Flowing your way through equations:
Putting the decisions in the hands of the students

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Abstract
In the teaching of thermodynamics and heat transfer, there are two subject matters that baffle and bewilder students, obscuring the education process. In thermodynamics it is property evaluation and in heat transfer it is transient conduction. Property evaluation becomes a mass of tables and interpolation. Transient conduction is several different sets of differential equations and dimensionless numbers that look like a bunch of z's and w's all strung together. In an attempt to clarify this for the students a set of flowcharts and decision trees have been designed to guide the selection of the appropriate model for both property evaluation and transient conduction. This further fosters the solution methodology that is emphasized in both courses. It also emphasizes a pattern to problem solving that is essential for successful engineering. In the paper that follows, the methodological approach to both these befuddled topics is outlined.

Introduction
Some students look back on their courses and recall only the mundane; for instance the response to a question of a 5-year graduate concerning what they remember of thermodynamics, might be the interpolation and not the modeling decisions they were making with regards to the properties. The same may be said of transient heat conduction, the alum may only recall using dimensionless numbers with different characteristic length, but not why the problems were solved in that manner. The essentials of engineering education include the ability to make informed modeling decisions during problem solving. To this end, several flowcharts and decision trees have been designed to clarify these concepts to the students with regards to these to topics.

Thermodynamic Property Evaluation
Students need to begin to formulate decisions on modeling very early in their engineering curriculum. Thermodynamics, taken in either the sophomore or junior year, is probably the first course for many students where these decisions are expected to be made by the individual and probably one of the greatest reasons that students initially dislike the course. For instance, students must be able to decide when something can be modeled as an ideal gas. This is typically the only equation of state that they are familiar with when they enter the course and would prefer to apply it for all circumstances. To assist students in learning decision making, we have found the flowchart in Appendix 1 to be very helpful. The flowchart is used in conjunction with the information presented in Appendix 2, to complete the learning of property evaluation. The flowchart emphasizes the thought process students should be following as they approach thermodynamics

problems and are making modeling choices. The flowchart aids in developing the decision making processes that must go into solving engineering problems. The use of the flowchart also assists students who are more inclined to be visual learners.

Appendix 2 shows a summary of equations used in property evaluation, with reference to the textbook for the course. This handout clarifies the conditions for certain assumptions, for instance what equations to be used when allowing specific heat to vary with temperature for a substance modeled as an ideal gas.

Heat Transfer Transient Conduction

In heat transfer, students are faced with the application of not only mathematics as they have seen in previous courses, but ordinary and partial differential equations: an intimidating prospect for students. One of the most powerful tools in heat transfer analysis is non-dimensional numbers. Specifically, in transient conduction, the Biot number and the Fourier number are significantly used in analysis. A key feature in evaluating the Biot and Fourier numbers is using the appropriate characteristic length for the problem. It has been the authors' experience that this decision is one with which students struggle. The characteristic length is typically dependent on the derivation and solution of the energy equation. Typically, this is derived and shown to the students; however, after an hour and half lecture with the magic of mathematics, students are often left wondering what they need to know and what they can take away and use for problem solving (i.e, doing the homework). The flow chart in Appendix 3 comes into play at this point. The solution methodology that is emphasized shows that when faced with a one-dimensional transient heat conduction problem, the simplest approach would be to use the lumped capacitance model. This also provides the easiest solution. If the Biot number fails the litmus test, then the flow chart indicates a move on to the next simplest solution, the one-term approximation. Finally, before trying to use the full series solution of the PDE, the possible use of the semi-infinite media solution is considered. The handout presented in Appendix 4, gives the same information as the flowchart, with a few more details and emphasis on boundary conditions.

Student Feedback

Two sets of students were surveyed with regards to the use of the flowchart for thermodynamics property evaluation: students at the University of Portland in ME 331, Introduction to Thermodynamics, a junior level course consisting of mechanical and electrical disciplines and graduate students at Michigan State University taking ME 802, Advanced Classical Thermodynamics. In the graduate course, the flowchart is used during the review of undergraduate thermodynamics in the beginning of the course. The survey form can be found in Appendix 5. Overall comments were very positive. From the undergraduate students written comments were minimal. Students either felt that approach was very logical and helpful or the visual method didn't appeal to them and they preferred the handout in Appendix 2. This is attributed to the different learning styles of students as to which they prefer, thus reinforcing the authors approach to supply to different forms of the same handout. The numerical scores are found in Table 1.

Table 1
Undergraduate student response

ME 331	**Q1**	**Q2**	**Q3**	**Q4**
Average	2.83	4.03	3.27	3.57

One interesting result of the numerical data is that the average response to question 3 (Do you think the flow chart approach to the decision making associated with learning engineering problem solving is useful?) is higher then question 1 (Did you find the flow chart helpful in identifying the substance model to be used in the evaluation of thermodynamic properties for different substance?). And the response to question 2 is the highest (How useful did you find the handout on evaluating the properties for certain substance types in learning thermodynamics property evaluation?). The authors believe that the students find the flowchart model useful for understanding problem solving; however, when it comes to working out problems they are referencing the handout from Appendix 2 since it is a comprehensive summary of the appropriate equations to be utilized.

The graduate students found the flowcharts to be very useful and in general rated the methodology very high. For the 20 respondents, the averages are shown in Table 2. The most positive results are in response to question 3 (How do you think undergraduate students would react to this flow chart…) and question 4 (Would you use the flow chart in teaching….). The averages are approximately 4.5 out of 5 for both questions. The written comments from the graduate students are mostly positive. In general, the approach is found to be very logical and educational. Negative comments focused on the approach being the equivalent to 'spoon feeding' and the perception that students would become too dependent on the handouts and not learn the material.

Table 2
Graduate student response

ME 802	**Q1**	**Q2**	**Q3**	**Q4**
Average	4.2	4.65	4.55	4.45

Student survey results are not available for the transient conduction flow chart at this time. Anecdotally, undergraduate students have found the flowchart approach to be very logical and helpful in clarifying the mathematics.

Conclusions

The flow chart methodology presented works well with the applicable topics, property evaluation and transient heat conduction. Student response to the approach has been positive. The authors feel that the flowcharts enhance the educational process and reinforce the thought processes and problem solving methodology emphasized in lecture. There may be even more opportunities in engineering to use the flow chart as a valuable tool in the teaching of decision making and problem solving.

References

Moran, Michael and Howard Shapiro, Fundamentals Of Engineering Thermodynamics, 5th Edition, Wiley 2004.

Incropera, Frank and De Witt Introduction to Heat Transfer 4th Edition, Wiley 2002.

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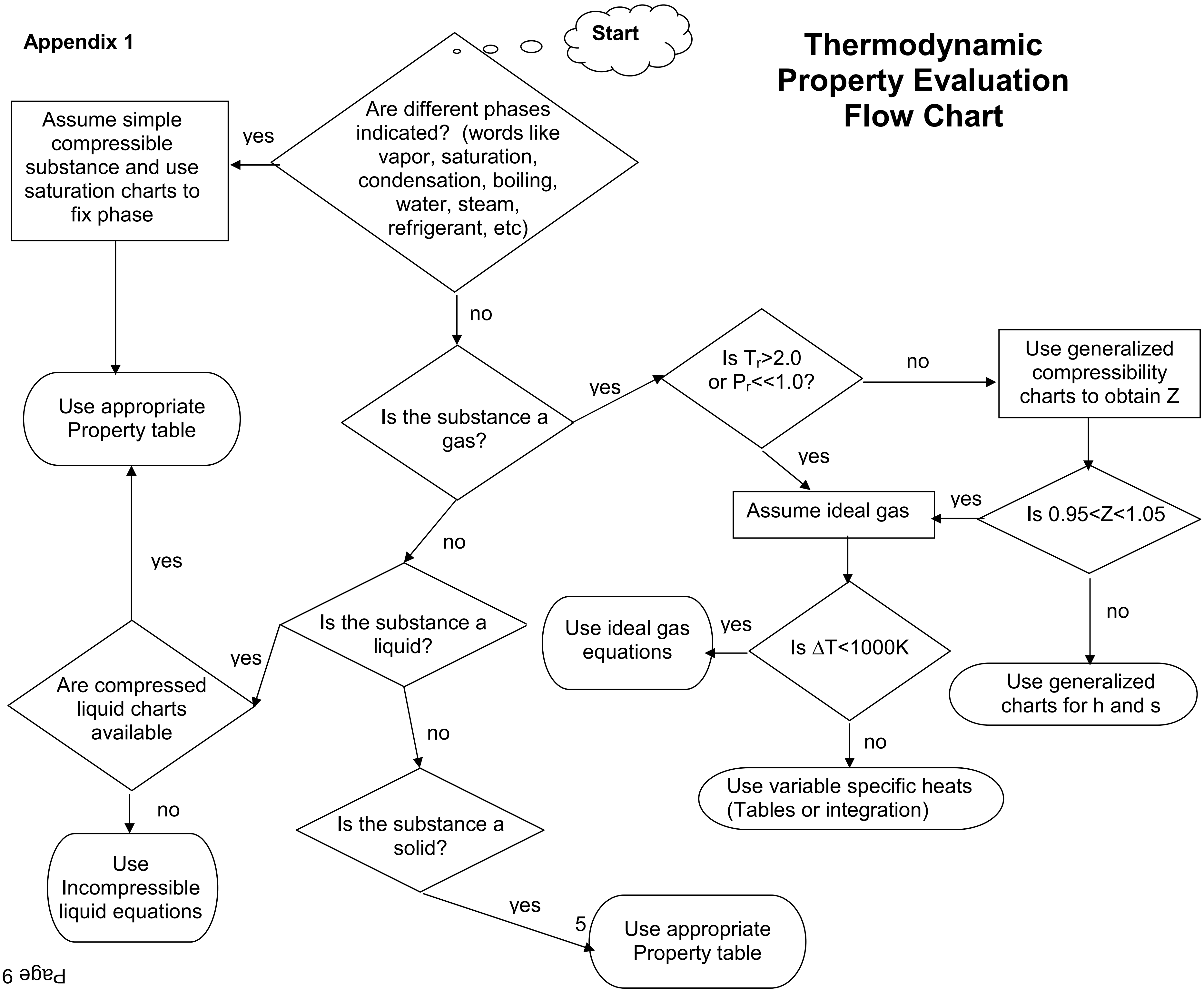
Appendix 1

Thermodynamic Property Evaluation Flow Chart



Appendix 2
Guidelines for Property Evaluation

The following methods are for property evaluation when a substance goes from state 1 to state 2. References are to Fundamentals of Engineering Thermodynamics, 5th Edition, Moran and Shapiro.

I. Ideal Gas (Air for instance)

A. First choice is to use tables

Internal energy (u) and enthalpy (h) are read directly from the tables, entropy, (s), is not.

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT - R\ln\left(\frac{P_2}{P_1}\right) \qquad \text{(6.19) Moran \& Shapiro}$$

On the tables is s^0, which is the temperature variation of entropy and is defined as

$$s_1^\circ - s_{ref}^\circ = \int_{T_{ref}}^{T_1} \frac{C_p(T)}{T} dT \qquad s_{ref}^\circ = 0 \text{ at } T_{ref} = 0 \qquad \text{(6.20) Moran \& Shapiro}$$

Then

$$s_2 - s_1 = s_2^o - s_1^o - R\ln\left(\frac{P_2}{P_1}\right) \qquad \text{(6.21a) Moran \& Shapiro}$$

where $s_1°$ and $s_2°$ are taken directly from the tables. If we want Δs in terms of temperature and specific volume, we write

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v(T)}{T} dT + R\ln\left(\frac{v_2}{v_1}\right) \qquad \text{(6.18) Moran \& Shapiro}$$

Since polynomial expansions of $c_V(T)$ are not readily available we can use

$$c_v = c_p - R \qquad \text{(3.44) Moran \& Shapiro}$$

so that

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT - R\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$

By definition

$$s_2 - s_1 = s_2^o - s_1^o - R\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$

so we can still use the air tables.

B. For $|T_2 - T_1| < 1000K$ it is a good approximation to assume constant specific heat and write the following:

$$h_2 - h_1 = c_p(T_2 - T_1) \qquad \text{(3.51) Moran \& Shapiro}$$

$$u_2 - u_1 = c_v(T_2 - T_1) \qquad \text{(3.50) Moran \& Shapiro}$$

$$s_2 - s_1 = c_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) \qquad \text{(6.23) Moran \& Shapiro}$$

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)$$ (6.22) Moran & Shapiro

where c_p and c_v are evaluated at T_{avg}

$$T_{avg} = (T_1 + T_2)/2$$

Note: **<u>If</u>** the change in entropy is zero (**<u>$\Delta s = 0$</u>**) then the above equations for **<u>ideal gases</u>** result in the following relationships, where $k = \frac{c_p}{c_v}$ {(3.10) Moran & Shapiro}:

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$ (6.45) Moran & Shapiro

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1}$$ (6.46) Moran & Shapiro

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^{k}$$ (6.47) Moran & Shapiro

C. For $|T_2 - T_1| > 1000K$, use $c_p(T)$ in polynomial form and perform the appropriate integration .

$$h_2 - h_1 = \int_{T_1}^{T_2} c_p(T)dT$$ (3.43) Moran & Shapiro

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T)dT = \int_{T_1}^{T_2} (c_p(T) - R)dT$$ (3.40) Moran & Shapiro

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT - R\ln\left(\frac{P_2}{P_1}\right)$$ (6.19) Moran & Shapiro

II. Real Gases

A. Use compressibility factor to determine departure from ideal gas behavior.

B. Use generalized compressibility charts for $\Delta h \ or \ \Delta s$.

III. Simple Compressible Substances (steam for instance)

Determine fluid phase

1. Both T and P are given

Go to saturation pressure table and get T_{sat} (@P)

If $T > T_{sat}$ (@P), superheated vapor
If $T < T_{sat}$ (@P), subcooled or compressed liquid
If $T = T_{sat}$ (@P), saturated condition (need another property to fix the state)
Or go to saturation temperature table and get P_{sat} (@T)
If $P > P_{sat}$ (@T), subcooled or compressed liquid
If $P < P_{sat}$ (@T), superheated vapor
If $P = P_{sat}$ (@T), saturated condition (need another property to fix the state)

2. P or T and one other property, β, are given. β may be u, v, h, or s

Go to saturation pressure or temperature table and find β_f and β_g

If $\beta < \beta_f$, subcooled or compressed liquid
If $\beta = \beta_f$, saturated liquid
If $\beta > \beta_g$, superheated vapor

If $\beta = \beta_g$, saturated vapor

If $\beta_f < \beta < \beta_g$, two-phase mixture with quality x

$$x = \frac{\beta - \beta_f}{\beta_g - \beta_f} = \frac{m_{vapor}}{m_{liquid} + m_{vapor}}$$ (3.1), (3.2), (3.6), (3.7), (6.6) Moran & Shapiro

IV. Incompressible Liquids

A. Using equations

$$c_p = c_v = c$$ (3.17) Moran & Shapiro

$$v = v_f(T)$$ (3.11) Moran & Shapiro

$$\Delta u = \int_{T_1}^{T_2} c\,dT = c(T_2 - T_1)$$ (3.18), (3.20a) Moran & Shapiro

$$\Delta h = \int_{T_1}^{T_2} c\,dT + v(P_2 - P_1) = c(T_2 - T_1) + v(P_2 - P_1)$$ (3.19), (3.20b) Moran & Shapiro

$$\Delta s = \int_{T_1}^{T_2} \frac{c}{T} dT = c \ln\left(\frac{T_2}{T_1}\right)$$ (6.24) Moran & Shapiro

B. Approximation using saturation values for substance (modeling a compressible liquid as an incompressible liquid).

$$v = v_f(@T)$$ (3.11) Moran & Shapiro

$$u = u_f(@T)$$ (3.12) Moran & Shapiro

$$h = h_f(@T) + v_f(@T)\{P - P_{sat}(@T)\}$$ (3.13) Moran & Shapiro

$$s = s_f(@T)$$ (6.7) Moran & Shapiro

Appendix 3

Flow Chart for 1-D Transient Heat Conduction Solutions

$$Bi = \frac{hL_c}{k} \le 0.1$$

$$L_c = \frac{V}{A}$$

Yes →

Lumped Capacitance

$$Fo = \frac{\alpha t}{L_c^{\,2}} \qquad L_c = \frac{V}{A}$$

↓

$$Fo = \frac{\alpha t}{L^2} \ge 0.2$$

L is the half-plane thickness or radius

Yes →

One-term approximation

$$Bi = \frac{hL}{k}$$

L is the half-plane thickness or radius

↓

$$Fo = \frac{\alpha t}{L^2} < 0.05$$

L is the distance from the surface of influence

Yes →

Semi-infinite media

$$Bi = \frac{hL}{k}$$

L is the distance from the surface of influence

↓

Full Series Solution or solve PDE

Appendix 4

Solution Methodology for One-Dimensional Transient Conduction

There are several different methods for solving 1-D, transient conduction problems that begin as a uniform temperature and are suddenly exposed to convection.

Lumped Capacitance Method (LCM)

A solid body changes temperature with time in a spatially uniform manner

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt} \qquad b=\frac{hA}{\rho V C_p}$$

The density, ρ, and the specific heat, C_p, are of the solid body. The Biot number is a non-dimensional number that expresses the ratio of convection away from a solid body to the conduction within the solid body.

Criteria for **LCM**: **Biot Number** based on volume to surface area ratio must be **less than 0.1**

$$Bi=\frac{hL_c}{k}\le 0.1 \qquad L_c=\frac{V}{A}$$

Looking a little more closely at the exponent of the solution:

$$bt=\frac{hA}{\rho V C_p}t=\frac{h\alpha}{L_c k}\frac{L_c}{L_c}t=BiFo$$

where

$$\alpha=\frac{k}{\rho C_p} \qquad Fo=\frac{\alpha t}{L^2_c}\text{, Fourier Number}$$

The Fourier number is a non-dimensional time.

If LCM is not valid, we must return to the heat conduction equation and solve the PDE for one-dimensional transient conduction. This typically leads to a lengthy, series solution. For one-dimensional, transient problems that are at a known uniform intital temperature [T(x,0) = T_i] that are suddenly exposed to convection on all sides

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=\theta(x,t)$$

$$Bi=\frac{hL}{k} \qquad Fo=\frac{\alpha t}{L^2}$$

where L is the half plane thickness of a large plane wall or the radius of a cyclinder or

sphere. For large time, **Fo > 0.2**, the solution, $\theta^*(x^*,Fo)$, maybe approximated by the first term only of the series solution to the PDE. This is known as the **one-term approximation**.

If both lumped capacitence and the one term approximation are invaild, there is one more option prior to using the full solution. That option is to model the finite solid body as a **semi-infinite media**. A semi-infinite media assumes that a body is so large that the heat conduction on one side is not influenced by the other side of the body. This will be true if **<u>Fo < 0.05</u>**, where the Fourier number is calculated based on the length from the surface of influence.

Appendix 5

Survey on Property Evaluation

Recall the property evaluation flow chart and handout used when we were introducing property evaluation in the first part of this course. Please answer the follow questions concerning these handouts.

1. Did you find the flow chart helpful in identifying the substance model to be used in the evaluation of thermodynamic properties for different substance?

Very Much		Some		None
5	4	3	2	1

2. How useful did you find the handout on evaluating the properties for certain substance types in learning thermodynamics property evaluation?

Very Much		Some		None
5	4	3	2	1

3. Do you think the flow chart approach to the decision making associated with learning engineering problem solving is useful?

Very much		Some		None
5	4	3	2	1

4. Would you recommend the flow chart approach, including the property evaluation handouts, in teaching undergraduate thermodynamics?

Definitely		Maybe		Never
5	4	3	2	1

5. Please share any other comments you have about the property evaluation flow chart and supporting web postings.

Recall the property evaluation flow chart and handout used when we were reviewing property evaluation in the first part of this course. Please answer the follow questions concerning this flow chart.

1. Did you find the flow chart helpful in identifying the substance model to be used in the evaluation of thermodynamic properties for different substance?

Very Much		Some		None
5	4	3	2	1

2. How useful did you find the web posting on evaluating the properties for certain substance types in relearning thermodynamics property evaluation?

Very Much		Some		None
5	4	3	2	1

3. How do you think undergraduate students would react to this flow chart approach to the decision making associated with learning engineering problem solving is useful?

Very much		Some		None
5	4	3	2	1

4. Would you use the flow chart approach, including the supplementary web postings in teaching undergraduate thermodynamics?

Definitely		Maybe		Never
5	4	3	2	1

5. Please share any other comments you have about the property evaluation flow chart and supporting web postings.