AC 2007-353: USING FINITE DIFFERENCE METHODS INSTEAD OF STANDARD CALCULUS IN TEACHING PHYSICS

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1. Introduction

Physics is the basis of innumerable technological applications. It has shaped the face of contemporary society and represents the paradigm of all exact sciences. For a professor, making physics more accessible to students is not only of extreme importance but also one of the most challenging and rewarding tasks. The method of teaching physics has gone remarkably unchanged for decades even when the contents of the subject are up to date. Sometimes modern teaching technologies are adopted, but most often only in laboratories, lecture delivery and presentations. In the last few decades several authors have stressed the importance of using numerical methods in introductory physics courses due to the increasing availability of personal computers and computer algebra systems. A rigorous understanding of physics presumes a rigorous understanding of standard calculus. Classical physics, however, can be reformulated using finite difference calculus instead of standard calculus. This reformulation is rigorous and in the case of classical mechanics it avoids assuming that space-time is differentiable and thus is conceptually more consistent with the intrinsic discrete nature of time and space[6,7,16]. **On the other hand physics in its intimate nature is a discrete science and most engineering and physics problems have no analytic solutions[1-7, 12-15].** This raises the question of whether physics can be understood without using standard calculus. In answering this question we notice that an alternative to standard calculus is *finite difference approximations,* which, has been widely used to solve physics and engineering problems, especially, when we are dealing with problems that have no analytic solutions. From an educational point of view this can dramatically enlarge the base of the examples used to support courses especially in mechanics, acoustics, electrodynamics, fluid mechanics, and modern physics. This motivates our interests in the reformulation of classical mechanics, electrodynamics and acoustics by using finite difference approach instead of standard calculus.

This reformulation, based on finite differences, together with a discussion of some of the educational aspects is presented in this paper. The finite difference techniques are intuitive modeling techniques easily understood and applied by the vast majority of students. In addition to the insight that can be gained on classical mechanics, acoustics, electrodynamics as within other branches of physics there is also the possibility of studying real-world physics and engineering problems that cannot be resolved analytically[10, 11, 12-14]. Also this approach accords the benefit of exploring many other areas of physics that otherwise require advanced mathematical techniques and knowledge. Last but not least we have to mention that in order to take the full advantage of these potentialities the numerical methods that should be understandable to the students should be easily programmable (preferable by using computer algebra systems) and efficient so that accurate results can be obtained without excessive computational resources and time.

The paper is organized as follows. Section 1 of this paper is reserved for introduction and we will sketch the finite difference methods, section 2 and 3 are reserved for the presentation of the discrete formulation of mechanics and electrodynamics, and we review some of the engineering applications of this approach in section 4. The pedagogical implementation of such formulations is discussed in section 5, and the last section is reserved for conclusions, discussion and future work.

1.1 Finite Difference Methods

The finite difference methods was developed by A. Tom in the early 1920s under the title "methods of squares" to solve non-linear hydrodynamic equations[9,13]. Since then the method has found several applications in solving different physics and engineering problems. The finite difference methods are based upon an approximation that permits the replacement of differential equations by finite difference forms. These finite difference approximations are algebraic in form; they relate the value of dependent variable at a point in the solution region to the values at some neighboring points. The finite difference methods have been applied successfully to solve many problems of mechanics, acoustics, electrodynamics, fluid mechanics, etc. Any approximation of a derivative in terms of values at a discrete set of points is called ***finite difference approximation***. The approach used in obtaining finite difference approximations is based on the using of Taylor series to approximate the derivative of a function[6-8,12]. The time derivative of particle position *x(t)* is expressed by:

$$\frac{dx}{dt} = \lim_{T \to 0} \frac{x(t+T) - x(t)}{T}$$

Virtually all-classical equations of physics are defined in terms of position, energy, potential, fields or other quantities and their first and second order derivatives, so rigorous understanding of classical mechanics or electrodynamics requires knowledge of calculus. ***If it is decided that calculus and advanced mathematics are necessary to understand, as well as to present and teach physics, that than it is also, in the opinion of the authors, beneficial to have alternative approaches in making physics more accessible to the students***. Our attempt in teaching physics is based on the use of the finite difference calculus instead of standard calculus. This involves replacing derivatives of physical quantities by their finite difference counterparts, as for example, the time derivative of the particle position is replaced by:

$$Dx(t) = \frac{x(t+T) - x(t)}{T} \tag{1}$$

where, T is the smallest time interval. Notice that the finite difference operator converges to time derivative as T goes to zero. This replacement will lead, in the case of classical mechanics, to a minor reformulation of energy, momentum and acceleration, while the discrete electrodynamics will remain quiet straightforward. This provides the basis for a rigorous mathematical treatment of classical mechanics or electrodynamics that is more accessible to the students. According to Lakshminkanthan and Trigante, 1988 or Greenspan, 1975 this operator has the following properties:

$$D[\alpha \cdot f(x) + \beta \cdot g(x)] = \alpha \cdot f(x) + \beta \cdot g(x) \qquad (2)$$

This is the well-known linearity property.

$$D[f(x) \cdot g(x)] = f(x) \cdot Dg(x) + g(x) \cdot Df(x) + TDf(x) \cdot Dg(x) \qquad (3)$$

And

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot Df(x) - f(x) \cdot Dg(x)}{g(x)\left(g(x) + TDg(x)\right)} \qquad (4)$$

The second-order time difference operator is defined as:

$$D^2 x(t) = \frac{x(t+T) - 2x(t) + x(t-T)}{T^2} \qquad (5)$$

Similar operators can be formulated, as we will show in section 3, in the case of discrete electrodynamics for any quantities used in classical physics.

Notice that we use the rescaled operator instead of the standard finite-difference operator. This operator converges to the standard derivatives as T goes to zero. To define an operator Σ, which is the inverse of D-operator, let:

$$DF(x) = f(x)$$

i.e. $f(x) = [F(x+T) - F(x)]/T$. If $C(x)$ is a periodic function with $C(x+T) = C(x)$, then

$$D[F(x) + C(x)] = f(x)$$

This means that we can infer $F(x)$ up to an arbitrary periodic function, $C(x)$. The rescaling summation operator, Σ is now defined as

$$\Sigma[f(x)] = F(x) + C(x) =$$
$$= T[f(x-T) + f(x-2T) + \Lambda + f(0)] + C(x) \qquad (6)$$

And the summation operator, Σ^*, of the finite difference calculus is given by:

$$\Sigma^{*} f(x) = f(x-T) + f(x-2T) + \Lambda + f(0) + C(x)/T$$

so,

$$\Sigma[f(x)] = T\Sigma^{*}[f(x)]$$

We can use the known summation operator properties to infer the following properties for the rescaled summation operator, *Σ*:

$$\Sigma[\alpha f(x) + \beta g(x)] = \alpha\Sigma f(x) + \beta\Sigma g(x) \tag{7}$$

This is the well-known linearity property for the summation operator. The discrete form of integral by parts is given by:

$$\Sigma[f(x)Dg(x)] = f(x)g(x) - \Sigma[Df(x)g(x) + TDg(x)] \tag{8}$$

where, we suppressed the arbitrary period function, *C(x)*. These formulae reduce to the standard calculus formulae when *T* goes to zero. We also have

$$\lim_{T\to 0}[\Sigma[f(x)] + C(x)] = \int f(x)dx + const \tag{9}$$

This paper will only focus on rescaled summation operators with specified limits of integration so that we can drop the arbitrary function *C(x)*.

2. Classical Mechanics

With the approach presented in the previous section we will reformulate some of the equations and theorems of classical mechanics, following the approaches developed in references [1, 3, 4, and 5].

2.1 Newton's Laws

The standard Lagrangian formulation of the classical mechanics leads in the discrete-tine case to the following version of Newton's laws, with standard notation of mechanics:

$$F(t) = m \cdot a(t) = Dp(t) \tag{10}$$

with the acceleration, using (5) defined by:

$$a(t) = \frac{x(t+\Delta t) - 2x(t) + x(t-\Delta t)}{(\Delta t)^2}$$

and with momentum defined by

$$p(t) = m \cdot Dx(t - \Delta t)$$

For the classical Lagrangian $L = \frac{1}{2}m(Dx)^2 - V$ (where V account for the potential energy), the energy function is defined by

$$E = \frac{1}{2}mDx\,(t)Dx\,(t-\Delta t) + V\,(x)$$

The energy is conserved when L is not an explicit function of time. Thus, all the standard quantities of classical mechanics have their counter-parts in the discrete mechanics. Moreover, several formulations of discrete mechanics have been studied, during the last four decades, both physically and mathematically [1-8, 16]. *These formulations were explicit, and for* $\Delta t = T > 0$ and $t_k = k{\cdot}\Delta t$, *and* $k = 1, 2, \cdots, N$, utilized, in one dimension, the formulae:

$$m \cdot a_k = F\left(x_k, v_k, t_k\right) \tag{11}$$

$$a_k = \frac{v_{k+1} - v_k}{\Delta t} \tag{12}$$

$$\frac{v_{k+1} + v_k}{2} = \frac{x_{k+1} - x_k}{\Delta t} \tag{13}$$

for $k = 0, 1, \cdots$. (11) is another discrete from of Newton's equation, (12) relates velocity and acceleration, and (13) relates distance and velocity. The feasibility of this formulation is derived from the proof given by the classical conservation law [6,7], and from the applicability to several non-linear problems of physical interests [7,8,9].

2.2 Newton's Equation and Initial-Value Problem

In a more general case, following the approach in Greenspan 1975, we consider a particle P of mass m being in motion on x-axis at x_k and with a velocity v_k, then the equation

$$F\left(x_k, v_k, t_k\right) = ma_k \tag{14}$$

which, at time t_k relates the form acting on P to its acceleration, is called a discrete Newton's equation. The actual determination of the motion of P from dynamical

Equation (14) when the initial values x_0 and v_0 are known is called ***an initial-value problem***. Before developing additional physical concepts, let us illustrate the solutions of some of the well-known dynamical problems.

a) Dynamics given a constant force: Consider a constant force $F = -k/m$. This implies

$$x(t+\Delta t) - 2x(t) + x(t-\Delta t) = -\frac{(\Delta t)^2 k}{m} \tag{15}$$

The solution is:

$$x(t) = -\left(\frac{k}{2m}\right)t^2 + v_0 t + x_0$$

and

$$Dx\ (t) = -\left(\frac{k}{m}\right)t + v_0$$

For the corresponding problem in classical mechanics:

$$\ddot{x}(t) = -\frac{k}{m}$$

which is the same solution to that given by the calculus-based dynamics.

b) The harmonic oscillator: Consider a linear force $F = k(y+a)$, where parameter a is a constant. Newton law in the case of harmonic oscillator becomes:

$$mD^2 y(t) = -k\big(y(t) + a\big) \tag{16}$$

Be defining $x(t) = y(t) + a$ the problem simplifies to:

$$x(t+\Delta t) - \left[\frac{(\Delta t)^2 k}{m} - 2\right]x(t) + x(t-\Delta t) = 0$$

The solution of this equation is given by:

$$x(t) = K_1 \cos\left(\frac{t}{\Delta t}\theta\right) + iK_1 \sin\left(\frac{t}{\Delta t}\theta\right) + K_2 \cos\left(\frac{t}{\Delta t}\theta\right) - iK_2 \sin\left(\frac{t}{\Delta t}\theta\right)$$

where $\theta = \cos^{-1}\left(1 - 1/2\nu^2\right)$ and $\nu = \Delta t\left(k/m\right)^{1/2}$

By choosing the complex coefficients of K_1 and K_2 to make $x(t)$ real, we finally get:

$$x(t) = c_1 \cos\left(\frac{t}{\Delta t}\theta + \Phi\right) \tag{17}$$

Now if we had solved the continuous version of the problem, we would have found

$$x(t) = c_1 \cos\left(\frac{t}{\Delta t}\left[2\left(1 - \cos\theta\right)^{1/2}\right] + \Phi\right)$$

Notice that discrete oscillator has a higher frequency than the continuous oscillator when $\theta > \left[2\left(1 - \cos(\theta)\right)\right]^{1/2}$. Thus discreteness causes the oscillator to move at a slightly increased frequency.

c) Nonlinear and damped pendulum: Motion of a nonlinear, damped pendulum, characterized by the dynamical equation:

$$a_k = -\alpha v_k - \sin x_k, \qquad k = 0,1,2,\Lambda \tag{18}$$

where α is the damping coefficient. The last equation can be rewritten as:

$$v_{k+1} = v_k + \frac{\Delta t}{2}\left[\alpha\left(v_{k-1} - 3v_k\right) + \left(\sin x_{k-1} - 3\sin x_k\right)\right], \qquad k = 1,2,\Lambda \tag{19}$$

Considering a particular initial value problem, in which a pendulum is released from a rest position at angle $\pi/4$, then:

$$\begin{aligned} v_1 &= -\left(\Delta t\right)\sin\frac{\pi}{4} \\ v_{k+1} &= v_k + \frac{\Delta t}{2}\left[\alpha\left(v_{k-1} - 3v_k\right) + \left(\sin x_{k-1} - 3\sin x_k\right)\right], \quad k = 1,2,\Lambda \end{aligned} \tag{20}$$

and

$$x_{k+1} = x_k + \frac{\Delta t}{2}\left[v_{k+1} + v_k\right], \qquad k = 0,1,2,\Lambda \tag{21}$$

The motion of the pendulum is generated recursively by using equations (20) and (21). Moreover, one can prove theoretically that the solution of this initial-value problem exists, is unique, and is given constructively by (20) and (21). The immediate availability of existence, of uniqueness, and of the solution itself, without any topological considerations, extends to all initial value problems formulated in terms of (12), (13) and (14). In this fashion we can therefore resolve nonlinear problems directly, without linearization, using only explicit, arithmetic formulae.

2.3 Conservation of Energy and Momentum

For the discussions in the previous section, the general procedure to be used in the general initial-value problem is given by:

$$v_1 = v_0 \left(\frac{\Delta t}{m} \right) F\left(x_0, v_0, t_0\right)$$

$$v_{k+1} = v_0 \left(\frac{\Delta t}{m} \right) \left[\frac{3}{2} F\left(x_k, v_k, t_k\right) - \frac{1}{2} F\left(x_{k-1}, v_{k-1}, t_{k-1}\right) \right] \qquad k = 1,2,\Lambda$$

$$x_{k+1} = x_k + \frac{1}{2} \Delta t\left(v_{k+1} + v_k\right) \qquad k = 1,2,\Lambda$$

The work done by a force F on a particle P moving from x_0 to x_k (1-D case) is given by

$$W = \left(x_1 - x_0\right) F\left(x_0, v_0, t_0\right) + \sum_{k=1}^{n-1} \left(x_{k+1} - x_k\right) \left(\frac{3}{2} F_k - \frac{1}{2} F_{k-1} \right)$$

or

$$W = \frac{m}{2} \left(v_1^2 - v_0^2\right) + \frac{m}{2} \sum_{k=1}^{n-1} \left(v_{k+1}^2 - v_k^2\right) \tag{22}$$

So that,

$$W = \frac{m}{2} \left(v_n^2 - v_0^2\right) = K_n - K_0 \tag{23}$$

Which is the discrete form of conservation law of the energy, and in a similar way one can get the conservation of momentum. Notice that these concepts can be generalized to higher dimensions by similar approaches.

2.4 Discrete Form of the Virial Theorem

If *E[x]* denotes the average value of x over a long period of time, then the virial theorem states that a system of *n* particles:

$$E[T] = E\left[\sum_{i=1}^{n} m_i \left(\frac{dx_i}{dt}\right)^2\right] = -\frac{1}{2}\sum_i E\left[F_i x_i\right] \qquad (24)$$

where F_i is a force on particle *i*. This result also holds using the finite-difference calculus. Let:

$$G = \sum_i p_i x_i \Rightarrow DG = \sum_i \left[Dp_i x_i + \left(p_i(\Delta t)Dp_i\right)Dx_i\right]$$

When: $L = \frac{1}{2}m(Dx)^2 - V(x,t)$, and $F_i = Dp_i$ and

$$p_i + (\Delta t)Dp_i = mDx_i$$

Thus,

$$DG = \sum_i \left[F_i x_i + m\left(Dx_i\right)^2\right]$$

Requiring $DG = 0$ yields:

$$E[\Delta T] = E\left[\frac{1}{2}m(Dx_i)^2\right] = -\frac{1}{2}\sum_i E\left[F_i x_i\right]$$

which proves the virial theorem.

3. Discrete Electrodynamics.

In the introductory section of this paper it was argued that it is natural to introduce classical physics problems, such as field theory or classical electrodynamics, using a discrete point of view. This runs counter to the intuition of most physicists; we argue here that it is natural because we have all been indoctrinated in an unnatural approach, for reasons (namely; our lack of ability to solve realistic problems by hand using discrete equations) that are no longer relevant in the world of computers. As evidence for this point of view we pointed out that field theories are always introduced discretely at the very beginning.

In fluid mechanics one begins by considering a fluid element and counting the momentum, mass, and energy passing in or out through its faces. In electrodynamics, one derives the continuity equation for charge by counting the charges passing the faces of a discrete volume element. However, in traditional approach one immediately abandons the discrete picture, takes the continuum limit, and writes partial differential equations. One then learns to solve "textbook problems" using analytic techniques, and only much later (often not at all, in formal course work) learns the discrete methods that are usually used to solve real-world problems. This is a distortion of the natural sequence. In the discrete case one starts with a picture that is much easier for students to grasp than the symbolic manipulations of vector calculus; the natural sequence would be to do model calculations, and then calculations of realistic systems, using the discrete equations, and leave the highly abstract analytic methods for last, after some intuition has been developed for how electromagnetic fields behave. The distorted sequence has been placed upon physicists for the first couple of centuries of our experience with field theories because discrete calculations are tedious to do by hand, but since computers become widely available this distortion has persisted only through inertia.

Another commonly perceived advantage of the continuum approach is the existence of several useful integral relations (Ampere's law, Gauss' law, and Faraday's law), symmetries, and the law of conservation of energy (Poynting's theorem), which are not necessarily true in discrete theory. The discretization presented here has been chosen to have all these desirable properties. Furthermore, the properties are all straightforward and rigorously provable using algebra, whereas very few students/physicists ever learn to prove them in the continuum approach – they accept them on faith. The elegant integral relations that make electrodynamics attractive seems much more elegant if one can prove them. Once a result is obtained in the discrete theory, it is trivially true in the continuum limit; learning the corresponding continuum limit; learning the corresponding continuum result requires no additional effort.

Maxwell's equations are equations for determining the time evolution of two vector fields, the electric field ***E*** and the magnetic field ***B***.

$$\frac{d\vec{E}}{dt} = c^2 curl\ \vec{B} - \frac{1}{\varepsilon_0}\vec{j} \tag{25}$$

$$\frac{d\vec{B}}{dt} = -curl\ \vec{E} \tag{26}$$

And the continuity equation

$$\frac{d\rho}{dt} = -div\,\vec{j} \tag{27}$$

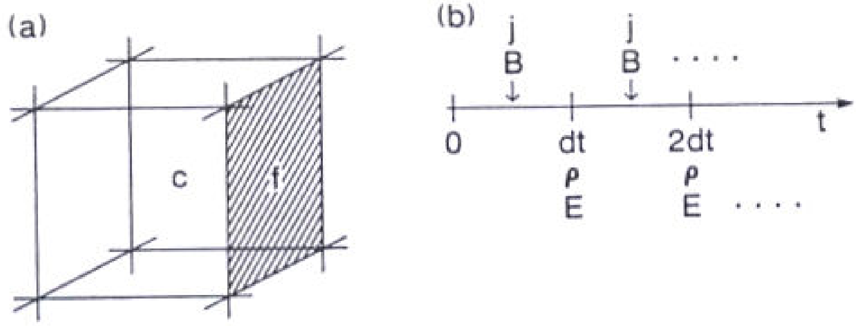

Figure 1: Cubic lattice

The discrete forms of these equations, considering an elementary cubic lattice (see Figure 1 for details of the computational cell) are:

$$\frac{\rho\left(c,t+\frac{1}{2}\Delta t\right)-\rho\left(c,t-\frac{1}{2}\Delta t\right)}{\Delta t}=-div\,\vec{j}=-\frac{1}{dr}\sum_{f}\vec{j}(f,t) \qquad (28)$$

$$\frac{E\left(f,t+\frac{1}{2}\Delta t\right)-E\left(f,t-\frac{1}{2}\Delta t\right)}{\Delta t}=c^{2}curl\,\vec{B}-\frac{1}{\varepsilon_{0}}\vec{j}(f,t) \qquad (29)$$

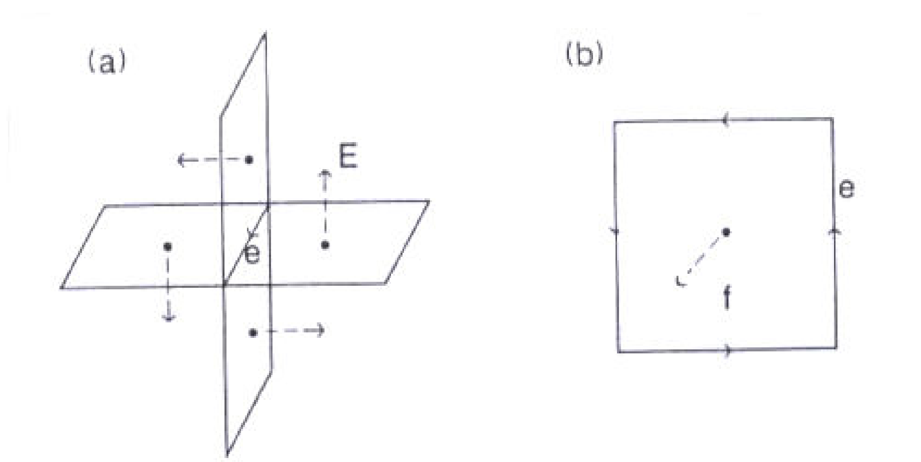

Figure 2: Geometry for electric and magnetic field computation

The discretization of curl, having an electric field for the diagram in Figure 2, yields to:

$$[curl\,\vec{E}](e,t)\equiv\frac{1}{dr}\sum_{j}E(f,t) \qquad (30)$$

The curl of E appears in the Maxwell's equations for ***dB/dt*** [Equation (26)], defined at the edges of the cell in Figure 2b, it is discretized as:

$$\frac{E\left(f, t+\frac{1}{2}\Delta t\right) - E\left(f, t-\frac{1}{2}\Delta t\right)}{\Delta t} = [curl\,\vec{E}](e,t) \qquad (31)$$

The curl of ***B***, appearing in Equation (24) is discretized as:

$$[curl\;\vec{B}](e,t) \equiv \frac{1}{dr}\sum_{j} B(e,t) \qquad (32)$$

Equations (28), (31), and (32) define the discrete electromagnetic system. This is not only the simplest discretization but also has some very nice properties. All the integral relations and other theorems that are true of the continuum electric and magnetic fields are exactly true of this discretization, and can be proved using simple algebra. For example, adding up Equation (28) over a set of cells comprising a region of space gives the integral form of the continuity equation, relating the sum of the charges in a region to the sum of the discrete currents at its surface.

The discrete electrodynamic system described above can be easily simulated on a computer. The computation of electromagnetic fields is needed for an abundance of everyday applications, such as antennas, radars, microwave devices, electric machines, transmission lines, radio, etc. The combination of discrete approach to Maxwell's equations with the opportunity to play with such computer simulations allows a student to acquire a physical understanding of a wide variety of electrodynamic phenomena with a minimum of formal mathematics. The interest in the algorithm described above is also based on its pedagogical simplicity.

3.1 Laplace Equation

One of the most used applications of finite difference in electromagnetics is to apply it to solve Laplace equation for electric potential [10 - 15].

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \qquad (33)$$

Applying the central difference formula, and using $x = i\Delta x, \quad y = j\Delta y, \quad i, j = 0,1,2,\ldots$ and assuming that $\Delta x = \Delta y = h$ (h is the mesh size) can be written as, for a computational molecule as in Figure 3:

$$\Phi(i,j) = \frac{1}{4}\left[\Phi(i+1,j) + \Phi(i-1,j) + \Phi(i,j+1) + \Phi(i,j-1)\right] \qquad (34)$$

It is worth to noting that (34) states that the value of *Φ* (electric potential) at each point is the average of those at the four surrounding points. The five-point computation molecule for the difference scheme in (34) is illustrated in Figure 3. The finite differences are well suited for computing the characteristic impedance, phase velocity and attenuation of different types of transmission lines, such as: bifilar lines, coaxial cables, micro-strips, striplines, etc. The knowledge of the basic characteristics of these transmission lines is a paramount importance in design and analysis of electronic circuits and systems.

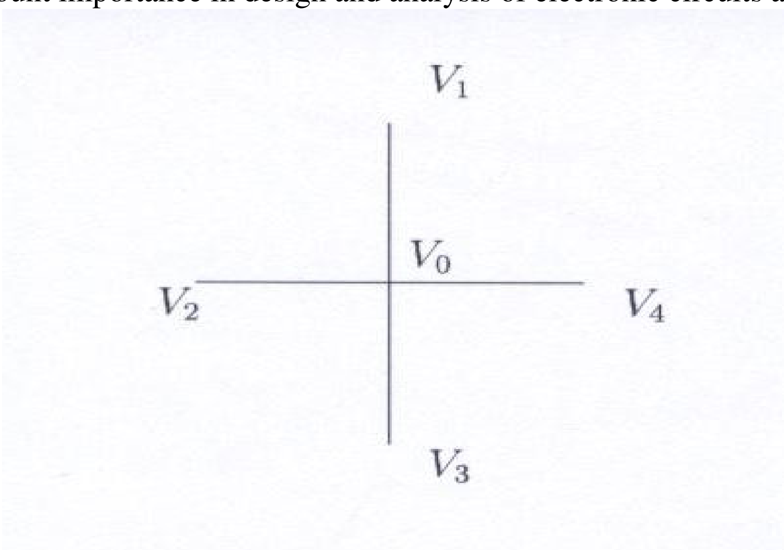

Figure 3: Computational molecule for Laplace's equation

For concreteness, following Sadiku, 1991[12], consider a micro-strip. Finite difference form of Laplace equation is applicable for this case, due to the fact that E and B have no components in the direction of propagation (so-called TEM mode, which is a good approximation if the dimensions of the line are much smaller than the wavelength). The techniques developed here is equally applicable to other transmission lines. Equation (34) can be written with the notation of Figure 3 as:

$$V_0 = \frac{1}{4}\left(V_1 + V_2 + V_3 + V_4\right) \qquad (35)$$

Equation (35) is a general formula to be applied to all nodes in the free space and the dielectric region of a micro-strip transmission line. At the dielectric boundary (Figure 4), the boundary condition is given in terms of the electric displacement vector:

$$D_{1n} = D_{2n} \qquad (36)$$

and must be imposed, based on the Gauss law. Expressed in terms of the geometry in Figure 4, the discrete form of this boundary condition can be expressed as:

$$V_0 = \frac{\varepsilon_1}{2(\varepsilon_1 + \varepsilon_2)} V_1 + \frac{1}{4} V_2 + \frac{\varepsilon_1}{2(\varepsilon_1 + \varepsilon_2)} V_3 + \frac{1}{4} V_4$$

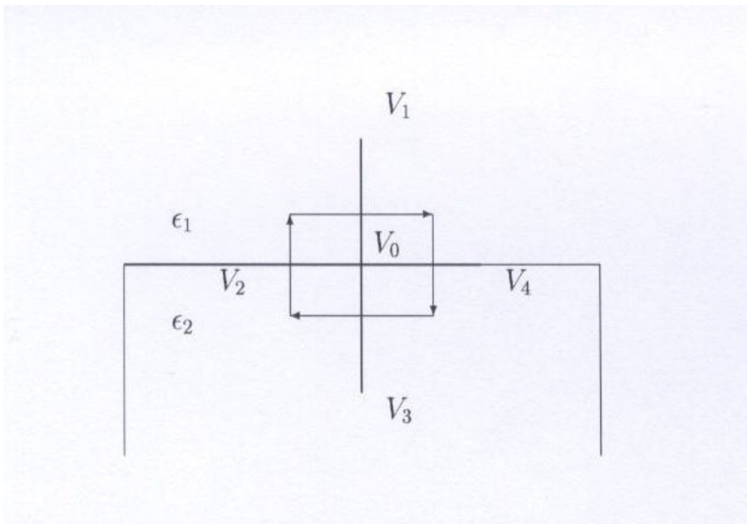

Figure 4: Interface between two media of dielectric permitivities.

On the line symmetry, we impose the condition

$$\frac{\partial V}{\partial n} = 0 \qquad (37)$$

This can be expressed in discrete form as:

$$V_0 = \frac{1}{4}\left(V_1 + 2V_2 + V_3\right)$$

By setting the potentials at fixed nodes equal to prescribed values and applying the finite difference relationships described above, one can determine the potentials at free nodes. Once this is accomplished the quantities of interests can be easily computed.

3.2 Transmission Lines – Finite Difference Approach

The usual mode of teaching about transmission lines is to use a circuit model[11-13] with a series of connected capacitors, inductors, and resistors. The starting point of any transmission line calculation is the two coupled "transmission line equations", as:

$$L\frac{\partial I}{\partial t}+RI+\frac{\partial V}{\partial z}=V_0$$

$$C\frac{\partial V}{\partial t}+GV+\frac{\partial I}{\partial z}=0 \tag{38}$$

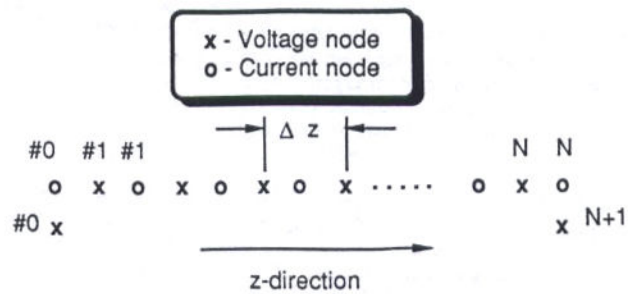

Figure 5: Illustration of the spatial finite-difference grid

I and V are the time and space-dependent current and voltage along the line, while L, C, R, and G are respectively the inductance, the capacitance, the series resistance, and shunt conductance per-unit of length. In general these parameters can also depend on position and time. The finite-difference formulation of equations (38), using the mesh in Figure 5, gives:

$$L\frac{I_k^{n+1}-I_k^n}{\Delta t}+RI_k^{n+1}+\frac{V_{k+1/2}^{n+1/2}-V_{k-1/2}^{n+1/2}}{\Delta t}=E_{0,k}^{n+1/2}$$

$$C\frac{V_{k+1/2}^{n+1/2}-V_{k+1/2}^{-1/2n}}{\Delta t}+GV_{k+1/2}^{n+1/2}+\frac{I_{k+1}^n-I_k^n}{\Delta t}=0 \tag{39}$$

One can simply solve equations (39) for the “newer” values of I and V with the results:

$$I_k^{n+1}=\frac{E_{o,k}^{n+1/2}-\left(V_{k+1/2}^{n+1/2}-V_{k-1/2}^{n+1/2}\right)}{\left(\frac{L}{\Delta t}+R\right)\Delta z}+\frac{I_k^n}{\left(1+\frac{R\Delta t}{L}\right)}$$

$$V_{k+1/2}^{n+1/2}=\frac{-\left(I_{k+1}^n-I_k^n\right)}{\left(\frac{C}{\Delta t}+G\right)\Delta z}+\frac{V_{k+1/2}^{n+1/2}}{\left(1+\frac{G\Delta t}{C}\right)} \tag{40}$$

One calculates the time-domain response by assuming some initial conditions (usually that currents and voltages are all zero at $t = 0$) and then calculating time steps in an iterative manner. In general numerical calculations are much less time consuming than if the traditionally circuit network approach is used.

4. Discussions and Conclusions

In this paper we presented in simple terms the basic concepts of finite difference methods, the presentation of classical mechanics, and electrodynamics via several examples. Algebra and calculus are two of the basic tools of mathematical physics. One of the main goals of any educator is to maximize the learning, while minimizing the teaching. Classical mechanics and electrodynamics can be reformulated using finite difference calculus instead of the traditional approach of using standard calculus. This reformulation is rigorous and in the case of classical mechanics it avoids assuming that space-time is differentiable and this is conceptually more consistent with intrinsic discrete nature of time and space. In the case of teaching electricity and magnetism, one can introduce Maxwell's equations at the beginning, such that an electric field is always a dynamical variable existing at each point of space. This is in generally not possible in a continuum approach, because the average physics or engineering student's lack of background in advanced calculus would make the continuum of Maxwell's equations meaningless. However, the discrete Maxwell's equations can be understood and applied to many problems through only basic algebra: problems become no more difficult or abstract than Coulomb law. Indeed, for a student who is uncomfortable with the idea of "action at a distance" they are in fact easier to understand than Coulomb's law. Using discrete approach, electrostatics can be introduced after Maxwell's equations and thus it becomes just the study of limiting form of the electric field at long times. Coulomb's law is a simple consequence of the discrete Maxwell's equations. If one uses a transient current that flows from infinity to a single cell (leaving a charge in that cell), the electric field will eventually settle down to a static value (i.e., Coulomb's law) and the magnetic field will settle down to zero.

Other advantages of the using finite difference formulation are: a) it can dramatically enlarge the number of examples that may be used during the lectures, and b) the opportunity of involving students in solving non-trivial "real-world" problems, which, would not only be very challenging but also appealing for them. In summary, a discrete approach to teaching classical mechanics and electrodynamics allows one to convey, at the introductory level, the conceptual simplicity of Newton's Laws or Maxwell's equations great synthesis of dynamics or electricity and magnetism into consistent theories described by the equations, such as the ones presented in this paper. Simple algebraically solvable problems and computer simulation of others make it possible to develop an intuitive understanding of the phenomena of classical mechanics and electrodynamics that does not depend of the abstract mathematics of advanced calculus. While teaching in Romania, one of the authors used this approach extensively in teaching classical mechanics and electromagnetics. Later, he used the same approach in the United States in teaching engineering electromagnetics. The response of the students to the finite difference approach in teaching mechanics and electromagnetics was surprisingly favorable.

5. References

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