AC 2010-1180: TEACHING LAPLACE CIRCUITS AND SYSTEM ANALYSIS WITH VARIOUS ENGINEERING APPLICATIONS IN MECHANICAL ENGINEERING PROGRAM

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Teaching Laplace Circuits and System Analysis with Various Engineering Applications in Mechanical Engineering Program

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Abstract

This paper presents new pedagogies developed from our experiences in teaching transform circuits and linear system analysis in the mechanical engineering curriculum. Linear Circuit Analysis II is a core course for sophomores with the mechanical engineering (ME) major. ME students are required to take this second electrical course with a focus on using Laplace Transform techniques to analyze linear circuits and various engineering systems. The course prerequisites to take this course include the knowledge of basic electrical DC and AC circuits, ordinary linear differential equations with constant coefficients, mass/spring systems, translational systems, rotational systems, and beam deflection equations. In this course, ME students will continue to explore advanced techniques to establish more complex math models for both electrical and mechanical systems. Then, students will solve them by using a direct method in the real domain and by applying the transform methods either in the frequency domain (Phasor Transform) or in the complex s-domain (Laplace Transform). Since the application of transform techniques is much quicker and more efficient, especially when a real system carries the initial condition(s) or boundary conditions, we will focus on the Phasor Transform to determine the steady-state response for an AC circuit and the Laplace Transform to derive the complete system solution, which includes transient and steady-state responses for both electrical and mechanical models. By offering a broad coverage of topics and case studies, this course could possibly be beneficial to the majority of engineering students.

Unlike a dynamical model describing an electrical circuit often accompanied with initial condition(s), many mechanical math models are subject to boundary conditions; therefore, the Laplace solutions for both cases are presented in the class. Meanwhile, more Laplace properties associated with derivatives, translations that include time domain shifting and frequency domain shifting, and piecewise-defined functions are covered in order to explain how to handle more complicated systems. In this paper, we will explain course prerequisites and describe our teaching methods. We will address the outcome of students' achievement, which will include applications of their acquired transform skills, and their motivation for continuing to pursue upper-level courses such as Dynamic System Modeling and Feedback Control Systems. Finally, we will examine the course assessment according to our collected data from grading students' course work, course evaluation, and learning outcome survey, and further address the possible course improvements based on our assessment.

I. Introduction

The transform techniques, including the Laplace transform, the phasor transform, the Fourier transform, and the Z- transform have continuously impacted the disciplines of electrical, mechanical, computer, and other engineering programs. This is due to the fact that the transform approach presents a much easier and more efficient method when we analyze and calculate responses for a complicated dynamic system. Mechanical engineering (ME) students should master these transform methods before they take upper-level ME design courses; however, due to time restraints, we only focused on the Laplace transform and the phasor transform and their applications in both electrical and mechanical fields in one semester. As for the Fourier transform and the Z-transform, they will be covered in future classes.

Our ME program is designed for sophomores with a focus on the Laplace transform, transform techniques and their applications in both electrical and mechanical areas. The course prerequisite entails that students have already acquired working skills of the basic DC series, parallel or series-parallel circuit structures, fundamental electrical laws, linear ordinary differential equations, translational systems, spring/mass systems, rotational systems, and beam deflection equations. ME students in this course will continue to explore the advanced electrical circuit analysis, such as phasor solutions for AC circuits, frequency responses for RLC networks, complete RLC network responses via Laplace circuits with initial conditions, etc. They will also learn to apply the Laplace transform as a shortcut to solve different types of mechanical systems with initial conditions or even boundary conditions. While offering a broad coverage of topics and case studies in various engineering fields, this course could be beneficial to all engineering students.

Since teaching advanced electrical analysis or networks to mechanical students has been a challenging task, we focused on the common and popular transform methods, which are widely applied in both electrical and mechanical areas, and present more electrical/mechanical application examples to motivate ME students. In addition, before the transform concept is introduced, a math study such as complex analysis, the Laplace transform and its properties are given. In this paper, we will present our experiences from teaching the subjects of advanced electrical and system analysis in the mechanical engineering curriculum.

The paper is organized as follows. We first explain the course prerequisites and describe our designed course content, and then we will introduce different types of engineering system solutions via different transform methods. We will also present the course assessment. Finally, we will address a possible improvement of the course content.

II. Course Prerequisites

In this section, we will explain the course prerequisites, which can be divided into three categories, as described below.

A. Electrical Engineering Course Requirement

The first electrical course covering the basic topics of electrical concepts, voltages and currents, sources, components, series circuits, parallel circuits, series-parallel circuits, and electrical fundamental laws is a prerequisite. ME students in the second electrical course will apply these established skills to analyze, implement, and verify various applications. Specifically, the skills of DC analysis will greatly help students to solve AC circuits or general RLC networks after the phasor transform and the Laplace transform are introduced, since basic DC laws will be expanded to impedance or s-circuits.

B. Math Requirement

After satisfying the prerequisite of the electrical course, students gain their maturity in the comprehension and application of math including calculus, linear algebra, and algebra. A firm grasp of calculus concepts is beneficial in understanding advanced course materials, such as how to establish and solve a differential equation from a basic electrical RLC network, a mass/spring system, a translational system, or a rotational system. The linear algebra skills will also help students to write and solve the network equations in a standard matrix format.

C. Mechanical Engineering Course Requirement

For ME students, analyzing and solving the fundamental mechanical systems by applying free body diagrams and Newton's Law/D'Alembert's Law is required; this requirement was enforced in the previous ME courses. Students should be able to handle the first order and the second order differential equations based on the fundamental engineering laws. Students are also encouraged to simulate their algorithms using C ++ or MATLAB.

III. Course Content and Typical Application Examples

We have divided the course content into three portions. First, complex numbers and phasors were reviewed, emphasizing on transforming a sine function in the time-domain to a phasor in the frequency domain. Then, the Laplace transform was introduced, and the inverse Laplace transform was performed using a method of partial fraction expansion. The second portion focused on the circuit impedance concepts and the benefits of applying the transform approach to solve circuits. Finally, in the third portion, we taught students various techniques using the Laplace transform tool to solve both electrical circuit responses and mechanical structure responses with side conditions (i.e. initial conditions plus boundary conditions).

The course was taught for 16 weeks with 3 lecture hours weekly. The textbook selected was "Basic Engineering Circuit Analysis", published by John & Wiley Sons, Inc. 2007 [1]. The book covers most of the topics required by the course, specifically AC circuits and power analysis, steady–state frequency responses, the Laplace circuits and their solutions, the determination of a transfer function, and the calculation of a circuit's step response or impulse response. The text presents course materials at an appropriate math level, uses an ample amount of accurate examples, adopts MATLAB and PSPICE programs to demonstrate simulations, and provides real application examples to motivate students. The book also covers materials taught in the first electrical circuit course (Chapter 1 through Chapter 7 [1]) and can serve as a comprehensive reference.

To minimize the amount of time used to learn different simulation tools, we simply selected MATLAB, a major simulation and design tool, which was familiarized by students when they took their first engineering course, "Engineering Fundamentals". However, other simulation tools, such as PSPICE, C++, Multisim, etc. were also welcomed when time permitted it.

A. The Steady-State Solutions of AC Circuits Using the Phasor Transform

We first briefly reviewed several key topics from the first electrical course: DC series/DC parallel/DC series-parallel combination circuits, Ohm's Law, Watt's Law, voltage divider rule, current divider rule, capacitors, inductors, etc. These topics were covered in Chapters 2, 3, 5, 6 and 7 [1]. Then we introduced the impedance (Z) concept for R, L, and C in its phasor format: $Z_R = R$, $Z_L = j\omega L$ and $Z_C = 1(j\omega C)$, we also explained how to convert all the sources, voltages or currents in an AC circuit to their phasor forms. Before some illustrative examples were given to solve the circuits, a general procedure was summarized.

1. Find the impedances for all of R, L, and C,
2. Convert all the sources, voltages, and currents into their phasor forms,
3. Redraw the AC circuit in its phasor equivalent,
4. Solve the obtained phasor circuit including its voltages, currents and powers using the generalized electrical laws.

Through our lecture examples and homework assignments, students understood and mastered the important *"transform approach"* which could be applied to find system solutions more easily and efficiently.

B. The Complete Response of Electrical Circuits with Initial Conditions Using the Laplace Transform

In the second portion of the course, The Laplace transform, which is a very important transform method, was introduced. We first defined the Laplace transform, and then discussed how to determine the Laplace transform for general signals and the inverse Laplace transform via the standard Laplace table or the partial fraction expansion [2]. Another type of impedance, which is called the Laplace or s-impedance was also developed for R, L and C: $Z_R = R$, $Z_L = sL$, and $Z_C = 1/sC$. In addition, if any reactive element (C or L) in the circuit contained the initial condition, it could be converted to either a voltage source ($v_C(0^-)/s$ for an initialized capacitor or $Li_L(0^-)$ for an initialized inductor) series with its corresponding Laplace impedance or a current source ($Cv_C(0^-)$ for C or $i_L(0^-)/s$ for L) paralleled with its corresponding Laplace impedance [3]. In the mean time, we took the Laplace transform to all the circuit sources, voltages, and currents before the transformed Laplace circuit was redrawn. In summary, a general procedure of applying the Laplace transform approach to solve an electrical circuit is described below:

1. Find the Laplace impedances and consider the possible initial condition(s) for all of R, L, and C,
2. Apply the Laplace transform to all the sources, voltages, and currents in the circuit,
3. Redraw the circuit in its Laplace equivalent,
4. Solve the obtained Laplace circuit including its voltages and currents using the generalized electrical laws,
5. Convert the related voltages or currents back to the time domain expressions via the inverse Laplace transform.

The following example illustrates how to apply the above procedure:

Consider the circuit shown in Figure 1, in which $v_{in}(t) = 5u(t)$, $v_{c1}(0^-) = 3\ V$, $v_{c2}(0^-) = 0\ V$ and $i_L(0^-) = 2\ A$. Find $v_{out}(t)$.

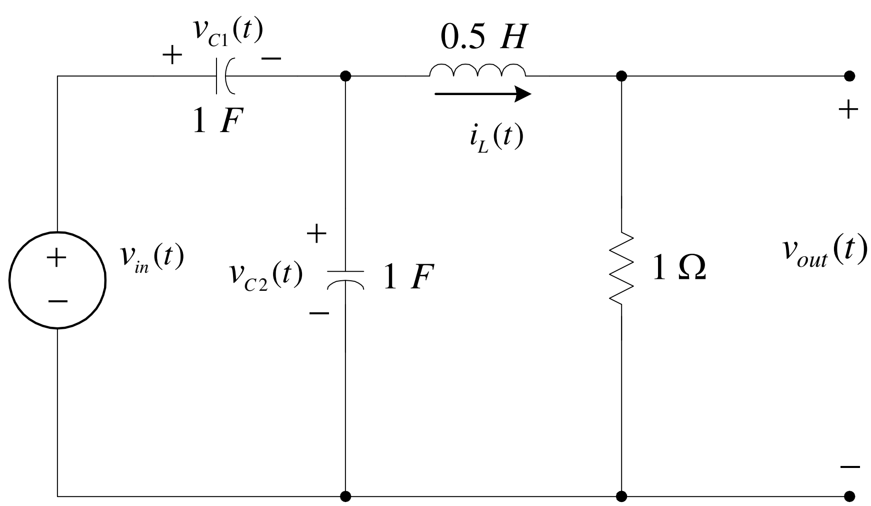

Figure 1. A RLC network with initial conditions.

Solution: After converting all of the circuit elements to their Laplace impedances and the initialized capacitor and inductor to voltage sources, we developed the Laplace equivalent for the circuit in Figure 2. Note that two (2) loop currents are also assigned in Figure 2.

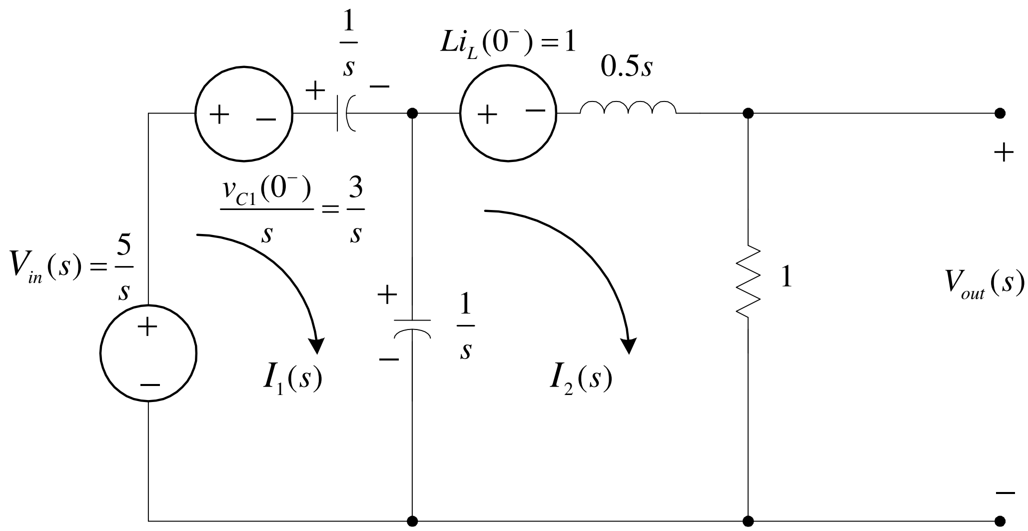

Figure 2. Laplace equivalent circuit in Figure 1.

Establish two (2) loop equations to solve $I_2(s)$,

For loop 1: $\frac{1}{s}I_1+\frac{1}{s}(I_1-I_2)=\frac{5}{s}-\frac{3}{s}$

For loop 2: $\frac{1}{s}(I_1-I_2)=(0.5s+1)I_2-1$

From the first equation, we obtain $I_1=1+0.5I_2$ and substitute it into the second equation,

$$\frac{1}{s}(1+0.5I_2-I_2)=(0.5s+1)I_2-1$$

Solving for I_2, we find

$$I_2=\frac{s+1}{0.5(s^2+2s+1)}=\frac{2}{s+1}$$

By Ohm's Law,

$$V_{out}(s)=I_2=\frac{2}{s+1}$$

Finally, the inverse Laplace transform gives the output voltage:

$$v_{out}(t)=L^{-1}\left\{\frac{2}{s+1}\right\}=2e^{-t}u(t).$$

C. The Complete Response of Mechanical Systems with Initial Conditions Using the Laplace Transform

The third portion of the course focused on solving various mechanical systems by using the Laplace transform. The Laplace solution for a mechanical structure is especially beneficial to ME students, since they often encounter higher order differential equation solution with certain initial conditions in vibration systems, translational systems, or rotational systems [4]. It is known that the Laplace transform can easily convert an ordinary differential equation *with any initial condition* into an algebraic equation with variable s. Therefore, the system solution can be determined easily and quickly. The general procedure to solve a mechanical structure is illustrated as follows:

1. Establish the necessary differential equation(s) based on its free-body diagram(s) and the fundamental laws, such as Newton's law, D'Alembert's law, Hook's Law, etc.
2. Take the Laplace transform to the differential equation(s) including possible initial conditions,
3. Solve the obtained Laplace transform equation(s) algebraically
4. Evaluate the inverse transform(s) to obtain the system solution(s) in the time-domain.

The following example demonstrates the application of the procedure:

The translational system shown in Figure 3 has the parameter values M=1 kg, B= 4 Ns/m, and K= 3 N/m. The applied force is $f_a(t) = 9u(t)$ N. The mass has no initial velocity, but it is released from a position 1 m to the right of its equilibrium position at the instant the force is applied. Decide the displacement $x(t)$ for all $t > 0$.

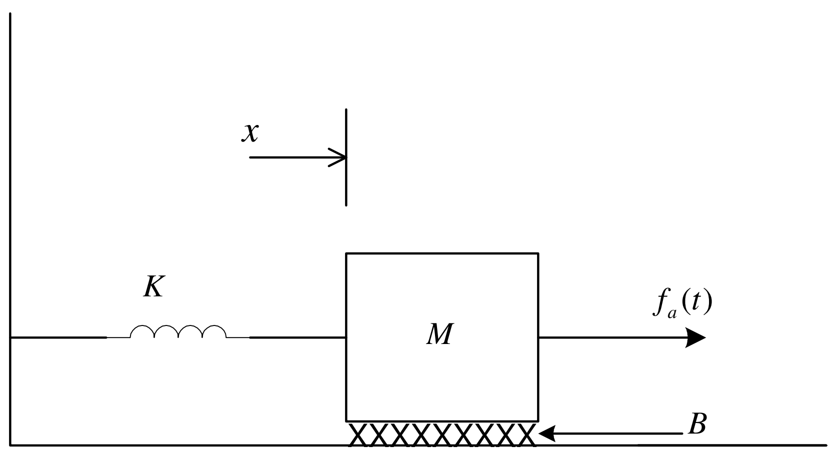

Figure 3. A translational system.

Solution: For $t > 0$, the differential equation describing the system from Newton's Law is

$$M\ddot{x} = f_a(t) - B\dot{x} - Kx$$

It becomes $\ddot{x}+4\dot{x}+3x=9u(t)$ when the parameter and input values are substituted. Transforming the differential equation term by term gives

$$[s^2X(s)-sx(0)-\dot{x}(0)]+4[sX(s)-x(0)]+3X(s)=9/s$$

Substituting the specified initial conditions $x(0)=1$ and $\dot{x}(0)=0$, we solve it algebraically

$$X(s)=\frac{s+4+9/s}{s^2+4s+3}=\frac{s^2+4s+9}{s(s+1)(s+3)}$$

Applying the partial fraction expansion, we have

$$X(s)=\frac{A_1}{s}+\frac{A_2}{s+1}+\frac{A_3}{s+3}$$

where residues are found to be $A_1=3$, $A_2=-3$ and $A_3=1$.

Taking the inverse Laplace transform of $X(s)$, we obtain the displacement in the time-domain

$$x(t)=(3-3e^{-t}+e^{-3t})u(t)$$

D. Boundary Conditions in the Laplace Transform

Since many mechanical math models not only contain initial values, but are also subject to boundary conditions, the Laplace solutions for both conditions are presented in the class. Thermal systems, vibration systems, beam deflections, and buckling columns are some practical examples with boundary conditions [4]. We only gave the introductory idea of handling boundary conditions via the Laplace transform due to the time restraints. However, students will study more complicated cases carrying the boundary conditions in future mechanical courses. Below is a simple example for solving the boundary value problem by using the Laplace transform.

For a system model $\ddot{y}+2\dot{y}+y=0$ with boundary values of $\dot{y}(0)=2$ and $y(1)=2$, find $y(x)$.

Solution: First, we take the Laplace transform term by term for the differential equation,

$$s^2Y(s)-sy(0)-\dot{y}(0)+2[sY(s)-y(0)]+Y(s)=0$$

Since $\dot{y}(0)=2$, but $y(0)$ is unknown, we assume $y(0)=c$ and rearrange the equation,

$$[s^2+2s+1]Y(s)-cs-2-2c=0$$

Solving for $Y(s)$, we obtain

$$Y(s)=\frac{cs+2+2c}{s^2+2s+1}=\frac{c(s+1)+(c+2)}{(s+1)^2}=\frac{c}{s+1}+\frac{c+2}{(s+1)^2}$$

Taking the inverse Laplace transform, we have

$$y(x)=ce^{-x}+(c+2)xe^{-x}$$

Applying the boundary condition $y(1)=2$ in $y(x)$ gives,

$$y(1)=2=ce^{-1}+(c+2)e^{-1}$$

Solving for c from the above equation to yield

$$c=e-1$$

Therefore, the final solution is given by

$$y(x)=(e-1)e^{-x}+(e+1)xe^{-x}$$

IV. Course Outcome and Assessment

Upon the completion of the course, a survey was conducted to ask each student to evaluate his or her achievement. Table 1 indicates the survey results. Note that the rating scale in Table 1 was based on the percentage of the overall students.

Table 1 Student Survey for achievements.

Rating scale	Level of understanding	Assignments	Excitement	Textbook
4 – excellent	85%	80%	80%	90%
3 – good	15%	20%	15%	10%
2 – fair	0%	0%	5%	0%
1 -unsatisfactory	0%	0%	0%	0%

Most of the students remained excited about the course, since the coverage of both electrical and mechanical fields had motivated them. Some students felt that more computer simulations were needed, because the course contained no lab sections based on the ME curriculum. The textbook also helped a great deal to develop electrical concepts using a large amount of numerical examples and software simulations.

V. Course Improvement and Future work

From our experiences in teaching the electrical course to mechanical students, we felt that the course contains well-established topics with suitable lectures and assignments. Students have the option to complete their MATLAB portions after class to enhance their simulations. We have also felt that a particular pre-requisite course, Linear Circuit Analysis I, plays a key role for success in the current class. We have decided to strengthen the topics such as KVL, voltage divider rule, and mesh analysis for this pre-requisite course. We would also like to see more comprehensive and challenging projects developed by students. In addition, more resonant circuits and the Fourier transform can be covered when time permits.

VI. Conclusion

We have enhanced the electrical course to include the general solution for both electrical circuits and mechanical systems, especially focusing on the applications of transform approach. From our course assessment, we have found that the transform method with various applications in multiple fields can motivate students not only to achieve the course objectives, but also to raise their levels of engineering analysis. With a well-established circuit and system background, students could continue to pursue our advanced control courses in their senior year. In addition, students can apply their system knowledge and skills in their senior designs.

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