Teaching Electrical Engineering by using Computer Algebra Systems.

by

Radian G. Belu[1], and Alexandru-Catalin Belu[2]
1) College of Engineering
2) Department of Mathematics
Wayne State University.
Detroit, Michigan, USA.

Abstract

In the past 40 years Computer Algebra Systems (CAS) has been used extensively in research and industrial applications. This growing use of "computer algebra" or "symbolic computation" makes it very attractive for employing in electrical engineering education. The objective of this paper is to draw the attention of broad engineering and educational circle to the enormous capabilities of symbolic computation to the teaching of electrical engineering. The role and the advantages of using of computer algebra systems in electrical education will be pointed out via a few significant examples. A brief description of computer algebra systems and the current status within electrical engineering education is given in this paper. There is authors' believe that will be an increase use of symbolic computation in higher education in the very near future, due to both computational and educational advantages.

1. Introduction.

Computer tools to help professors and students alike in the teaching and learning processes have become very popular in the last two decades. Scores of articles and technical papers can be found in the technical and popular literature referring to different software and hardware tools and the techniques to use them in educational environments[2-6], probably due to the incredible advances in software and hardware development, in the last two decades and their significant reduced costs. However the main use has been reported in numerical computation, simulation, and computer-aide design. Many authors have illustrated a wide range of applications in different areas, such as electrostatics, circuit analysis, electrical machine analysis and design, electromagnetics, etc. As long as computers were capable only working with numbers, their applications in engineering education was somewhat controversial. Many experts pointed out that the generality being lost if the information is represented in "discrete" forms, only. Anyway, for a student, numerical results may obscure the fact that they follow from a model, which is analytic in nature. Different and unique to their digital nature is using computers for symbolic analysis. In the past 40 years Computer Algebra Systems (CAS) has been used extensively in research and industrial applications. This growing use of "computer algebra" or "symbolic computation" makes it very attractive for employing in electrical engineering education.

There is another issue that makes symbolic computation very attractive for electrical engineering (EE) educational point of view. The extensive teaching experience of one of the authors of this paper and a number of articles published over the years[2,7,14] show that many EE students have difficulty in learning technical subjects because they lack sufficient competence in mathematics. This lack of mathematical knowledge makes it difficult for them analyze signals, circuits or in specially to deal with electromagnetic field problems, electric machines analysis and design, antennas and radio propagation, tasks that are heavily dependent on mathematical modeling and manipulation. Problem solving from physics and/or electrical engineering principles often starts with a verbal statement of the physical problem, and the solution can be regarded as a four-step process. The student's first task is to state the problem in mathematical terms, i.e. to formulate the equations representing the physical situation. The second step is to symbolically solve those equations for the desired terms. The third step is the easy part: plugging in the values and doing the numerical evaluations. Fourth, and frequently overlooked by many students, is to judge the correctness of the initial equations, the solutions, and the final numerical results. In short does the answer make sense?

Students frequently have difficulties stating a physical problem in mathematical terms. In addition, they often have in more or less degree lack the ability to do the symbolic manipulations necessary for solving the relevant equations. To really learn EE students must learn to do symbolic analysis of circuits, systems, EM field problems, antennas, etc, the first and the last steps in problem solving, and this should be the main focus of our teaching efforts. Our efforts we have as teachers are therefore how to handle the situation that many students have to a limited ability in symbolic manipulations. The literature is full of good suggestion s, one of which is to use Computer Algebra Systems (CAS) in engineering education [2, 3, 5-12]. Early uses of computer algebra systems in teaching focused on mathematics education, but in recent years CAS have also been used in teaching other subjects as well, including physics [5, 6, 10, 13, 15], electrical engineering[3, 7, 12], chemistry [9, 10], mechanical engineering [2, 9, 10], etc. Computers have, of course, been ubiquitous in EE education since the beginning of computer age, but their use focused on numerical computations, computer-aided design, visualization and simulation. Numerically based tools, such as PSpics/OrCad, Altera, etc. are very useful in many situations, but, where students are concerned, they often obscure an important fact – that behind any numerical result is a symbolic model of the physical world.

The objective of this paper is to draw the attention of broad engineering and educational circle to the enormous capabilities of symbolic computation to the teaching of electrical engineering. The role and the advantages of the using of computer algebra systems in electrical education will be pointed out via a few, but significant examples. A brief description of computer algebra systems and the current status within electrical engineering education will also be given in this paper. There is authors' believe that will be an increase use of symbolic computation in higher education in the near future.

2. Computer Algebra Systems in EE Education – Pedagogical Considerations.

Our purpose is to extend the use of computer algebra systems in EE education, first focusing on the subjects, such as: electric circuit analysis, electromagnetics, and communication electronics. Later we plan to move in other fields of EE education, such as signals and systems, control, etc. A computer algebra system, such as Maple, Mathematica or Mathcad is a powerful software program for doing symbolic algebra computations, numerical calculation, plotting graphs and diagrams, etc.[1-15]. The key advantages of using computer algebra systems in engineering education include: 1) CAS allows the focus to be on concepts and understanding principles rather than on large amount of "routine" mathematical manipulations; 2) motivation can be enhanced because more real-life engineering problems can be tackled; and 3) many more examples can be covered by students allowing them to be more active participants in discovery learning. On the other hand, the authors' experience and current literature show a widespread problem in electrical engineering (EE) education, and not only, namely that many students do not master the mathematical tools that are prerequisite for studying EE. This lack of mathematical knowledge makes it difficult for them to analyze electric circuits, signals and systems, or especially electromagnetic field problems, a task that is heavily dependent on mathematical modeling and manipulation. These aspects make very attractive the use of computer algebra systems in EE education, which enabling the students to focus on qualitative aspects rather than tedious calculations. It is also in the authors' believes that a balanced compromise between the analytic and numerical methods, especially in subject such as: electric circuit analysis and design, electromagnetics, signals and systems may be very useful in the learning and teaching processes.

2.1 Worksheet models and their use.

Our approach in using CAS in EE education will be on the use of worksheet as teaching aid. Over the years many worksheets have been written, as a result of use of CAS, such as Maple, MathCAD, or Mathematica, in engineering, physics, or mathematics education. We plan, and already start to use essentially three different kinds of worksheets: 1) supplementary lecture notes; 2) homework assignments in form of CAS worksheets; and 3) solutions of the homework problems completed by the teacher, or by a student. All these worksheets are or will be posted on the web. Writing good worksheets is a time-consuming endeavor, and use a CAS would become even more appealing if such documents could be shared among teachers and students at different institutions. Distribution of such documents among institutions could be another reason to pay careful attention to the details in worksheet design. The worksheets are divided in a number of sections, containing introduction/statement of the problem, bibliographical information, information about authors, solving methods, etc.

Each document will have a descriptive to that the user can refer to it easily. If it contains the solution of a problem given in a textbook, the title of the worksheet will be made by some combination of the author's name, the title of the book, and the problem number. Finding descriptive title is more of problem than it may first appear, especially in a set of connected worksheets. Next teachers as well as students may have questions or comments about the contents and may wish to contact the author, so the appropriate contact information must be included. The date and the version numbers of a worksheet are also important information, as well as the CAS programs used. If it is necessary to use one or more external library, software packages, etc. it is important to inform the document users about them, location, etc. For pedagogical reasons worksheets designed for education should encourage students to make modifications. One may assign additional problems for students to do, or there may be notes they themselves want to add to clarify a point. Soon there will be many versions of the same original worksheet, and it is important to know who made any particular modification. The above information should be kept in a single region at the beginning of the worksheet.

2.2 Pedagogical considerations.

Every worksheet will have an introduction. The content of this introductory section may differ according to the purpose of the worksheet. References should be given for every subject; supplementary class notes, together with short theoretical development may be also included. If the worksheet is the answer is an answer key for a homework assignment, should be stated in introduction. To save the student time, the worksheets containing only problems will made available electronically. The main part of the worksheet could be divided in two sections: 1) theoretical analysis, which should keep symbolic if at all possible; and 2) the second section will be reserved for numerical calculations, presentation and discussion of the results, and checking the correctness of the answers. Any symbolic computation should always start from the fundamental physics principles. It is very important to keep in mind, that the students never get enough experience in applying these laws to different physical situations.

Pedagogical considerations also make it important to keep calculations symbolic as long as possible. Symbolic results of a calculation are important for students to better judge, evaluate and understand the validity of obtained results. Students have a tendency to do numerical calculations using calculators and to resist doing symbolic manipulations. If numerical answers to a problem or application are required, as is often the case, they should obtain and checked at the end of the application. It is so important to stress the importance of doing symbolic computation before obtaining the numerical results, and as teachers we should always try to require this to our students. Another important issue: students should always require to judge and check the correctness of the obtained results, and this can be done in different ways depending on the particular subject. Such checks have a dual purpose in education. First, the symbolic solution along with the estimates provides a deeper insight into the behavior of an EE problem and is therefore an important learning experience. Second, checking results is important in any workplace, and it should be practiced with regularly.

Worksheets need to encourage students to do future work on a subject, via questions and problems. First, the worksheet may have short questions to help students to focus on important aspects of the problem. Questions could require student to elaborate further in writing on a point of understanding, or could require them to do calculations on similar problems or other aspects of the same problem. Second, a set of elaborate problems relating to one topic could be posted at the end of worksheet, or on the web as additional material. These should be design in such way that the students are required on a problem in several ways, theoretical, numerical and practical.

In EE education there are some other important issues, such as: circuit representation by schematics and diagrams, plotting and visualization of the results, etc. And it is important to include them in worksheets. Good instructional materials need also to include references and links to additional sources on the subject. Although, the courses in which our worksheets will be used include textbooks, we consider that it is important to encourage students to use library in order to get different perspectives and better understanding the topic. Finally, it is not realistic to expect students to learn more than a limited number of commands of a specific CAS program, and even these must be repeated frequently. Thus it is important that the worksheets to be used by students contain, as far as possible, only this limited number of commands. The specific commands will vary from subject from subject, but it is a good pedagogical approach to consider this problem before starting a specific CAS program in teaching. Unusual commands used in, or output from, a worksheet may be explained in the text for the benefits of the students.

3. Illustrative Examples.

In this section will present and comment a number of illustrative examples of the application of CAS in EE teaching and learning processes.

3.1 Impedance matching network,

This worksheet contains a brief introduction to the theory of simple impedance matching network, along with some theoretical and practical student problems. Assume a given source with source impedance R_2 and a load R_1, both impedances being real. If the two impedances are different, for maximum power transfer we may insert an impedance matching LC network between the source and the load. We will construct an impedance network starting with the circuit configuration of Figure 1. We assume that $Z_S = jX_S$ and $Z_P = jX_P$ are imaginary. For matching to occur, the impedance of the network as seen from the terminals must equal the source impedance R_2. This will insure that the maximum energy will be transferred from the source to the load, since Z_S and Z_P have purely imaginary impedances and thus can absorb no energy.

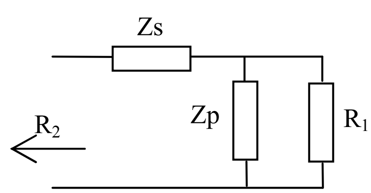

Figure 1 – Impedance matching network.

Calculations and Derivations – The impedance of this network is equal to the source impedance; thus we have the following equations (using Maple):

- Restart

➢ $E_1 := R_2 = Z_S + Z_P*R_1/(Z_P+R_1)$; $Z_S = IX_S$; $Z_P = IX_P$;

$$E_1 := R_2 = Z_S + Z_P*R_1/(Z_P+R_1$$
$$Z_S = IX_S$$
$$Z_P = IX_P$$

➢ $E_2 := \text{evalc}(E_1)$;

$$E_2 := R_2 = X^2_P R_1/(RI^2 + X^2_P) + I(X_S + X_P RI^2/(RI^2 + X^2_P))$$

We have obtained an equation relating the unknowns X_S and X_P to the known resistances R_1 and R_2. Since this is a complex equation, the real and the imaginary part must be equal, so we may rewrite equation E_2, as:

➢ $Eq1 := \text{evalc}(\text{Im}(\text{lhs}(E_2))) = \text{evalc}(\text{Im}(\text{rhs}(E_2)))$;

$$Eq1 = 0 = X_S + X_P RI^2/(RI^2 + X^2_P)$$

➢ $Eq2 := \text{evalc}(\text{Re}(\text{lhs}(E_2))) = \text{evalc}(\text{Re}(\text{rhs}(E_2)))$;

$$Eq2 = R_2 = X_S + X_P^2 RI/(RI^2 + X^2_P)$$

Now we got two purely real number equations with unknowns X_S and X_P, and solving them we get:

➢ solve({Eq1, Eq2},{ X_S,X_P})

The worksheet also the entire solving, determining the impedance matching networks, the plots of the results, a discussion of high pass filter networks obtained from impedance matching calculations, comments, numerical evaluations, and problems.

3.2 Electrostatic Potential Problem.

Another worksheet focused on electrostatic potential computations. The geometry of the problem is shown in Figure 2, and it consists of two grounded semi-infinite electrode separated by a distance b. A third electrode located at x = 0 is maintained at potential V_0. The potential in the xy plane satisfies Laplace equation:

$$\Delta V(x,y) = 0, \qquad (1)$$

subject to boundary conditions:

$V(x,0) = 0;\ V(x, 1) = 0$

$V(0,y) = 1;$ and $\lim V(x,y) = 0$, as $x \rightarrow \infty$

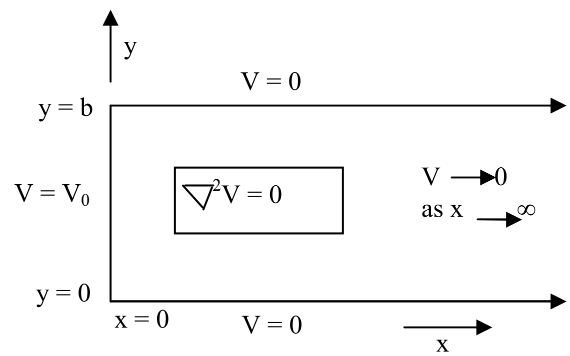

Figure 2: The geometry of the Electrostatic Potential Problem.

Note: the spatial dimensions are scaled in units of b, and the potential V(x,y) is measured in units of V_0. The solution of this boundary value problem is given by a Fourier series (9):

$$V(x,y) = \sum(1/(2m-1))\sin[(2m-1)\pi y]\exp(-(2m-1)\pi x) \qquad (2)$$

This problem require to obtain a plot showing V(x,y) over the ***xy*** *plane in the region*

$0 \leq 1$, and $y \leq 1$.

Comments: Generation of the desired plot involves the evaluation of the truncation series of equation (2) over a grid of points in the **xy** plane.

Approach Using Mathematica*:*

```
V(x_, y_, nMax_): = 4/Pi)Sum[Sin(2m-1)Pi y]Exp[-(2m-1)Pi x]/(2m-1). {m, 1,
nMax}], Plot[Evaluate[ V[0,y,50], {y,0,1}, PlotDivison  -> 1, PlotPoints -> 101,
PlotRange -> {{0,1}, {0, 1.5}}] ;
```

Comment: To generate a 3-D plot

```
Plot3D[Evaluate[ V[x,y,50], {y,0,1}, PlotPoints -> 21, Sahding -> False];
```

Approach Using Maple:

- V := proc(x,y,n);
- 4/Pi*Sum(sin((2*m-1)*Pi*y)*Exp(-(2*m-1)*Pi*x)/(2*m-1), m=1,..n), end;
- Plot(V(0,y,50), y=0..1, numpoints=101, style=Line);
- Plot3d(V9x,y,50),x=0..1,y=0..1,grid=[21,21]);

Other approaches may include another CAS programs, such as: MathCAD, Derive, etc

This worksheet also includes: comments, theoretical considerations, references, problems, questions, etc.

4. Conclusions, Discussions and Future Work.

1. Use of CAS, such as Maple, MathCAD or Mathematica adds new tools to engineering education. They have the ability to reduce the drudgery of symbolic manipulations that many students find often so tedious. The CAS programs also include graphic and numeric capabilities, text editors, etc. that are adequate of present day learning and teaching processes.
2. This approach of EE education that resorts on symbolic computation enables to the students to focus on the ideas of theoretical approach rather than on computational difficulties. One could claim that the use of such tools may prevent a good command of computational techniques. However, the task of engineering schools to provide technological knowledge, not computational skills.
3. However, if such CAS worksheets are to be used successfully in education it is important that simple design guidelines to be observed to make them easier for students and instructors to use. Worksheets must have comprehensive bibliographic information such as a descriptive title, information about author(s), date, version, file name of the worksheet, CAS release number, etc. For pedagogical reasons calculations in the worksheets should start from fundamental principles, and kept symbolic as far as possible. Strong emphasis should be placed on assigning students problems that enlarge the worksheets. A key to effective learning is that students concentrate on modeling and testing results. To do this they should include both mathematics and written discussions in worksheets, and in this respect the CAS programs are very suitable.
4. Instructors before selecting what CAS programs will use need to do detailed study of the advantages and disadvantages of the each symbolic software in a specific course, their pitfalls,

domain validity, etc[1-4]. Systematic surveys are needed in order to determine the students' acceptance of various symbolic tools, their usage in learning and teaching processes, and their effects in increasing quality of EE education. However, one of the authors taught, over the years several electrical engineering and physics courses, by using computer algebra system as teaching aid. From his experience, the students learned quite fast to use CAS programs and to apply them in solving problems, simulation, plotting graphs, etc. Students' comments on the use of CAS programs in these courses have been generally very favorable.

5. It is clear that symbolic computations enable the instructors to derive almost everything and to solve quite easily many difficult problems. This makes the usage of this CAS programs very fruitful at the final stage of undergraduate or during the graduate studies. Future work may include the extension of symbolic computation to earlier stages of undergraduate programs, or to EE fields, such as electric machines and power systems, electromagnetics, antennas and radio propagation, etc.

References

1. J.M. Aguirregabiria et al, *Are we careful enough when using computer algebra,* Computers in Physics, Vol. 8(1), 1994, pp. 56-62.
2. A.I. Beltzer and A.L. Shenkman, *Use of Symbolic Computation in Electrical Engineering,* IEEE Trans, on Educ., Vol. 38(2), 1995, pp. 177-184.
3. A.I. Beltzer, *Engineering Analysis via Symbolic Computation,* Applied Mech. Rev, Vol. 43(6), 1990, pp. 119-128.
4. C. Casniazares and Z.T. Faur, *Advantages and Disadvantages of using Various Computer Tools in EE Courses,* IEEE Trans, on Educ., Vol. 40(3), 1997, pp. 166-172.
5. D.M. Cook et al., *A Comparison of Several Symbolic Manipulating Program: Part I,* Computers in Physics, Vol. 6(4), 1992, pp. 411-419.
6. D.M. Cook et al., *A Comparison of Several Symbolic Manipulating Program: Part II,* Computers in Physics, Vol. 6(5), 1992, pp. 530-541.
7. R.R. DeLyser, *Using Mathcad in Electromagnetics Education,* IEEE Trans, on Educ., Vol. 39(2), 1996, pp. 198-209.
8. J. Hillel et al., *Basic Functions through the lens of Computer Algebra Systems*, J. of Mathematical Behavior, Vol. 11, 1992, pp. 119-156.
9. T. Lee (ed), Mathematical computations with Maple V: Ideas and applications, Birkhauser, Bassel, 1993.
10. R. Lopez (ed), Maple V: Mathematics and its applications, Birkhauser, Bassel, 1994.
11. P. Lorrain et al, *Electromagnetic Fields and Waves,* WE Freeman, NY, 1988.
12. N. MacDonald, *Introduction physics students to symbolic education,* Eur. J. of Physics, Vol. 12, 1992, pp.10-15.
13. H. Pota, *Computer-aided Analog Electronics Teaching,* IEEE Trans, on Educ., Vol. 40(1), 1997, pp. 22-35.
14. O. Royrvik, *Teaching EE using Maple,* Int. J. of El. Eng. Educ. Vol. 39(3), 2003, pp. 297-310.
15. C. Wooff and D. Hodgkinson, *Computer algebra systems in physics teaching ,* Eur. J. of Physics, Vol. 9, 1988, pp. 145-149.

RADIAN BELU

Dr. Belu received the MSc in Physics from University of Bucharest, M.Sc in Electrical Engineering and PhD in Power Engineering from Polytechnic University of Bucharest, Romania, and PhD in Physics, from The University of Western Ontario, London, Canada. He is currently an assistant professor at the Wayne State University. His research interests include, space and atmosphere physics, power quality, radar and remote sensing, electromagnetic compatibility, radio propagation, and engineering education.

ALEXANDRU CATALIN BELU

Mr. Belu received B.Sc. and M.Sc. in Software and Electrical Engineering from The University of Western Ontario, London, Canada, He se currently graduate student in Applied Mathematics at Wayne State University, Detroit, Michigan, USA.