus, the ratio of diametral pitches used in each stage is found. Ideally, the ratio of diametral pitches should be either 3/4, 2/3, or 1/2. Examining Table 8-2 of Mott[1] one sees why. There are eight combinations of standard diametral pitches that produce a ratio of 3/4, ten combinations providing a 2/3 ratio, and fifteen combinations producing a 1/2 ratio. Selecting a ratio of diametral pitches having numerous combinations provides the student with greater flexibility and – in effect – makes the design scaleable so that stress levels and material strengths can be more easily matched.

One problem remains though. That is, to calculate the numbers of teeth on all the gears. For the ratios of 13/3 and 3 with the same number of teeth on the pinions in each stage, the smallest pinion that can be used has 16-teeth. For the second stage, the gear would have 3 x 16 = 48 teeth. However, the number of teeth on the first stage gear would be (13/3) x 16 = 69 1/3. Obviously, this would not work.

Such a difficulty is easily worked around though by using a pinion where – in this case - three can be factored out of the number of teeth. For instance, 18-tooth pinions would result in gears having 78-teeth and 54-teeth in the first and second stages, respectively.

**Gear Design Analysis Spreadsheet**

Numerous calculations are required to analyze the bending and contact stresses in a gear design. Consequently, the spreadsheet depicted in Figure 2 was coded by the author to avoid much of the tedium involved with analyzing each student's design. The students can also benefit from developing a similar tool. As such, the students are encouraged to code their own spreadsheet.

| | |
|---|---|
| Student: | John Doe |

| Design Specifications: | | |
|---|---|---|
| TV = | 13 | |
| $P_{in}$ = | 16 | hp |
| $n_{in}$ = | 1150 | rpm |

| Kinematic Data: | $Q_v$ | $N_P$ | $N_G$ | $P_d$ ($in^{-1}$) | φ (deg) |
|---|---|---|---|---|---|
| Stage 1 | 10 | 18 | 78 | 8 | 20 |
| Stage 2 | 10 | 18 | 54 | 6 | 20 |

| Kinematic Verification: | | |
|---|---|---|
| Train Value? | | OK |
| $C_1 = C_2$? | | OK |
| Standard $P_d$ in Stage 1? | | OK |
| Standard $P_d$ in Stage 2? | | OK |
| Interference in Stage 1? | | OK |
| Interference in Stage 2? | | OK |
| Contact Ratio in Stage 1? | 1.68 | OK |
| Contact Ratio in Stage 1? | 1.65 | OK |

| | Stage 1 | Stage 2 |
|---|---|---|
| Velocity Ratio | 4.33 | 3 |
| Input Torque (in-lb) | 877 | 3800 |
| Input Speed (rpm) | 1150 | 265 |
| Tangential Force (lb) | 779 | 2533 |
| Radial Force (lb) | 284 | 922 |
| Efficiency (%) | 100 | 100 |
| Output Torque (in-lb) | 3800 | 11399 |
| Face Width (in) | 1.5 | 2 |

Inputs for Stress Calculations: (Stage 1)

| | Geometry Factor (J) | Geometry Factor (I) | $s_{at}$ (psi) | $s_{ac}$ (psi) | $C_p$ ($psi^{1/2}$) |
|---|---|---|---|---|---|
| Pinion | 0.32 | 0.108 | 55000 | 180000 | 2300 |
| Gear | 0.415 | 0.108 | 55000 | 180000 | 2300 |

Inputs for Stress Calculations: (Stage 2)

| | Geometry Factor (J) | Geometry Factor (I) | $s_{at}$ (psi) | $s_{ac}$ (psi) | $C_p$ ($psi^{1/2}$) |
|---|---|---|---|---|---|
| Pinion | 0.318 | 0.1 | 55000 | 180000 | 2300 |
| Gear | 0.4 | 0.1 | 55000 | 180000 | 2300 |

| Application Factors: | Stage 1 | Stage 2 | |
|---|---|---|---|
| $K_o$ = | 1.00 | 1.00 | |
| $K_s$ = | 1.00 | 1.00 | |
| $C_{pf}$ = | 0.048 | 0.054 | |
| $C_{ma}$ = | 0.15 | 0.16 | |
| $K_m$ = | 1.198 | 1.212 | |
| $K_b$ = | 1.00 | 1.00 | |
| $v_t$ = | 677 | 208 | ft/min |
| B = | 0.397 | 0.397 | |
| A = | 83.77 | 83.77 | |
| $K_v$ = | 1.11 | 1.07 | |
| $K_R$ = | 1.00 | 1.00 | |
| SF = | 1.00 | 1.00 | |
| $C_H$ = | 1.00 | 1.00 | |

| Life Requirements: | Life = | 20000 | hr |
|---|---|---|---|
| | | $Y_n$ | $Z_n$ |
| $N_{P1}$ = | 1.38E+09 cycles | 0.93 | 0.89 |
| $N_{G1}$ = | 3.18E+08 cycles | 0.96 | 0.92 |
| $N_{P2}$ = | 3.18E+08 cycles | 0.96 | 0.92 |
| $N_{G2}$ = | 1.06E+08 cycles | 0.98 | 0.95 |

Bending Stress Number: ($s_t$)

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 17333 | 13365 | psi |
| Stage 2 | 30860 | 24533 | psi |

Adj. Allowable Bending Stress Number: ($s_{at'}$)

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 51271 | 52626 | psi |
| Stage 2 | 52626 | 53666 | psi |

Contact Stress Number: ($s_c$)

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 122853 | 122853 | psi |
| Stage 2 | 169825 | 169825 | psi |

Adj. Allowable Contact Stress Number: ($s_{ac'}$)

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 160718 | 166230 | psi |
| Stage 2 | 166230 | 170484 | psi |

Strength Verification: (% of appropriate adj. allowable stress number if inequality is satisfied)

| | Stage 1 Pinion | Stage 1 Gear | Stage 2 Pinion | Stage 2 Gear |
|---|---|---|---|---|
| $s_t \le s_{at'}$ ? | 33.8 | 25.4 | 58.6 | 45.7 |
| $s_c \le s_{ac'}$ ? | 76.4 | 73.9 | OVERLOAD | 99.6 |

| | | |
|---|---|---|
| Size = | 15.375 | in |

**Figure 2:** Gear Design Analysis Spreadsheet with $VR_1 = 13/3$, $VR_2 = 3$, and $P_{d2}/P_{d1} = 3/4$.

Notice that certain fields of the spreadsheet are shaded. The shaded fields are those where data must be input. All other fields either contain data consistent with the design specifications or are fields with imbedded formulas. As one can see, using the project specifications and automating most of the calculations means that a minimal amount of input is needed to assess a design. The needed inputs include the assigned train value and input power specifications, the numbers of teeth, diametral pitch, and geometry factors for each gear, and the size factor ($K_s$) for each stage of gearing. As coded, the spreadsheet in Figure 2 requires that the use of standard diametral pitches and the lack of interference be checked manually using Table 8-2 and Table 8-6, respectively, of the Mott[1].

**Examining the Kinetic Challenge of the Project**

The challenge here is for the student to create a design that is most size efficient. Figure 3 defines what is meant by the size of the gearing. Notice that the gear train size equal is the common center distance, plus the outside radii of the gears in both stages.

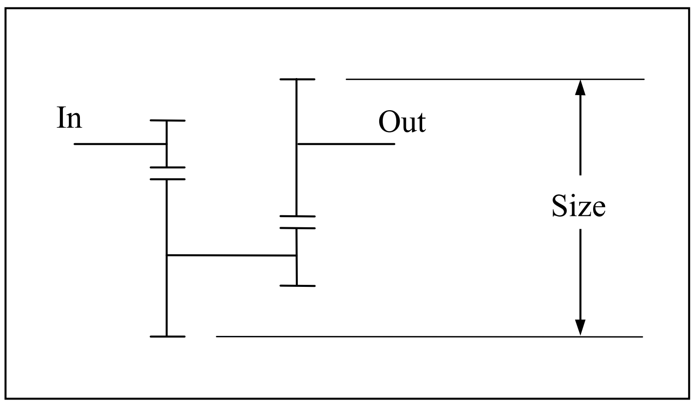


**Figure 3:** Definition of size for the gear train.

Since the gear material/strength is the same for all the gears, minimizing the size of the design requires the students to produce a "balanced" design. By "balanced" it is meant that 1) the bending stress levels in each stage are nearly equivalent, 2) the contact stress levels in each stage are nearly equivalent, and 3) all the design stress levels should almost fully utilize the capability of the gear material. Stated differently, the bending stress number should be nearly equal to the adjusted allowable bending stress and the contact stress number should be nearly equal to the adjusted allowable contact stress.

In Figure 2, it is observed that with regard to both the bending stress and contact stress criteria, the second stage of the design is much more highly stressed. Such a result suggests that more velocity ratio is needed in the first stage to balance the stress levels between the two stages. After several fruitless attempts, the design used in Figure 4 was obtained. In this design the velocity ratio of the first stage and second stage are 5 and 13/5, respectively. Also notice that the ratio of diametral pitches is now 1/2.

| Student: | John Doe |
|---|---|

**Design Specifications:**

| | | |
|---|---|---|
| TV = | 13 | |
| $P_{in}$ = | 16 | hp |
| $n_{in}$ = | 1150 | rpm |

**Kinematic Data:**

| | $Q_v$ | $N_P$ | $N_G$ | $P_d$ ($in^{-1}$) | φ (deg) |
|---|---|---|---|---|---|
| Stage 1 | 10 | 24 | 120 | 12 | 20 |
| Stage 2 | 10 | 20 | 52 | 6 | 20 |

| | Stage 1 | Stage 2 |
|---|---|---|
| Velocity Ratio | 5.00 | 2.6 |
| Input Torque (in-lb) | 877 | 4384 |
| Input Speed (rpm) | 1150 | 230 |
| Tangential Force (lb) | 877 | 2631 |
| Radial Force (lb) | 319 | 957 |
| Efficiency (%) | 100 | 100 |
| Output Torque (in-lb) | 4384 | 11399 |
| Face Width (in) | 1 | 2 |

**Kinematic Verification:**

| | | |
|---|---|---|
| Train Value? | | OK |
| $C_1 = C_2$? | | OK |
| Standard $P_d$ in Stage 1? | | OK |
| Standard $P_d$ in Stage 2? | | OK |
| Interference in Stage 1? | | OK |
| Interference in Stage 2? | | OK |
| Contact Ratio in Stage 1? | 1.74 | OK |
| Contact Ratio in Stage 1? | 1.66 | OK |

**Application Factors:**

| | Stage 1 | Stage 2 | |
|---|---|---|---|
| $K_o$ = | 1.00 | 1.00 | |
| $K_s$ = | 1.00 | 1.00 | |
| $C_{pf}$ = | 0.025 | 0.048 | |
| $C_{ma}$ = | 0.14 | 0.16 | |
| $K_m$ = | 1.168 | 1.206 | |
| $K_b$ = | 1.00 | 1.00 | |
| $v_t$ = | 602 | 201 | ft/min |
| B = | 0.397 | 0.397 | |
| A = | 83.77 | 83.77 | |
| $K_v$ = | 1.11 | 1.06 | |
| $K_R$ = | 1.00 | 1.00 | |
| SF = | 1.00 | 1.00 | |
| $C_H$ = | 1.00 | 1.00 | |

**Inputs for Stress Calculations: (Stage 1)**

| | Geometry Factor (J) | Geometry Factor (I) | $s_{at}$ (psi) | $s_{ac}$ (psi) | $C_p$ ($psi^{1/2}$) |
|---|---|---|---|---|---|
| Pinion | 0.365 | 0.118 | 55000 | 180000 | 2300 |
| Gear | 0.44 | 0.118 | 55000 | 180000 | 2300 |

**Inputs for Stress Calculations: (Stage 2)**

| | Geometry Factor (J) | Geometry Factor (I) | $s_{at}$ (psi) | $s_{ac}$ (psi) | $C_p$ ($psi^{1/2}$) |
|---|---|---|---|---|---|
| Pinion | 0.33 | 0.1 | 55000 | 180000 | 2300 |
| Gear | 0.4 | 0.1 | 55000 | 180000 | 2300 |

**Life Requirements:** Life = 20000 hr

| | | | $Y_n$ | $Z_n$ |
|---|---|---|---|---|
| $N_{P1}$ = | 1.38E+09 | cycles | 0.93 | 0.89 |
| $N_{G1}$ = | 2.76E+08 | cycles | 0.96 | 0.93 |
| $N_{P2}$ = | 2.76E+08 | cycles | 0.96 | 0.93 |
| $N_{G2}$ = | 1.06E+08 | cycles | 0.98 | 0.95 |

**Bending Stress Number: ($s_t$)**

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 37277 | 30923 | psi |
| Stage 2 | 30678 | 25309 | psi |

**Adj. Allowable Bending Stress Number: ($s_{at'}$)**

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 51271 | 52761 | psi |
| Stage 2 | 52761 | 53666 | psi |

**Contact Stress Number: ($s_c$)**

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 159422 | 159422 | psi |
| Stage 2 | 163637 | 163637 | psi |

**Adj. Allowable Contact Stress Number: ($s_{ac'}$)**

| | Pinion | Gear | |
|---|---|---|---|
| Stage 1 | 160718 | 166778 | psi |
| Stage 2 | 166778 | 170484 | psi |

| Size = | 15.33333 | in |
|---|---|---|

**Strength Verification: (% of appropriate adj. allowable stress number if inequality is satisfied)**

| | Stage 1 Pinion | Stage 1 Gear | Stage 2 Pinion | Stage 2 Gear |
|---|---|---|---|---|
| $s_t \le s_{at'}$ ? | 72.7 | 58.6 | 58.1 | 47.2 |
| $s_c \le s_{ac'}$ ? | 99.2 | 95.6 | 98.1 | 96.0 |

**Figure 4:** Gear Design Analysis Spreadsheet with $VR_1 = 5$, $VR_2 = 13/5$, and $P_{d2}/P_{d1} = 1/2$.

From Figure 4 it is apparent that the revised design has much better balanced stress levels than the design depicted in Figure 2. The contact stress number as a percentage of the adjusted allowable contact stress is very high and nearly identical for all four gears with values ranging between 95.6% and 99.2%. The bending stress numbers as a percentage of the adjusted allowable bending stress have a greater variation and a lower utilization of the material capability. It is entirely possible that through subsequent iterations the design could be further improved.

## Conclusions

Technical challenge can be added to a gear reducer design project by requiring that a gear reducer have concentric shafts, double reduction, standard diametral pitches, and an exact train value. Finding an optimal design is made even more difficult by requiring that all the gearing possess identical material strengths. Individualized design specifications that yield comparably sized results can be created by basing them after performance specifications for a line of commercially available gear reducers. Because the gear design process is inherently iterative, investing time to code a spreadsheet to analyze the stresses in the design is a good investment of time. A spreadsheet specialized to the design specifications has been coded by the author and is available from the author.

## References


1. Mott, R. L., Machine Elements in Mechanical Design, 4e, Prentice-Hall, Upper Saddle River, New Jersey, 2004
2. Link-Belt and Stearns Power Transmission Products, Catalog LB 1300, PT Components, Inc., Indianapolis, Indiana, 1982.