Full Cycle Solution for 3-D Offset Slider Crank Kinematics: Pseudographics - A Pedagogic Examination of a Non-Traditional Computational Method

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Abstract

The slider crank is a mechanism that students encounter at an early stage in the study of both 2-D and 3-D kinematics. In the current paper this classic device is used as an exemplar for a coordinate geometry based method, with the coined name of "pseudographics", that provides an option to the more familiar textbook vectorial approach.

Pseudographics employs a commercial equation solving software to generate coordinates of the kinematic polygons for position, velocity and acceleration. The lines and arcs used to construct 2-D diagrams are replaced in 3-D pseudographics by equations for a straight line, a plane and the surface of a sphere. Because it avoids cross and dot products, matrices and repeated differentiations, the method has a lowered demand for skills in mathematics. The author sees pseudographics fulfilling the dual role of providing engineering students with an alternative to the prevalent textbook technique, and also opening a door to the understanding of mechanism kinematics to students who do not have a background in engineering mathematics.

A determination of the angular velocity of the connecting rod is emphasized. Lecture experience has shown that the visualization of the motion of this member provides a learning challenge. Pseudographics uses 3-D coordinate geometry in conjunction with motion limitations for a single rigid body to identify kinematics features of the slider crank. Students appreciate that information on full cycle behaviour is necessary for design work, so output plots of some kinematic features for a revolution of the input driving crank are presented. Computer codes are appended.

In closing, the paper summarizes the advantages and disadvantages of pseudographics in comparison to current textbook approaches to 3-D mechanisms. Student reaction is provided in brief, and future work in pseudographics is indicated.

Introduction

The purpose of the work presented in this paper is to demonstrate the application of a coordinate geometry based technique in 3-D mechanism kinematics.

Previous work[1-6] by the author on planar mechanism analysis and linkage dimensional optimization has demonstrated a computational method with the coined name “pseudographics”. The efficacy of the commercial software[7] employed for the technique has been discussed in these earlier papers. The present work extends the use of pseudographics to the kinematic analysis of the 3-D slider crank. The single driving crank angle solution for this mechanism is presented in a number of current textbooks, and the data from a typical problem[8], as in Fig. 1, are used to demonstrate pseudographics.

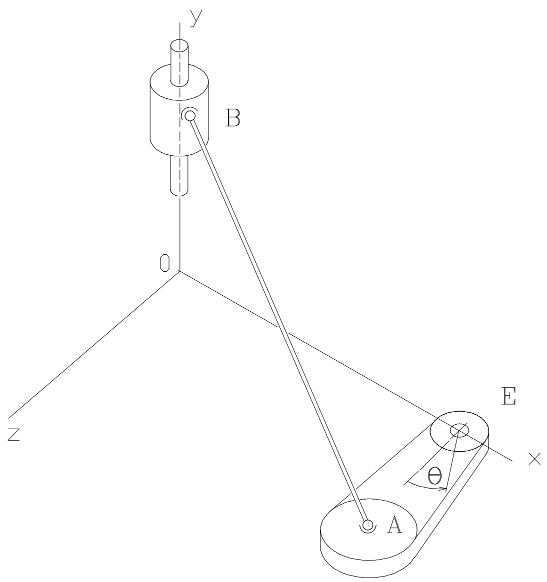


Fig. 1. The offset 3-D slider crank as utilized in the current work. The origin of the xyz axes is the lowest position of the slider B. Dimensions: OE = 250 mm, AB = 330 mm, AE= 80 mm. The driving crank rotates anti-clockwise when viewed from the outer end of the y axis. There are ball and socket joints at A and B.

Mechanical Analysis.

Three kinematic polygons for a 3-D slider crank are shown in Figs. 2, 3, and 4. Thirty coordinates completely define these three diagrams; thirteen of the coordinates are initially unknown. The objective of this analysis is to find these unknown coordinates, and hence to determine the velocity and acceleration of the slider and the angular velocity of the connecting rod AB. The following statements model the problem, along with the computer code, nomenclature and pseudographics protocol of Appendices A and B.

Position, Fig. 2. The locus of joint A is circular and horizontal with the coordinates (XA,YA, ZA) defined by the crank angle θ. Also joint B must have a location on the surface of a sphere, centered at A, with a radius equal to the length of the connecting rod AB. Noting that the line of action of the slider pierces this sphere at (0,YB,0) allows YB to be found. With point B located, the angular orientation, εx, εy, εz, of the line AB is calculated.

A table, as in Fig. 5, helps to keep track of the elements of the three kinematic polygons – particularly useful for error tracking and for extending the completed model to accommodate alteration in the mechanism geometry.

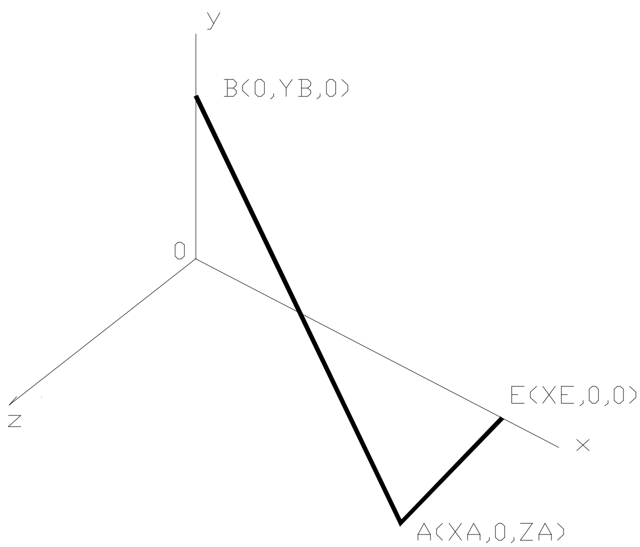


Fig. 2. Position diagram.
Uppercase letters indicate linkage joints.

Velocity, Fig. 3. The locus of vertex 'a' is circular and horizontal, with a radius equal to the speed, vA, of the joint A. The vertex 'b' lies on the intersection of the vertical line on which the slider velocity vector lies and the plane lying perpendicular to the connecting rod and containing the vertex 'a'. The line ab represents the velocity vector of joint B relative joint A, i.e., **vBA**.

The angular velocity, ω, of the connecting rod is determined using the now known components of **vBA**. This velocity vector has components in the x, y, z directions given by

$$\left.\begin{aligned} vBAx &= \omega z * ry - \omega y * rz \\ vBAy &= \omega x * rz - \omega z * rx \\ vBAz &= \omega y * rx - \omega x * ry \end{aligned}\right\} \tag{1}$$

where ωx, ωy, ωz are the components of the vector **ω**, and rx, ry, rz, are the components of the spatial vector **AB**. These three equations contain three unknowns, ωx, ωy, ωz, but the matrix of coefficients has a determinant of zero, and thus an additional equation is required. Assuming that rotation of the connecting rod about its centerline AB is zero provides another equation for angular velocity components. Thus noting that the angular velocity vector **ω** must be perpendicular to the spatial line AB gives the relationship

$$\omega x * \cos(\varepsilon x) + \omega y * \cos(\varepsilon y) + \omega x * \cos(\varepsilon z) = 0 \tag{2}$$

The code for angular velocity analysis, Fig. A5, can be modified to suit other types of connecting rod constraint. For example, the ball and socket joint at B could be replaced by a clevis, and then the connecting rod would have zero angular velocity about an axis created by the projection of AB onto a horizontal plane.

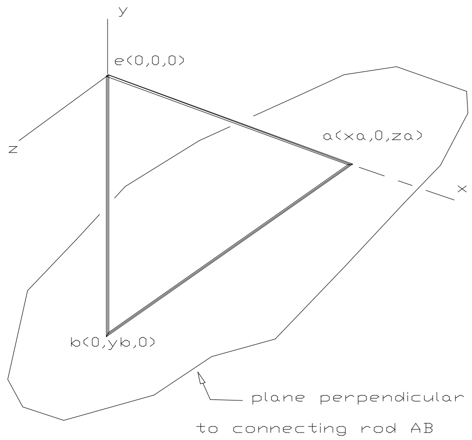


Fig. 3. Velocity diagram. Lowercase letters indicate the intersection of velocity vectors. Vertex 'b' is located at the intersection of the velocity vector **vB** ≡ eb, and a plane perpendicular to the connecting rod AB, and containing the vertex 'a'.

Acceleration, Fig. 4. The locus of vertex 'al' is circular and horizontal with a radius equal to the acceleration of joint A, aA. The acceleration vector, **aA**, is 180° out-of-phase with the spatial vector **EA**. The vertex, 'b11' is located at a distance equivalent to the normal component of acceleration of joint B relative to joint A. The acceleration line a1b11 has the same orientation as the spatial line BA, so 'b11' can be located.

The line b11b1 represents the tangential acceleration, aBAt, of joint B relative to joint A and lies in a plane perpendicular to the link AB. Thus the vertex 'b1' is located at the intersection of this plane and the vertical line e1b1 which represents the acceleration vector of the slider, **aB**. The acceleration polygon is now complete.

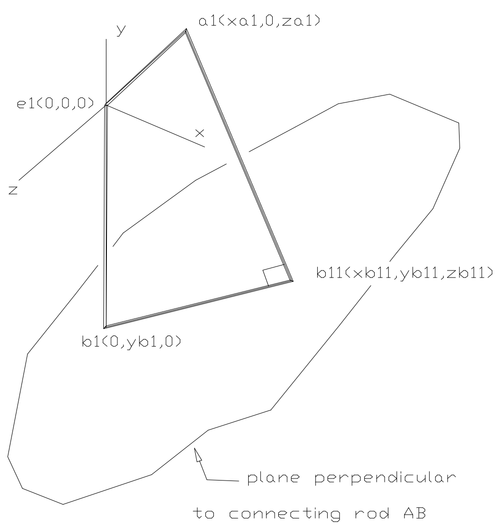


Fig. 4. Acceleration diagram. Lowercase letters followed by single or double numeral 1 denote the intersection of acceleration vectors.

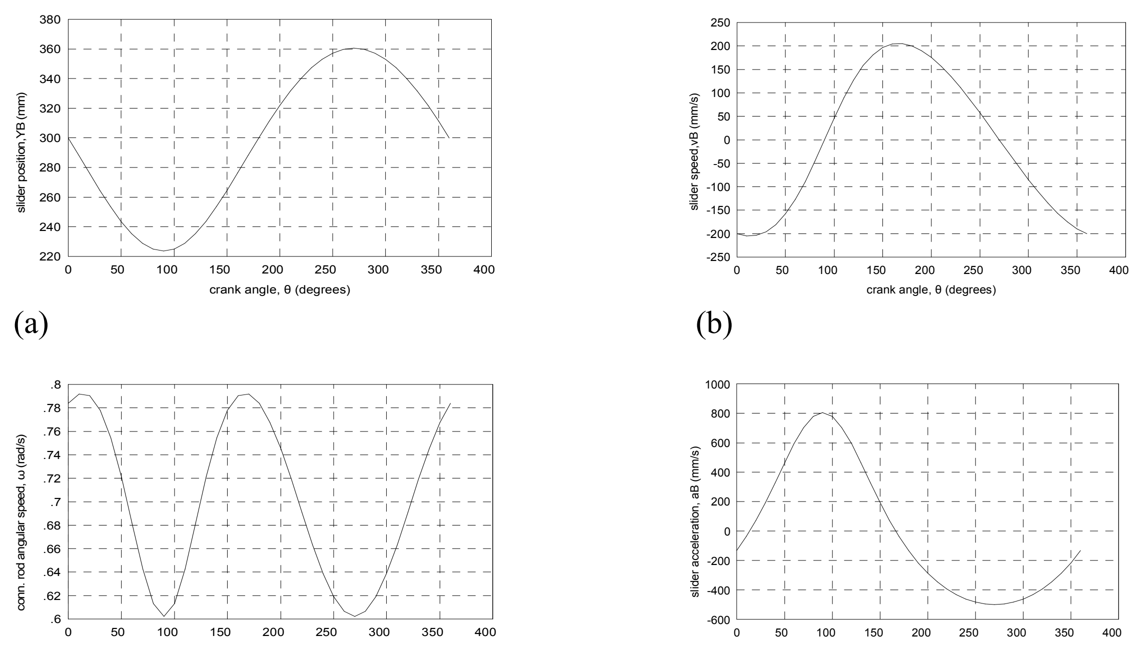
The pseudographical method requires a large number of modeling statements, so even with a small number of mechanism parts many coordinates must be determined. The table shown below is a useful 'aide-mémoire', and is most effective when employed *ab initio.*

	Vertex	x	y	z	Location as per TK model. Asterisks indicate known coordinates	
position	E	XE*	0*	0*	E is fixed.	position
	A	XA	0*	ZA	Locus of A is a circle of radius AE, centre E.	
	B	0*	YB	0*	B is located on y axis.	
velocity	e	0*	0*	0*	E is fixed.	velocity
	a	xa	0*	za	Locus of ‘a’ is a circle of radius vA = ωAE*AE, centre ‘e’.	
	b	0*	yb	0*	Velocity of B is vertical	
acceleration	e1	0*	0*	0*	E is fixed.	acceleration
	a 1	xa1	0*	za1	Locus of 'a1' is a circle of radius aAn = $(\omega AE)^2$ * AE, centre 'e', with the line ea1 180° out-of-phase with the line EA.	
	b11	xb11	yb11	zb11	'b11' is located at a distance along the inclination of BA equivalent to the normal component of acceleration of joint B relative to joint A, aBAn = ω^2 * AB	
	b1	0*	ya1	0*	Vertex 'b1' is located by finding the intersection of the vertical ‘b1’ line and a plane perpendicular to the line AB and passing through the point 'b11'	

Fig. 5. Coordinates of the position, velocity and acceleration polygons.

Results.

Some of the outcomes of the preceding analysis are shown in Fig. 6 for a cycle of the crank AE.



(a) (b) (c) (d)

Fig. 6. Some kinematic features of the 3-D slider crank.
(a) slider position YB, versus input link angle θ
(b) slider speed vB, versus input link angle θ
(c) connecting rod angular velocity ω, versus input link angle θ
(d) slider acceleration aB, versus input link angle θ

Conclusions.

- The 'pseudographic'method described here provides an alternative to the traditional vector algebra approach for the generation of full cycle kinematics of 3-D spatial mechanisms. It is tedious keeping track of the large number of coordinates to be found, but on the other hand the method reveals some interesting facets. For example, the angular orientation of the connecting rod AB is not apparent in a vectorial solution[8], but is more evident in pseudographics.

- The method described in this paper requires relatively little mathematical work – no matrices, vector algebra, complex numbers or repeated differentiations – the necessary ingredients of a traditional vector solution for the kinematics of 2-D and 3-D mechanisms.

- Students should check the data as found by pseudographics – it is easy to generate convincing, but incorrect results. Existing publications and software provide a double-check using either single position[8] or full cycle animated [9, 10] solutions.

- A pseudographical model for the 2-D slider crank, as discussed in an earlier work[6], is well tested and is a useful teaching tool. The 3-D version of the present paper is still "a work in progress", but students can benefit from a dissection of the code – the method offers a very thorough examination of mechanism kinematics and provides an instructor with an "endless" supply of 3-D slider crank solutions.

- Student feedback on pseudographics is about evenly split between those who see benefit in the method, and the less enthusiastic who may have been conditioned into more familiar equation solving approaches. Opportunity to use 3-D pseudographics has been limited to exploratory forays in an introductory course.

- Future plans for this work include making 2-D and 3-D mechanisms more accessible to non-mathematically inclined users via an online 'store front'.

Appendix A: TK Solver Code

Input	Name	Output	Unit	Comment
80	AE		mm	length of crank AE
250	OE		mm	slider line of action offset
330	AB		mm	length of connecting rod
0	Θ		degrees	crank angle, anticlockwise viewed from end of y axis
	XA	250	mm	x coordinate of joint A
0	YA		mm	y coordinate of joint A
	ZA	80	mm	z coordinate of joint A
0	XB		mm	x coordinate of joint B
	YB	200	mm	y coordinate of joint B
0	ZB		mm	z coordinate of joint B
	ϵx	139.25	degrees	angle between x axis and the connecting rod AB
	ϵy	52.69	degrees	angle between y axis and the connecting rod AB
	ϵz	104.02	degrees	angle between z axis and the connecting rod AB

3	AE		rad/s	angular velocity of crank AE
	vB	-300	mm/s	velocity of slider B
	ωx	-.2203	rad/s	angular velocity of connecting rod AB about the x axis
	ωy	.1763	rad/s	angular velocity of connecting rod AB about the y axis
	ωz	1.1294	rad/s	angular velocity of connecting rod AB about the z axis
	ω	1.1642	rad/s	angular velocity of connecting rod AB
	rx	-250	mm	x component of line AB
	ry	200	mm	y component of line AB
	rz	-80	mm	z component of line AB
	vBAx	-240	mm/s	x component of the velocity of B relative to A
	vBAz	0	mm/s	y component of the velocity of B relative to A
	vBAy	-300	mm/s	z component of the velocity of B relative to A
	aB	-450.00	mm/s/s	acceleration of slider B

Fig. A1. TK Variable Sheet. The results shown correspond to an input crank angle of zero. A guessed input is required for one component of the connecting rod angular velocity.

call position (Θ,XA,YA,ZA,YB, єx, єy, єz)
call velocity (Θ,єx, єy, єz ,XA,ZA,YB;vB,rx,ry,rz,vBAx,vBAy,vBAz)
call angvelocity (,rx,ry,rz,vBAx,vBAy,vBAz; ωx,ωy,ωz ,ω)
call acceleration(Θ, єx, єy, єz,,ω,rx,ry,rz;aB)

Fig. A2 TK Rule Sheet.

XA = OE + AE*sind(Θ); XA
YA = 0
ZA = AE *cosd(Θ) ;ZA
AB = sqrt((XB-XA)^2+(YB-YA)^2+(ZB-ZA)^2) ;YB
XB = 0
ZB = 0
cosd(єx) = (XB-XA)/AB ; єx
cosd(єy) = (YB-YA)/AB ; єy
cosd(єx) = (ZB-ZA)/AB ; єz

Fig. A3. TK Rule Function Subsheet for position. Unknowns appear after the semi-colons.

za = - vA *sind(Θ) ; za
xb = 0
zb=0
rx = XB-XA ; rx
ry = YB-YA ; ry
rz = ZB-ZA ; rz
rx*(xb-xa) + ry*(yb-ya) + rz*(zb-za) = 0 ; yb
vB = yb ;vB
vBAx = xb-xa ; vBAx
vBAy = yb-ya ; vBAy
vBAz = zb-za ; vBAz

Fig. A4. TK Rule Function Subsheet for linear velocity.

ωx*cosd(εx) + ωy*cosd(εy) + ωz*cosd(εz) = 0 ; ωx, ωy, ωz
vBAx = ωy*rz - ωz*ry ; ωx, ωy, ωz
vBAy = ωz*rx - ωx*rz ; ωx, ωy, ωz
vBAz = ωx*ry - ωy*rx ; ωx, ωy, ωz
ω = sqrt(ωx^2+ ωy^2+ ωz^2) ; ω

Fig. A5. TK Rule Function Subsheet for angular velocity.

xa1 = aAn *sind(Θ+180) ; xa1
aAn = AE^2*AE ; aAn
ya1 = 0
za1 = aAn*cosd(Θ+180) ;za1
xb11 = xa1 + aBAn*cosd(Θ+180) ; xb11
aBAn = ω^2*AB ; aBAn
yb11 = ya1 + aBAn*cosd(Θ+180) ; yb11
zb11 = za1 + aBAn*cosd(Θ+180) ; zb11
xb1 = 0
zb1 = 0
rx*(xb1-xb11) + ry*(yb1-yb11) + rz*(zb1-zb11) = 0 ; yb1
aB = yb1 ; aB

Fig. A6. TK Rule Function Subsheet for acceleration.

Appendix B: Protocol and nomenclature in pseudographics.

(1) Subscripts, superscripts, primes and bold facing, as a mathematical notation, are not available in TK Solver and so are not used in pseudographics.
(2) Position diagram: an uppercase letter denotes a joint.
(3) Velocity diagram: a lowercase letter denotes the head or tail of a velocity vector, e.g., the velocity of joint B relative to joint A ≡ ab ≡ vBA.
(4) Acceleration diagram: the labels mimic those of Morrison[11], an early text on dynamics with an emphasis on graphical methods – with the numeral 1 replacing primes. So a lowercase letter, followed by a single, double numeral 1 denotes the head or tail of an acceleration vector, e.g., the tangential acceleration of joint B relative to joint A ≡ b11b1 ≡ aBAt.
(5) All coordinates are relative to a fixed frame with the x, y, and z directions as shown in Fig. 1. So XA, xa, xa1 are horizontal coordinates of the position, velocity and acceleration vectors relative to an earth point.
(6) Angles are measured in degrees, taking anticlockwise as viewed from the outer end of the relevant axis as positive.
(7) Links are labeled by a pair of uppercase letters taken in alphabetical order.

Position nomenclature:

X/Y/ZA, B, E…………	x,y,z coordinates of A, B, E
θ………………………	input angle for driving crank AE
rx,y,z…………………	displacement in x, y, z direction of joint B relative to joint A
εx,y,z…………………	angles between connecting AB and the x,y,z axes

Velocity nomenclature:

vA, vB………………..	absolute velocity of joints A, B

vBA…………………… velocity of joint B relative to joint A
vBAx,y,z……………… velocity of joint B relative to joint A in x, y, z direction
ωAE………………….. angular velocity of input lin AE in rad/s
ω……………………… angular velocity connecting rod in rad/s
ωx,y,z………………… angular velocity of connecting rod AB about x, y, z axes in rad/s

Acceleration nomenclature:
aAn……………….. normal component of acceleration of joint A
aBAn……………… normal component of the acceleration of joint B relative to joint A
aBAt………………. tangential component of the acceleration of joint B relative to joint A

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Biographical Sketch.

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