AC 2011-16: DARING YOUNG ENGINEERS ON THE FLYING TRAPEZE: USING CIRCUS ARTS TO TEACH DYNAMICS

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Introduction

Anyone who has had the experience of watching a circus, after having studied dynamics, may have been struck by the thought that much of the equipment used is, in some ways, very similar to that used in an introductory dynamics class. One can view a flying trapeze as a large pendulum, a bungee trapeze as a mass and a spring, and a German wheel as a slipping (or non-slipping, depending on the simplifications made) disk rolling on a surface. "Dynamics with Circus Laboratory" was designed to explore these connections in a fun, intensive elective course.

This course was developed to supplement engineering students' exposure to dynamics, and to give them hands-on experience doing experiments related to dynamics. Mechanical Engineering students at the University of St. Thomas are required to take a traditional Mechanics course in which dynamics is taught, however this new course exposed/introduced students to additional topics, such as Lagrangian dynamics, that are not covered in the required class. This paper will describe the initial offering of the course, focusing on content and lessons learned.

Course Logistics

This course was a 2-credit elective course offered in an intensive four week format (required engineering courses at the University of St. Thomas are 4 credit classes). Class met for three hours a day, Monday through Friday. Lab sessions were held on Tuesdays and Thursdays at Circus Juventas. The balance of the five days were used for lecture and data analysis.

As a prerequisite for this course, students were required to have completed a calculus-based Physics class. Eleven registered for the class. Ten of the students were Mechanical Engineering majors, and one was an Electrical Engineering major. Students ranged from sophomores to seniors.

The syllabus description for the course is as follows.

> *This course will cover dynamics including rotating reference frames and rigid body dynamics. Unique to this course are the lab sessions, which will occur at Circus Juventas. Exercises and experiments involving the flying trapeze, Spanish web, and other circus equipment will be performed to strengthen understanding of the material covered in class. (Note that students will not be required to try out the circus equipment, however all students will have the option to do so.)*

The stated learning objectives for the course were that at the end of the four week session:

- Students should be able to write the equations of motion for a variety of systems using both momentum principles and Lagrangian dynamics.
- Students should be comfortable performing reference frame transformations.
- Students will have been exposed to a number of experimental methods for performing dynamics research.

Overall, it was hoped that this class would allow the students a chance to explore how real world dynamic systems relate to theoretical models and to gain an appreciation for some of the challenges of conducting experimental work on moving systems.

Course Schedule

This class took place during the university's J-Term, an intensive four week January term during which students typically take a single two or four credit class that meets on a daily, or almost daily, basis. Table 1 lists the schedule with the topics for each class (lecture or lab) and the assignment due dates associated with each. The details for each topic and assignment are described in the sections that follow. It should be noted that the course, for the most part, stayed on schedule, but that occasional extra computer lab time was added to give students more time to work on lab analysis.

Books and Equipment

The textbook for the class was *Fundamentals of Applied Dynamics*[1]. While this was a recommended, rather than required, textbook, most students in the course purchased it.

For the labs in this course, three different motion analysis tools were used.

- PASCO[2] "Amusement Park Physics Bundle" sets which consist of:
    - A data-logger
    - A 3-axis acceleration altimeter sensor
    - A vest to hold the sensor/data-logger
- KA Video[3] human movement analysis program
- Dartfish[4] image processing software

The KA Video and Dartfish programs, while new to our Engineering students, are taught and used in the university's Health and Human Performance department, which kindly allowed the students to use their facilities and equipment.

Course Grading

The course grading consisted of four elements:

- Problem Sets 60%
  Problem sets consisted primarily of questions in which the students were asked to derive the equations of motion for rigid body systems.

- Coaching Notebook 5%
  Each student was asked to bring a notebook with them to all lab sessions. They were to record advice and comments from the lab coaches in their notebook, as well as their written notes for the experiments that they performed.

- Lab Write-ups 15%
- Final Project 20%
  The lab assignments and final project will be discussed below.

It should be noted that as it became clear how much time the lab write-ups were taking, students were given the option of having the write-ups count for 30% of their grade, reducing the weighting for problem sets to 45%.

Table 1: Class schedule

Week	Day	Agenda	Work Due
1	**1**	Course logistics; kinematics; rotation	
	2	LAB: Introduction to Circus Arts	
	3	More kinematics	Problem Set #1
	4	LAB: German wheel	
	5	COMPUTER LAB: Introduction to DARTFISH and KAnalysis	Problem Set #2
2	**6**	Momentum	German wheel Lab Write-up
	7	LAB: Low-casting and flying trapeze	
	8	Variational dynamics (Lagrangian dynamics)	Problem Set #3
	9	LAB: Low-casting and flying trapeze	
	10	Variational dynamics (Lagrangian dynamics)	
3	**11**	NO CLASS (University holiday)	
	12	LAB: Bungee trapeze	Problem Set #4
	13	Dynamics of systems containing rigid bodies	Flying Trapeze Write-up
	14	LAB: Spanish web	
	15	Dynamics of systems containing rigid bodies	
4	**16**	More dynamics!	Bungee Trapeze Write-up
	17	LAB: Demo prep	
	18	Final Project Work Day	
	19	LAB: Demonstration day (for 6th grade audience)	Problem Set #5 (due the following day)

Course Staff

Due to the nature of this course, a larger than usual number of staff were involved. In addition to the lecturer, who is a professor in the Engineering department, there was an undergraduate teaching assistant from the Health and Human Performance department who was a skilled user of KA Video and Dartfish. Additionally, each lab session involved a varying number of coaches from the circus school.

Lab Units

As the main innovation for this course was the lab component, we will explore each of the labs in detail below. Each of the labs was tied to a course topic and used a specific set of equipment. Given the short duration (four weeks) of the course, we were limited in the number of labs that we could accomplish. There are an almost unlimited number of dynamics labs that could be designed for the circus, and we hope to have the chance to develop these in future iterations of the course.

Warm Up Lab: Introduction to Circus Arts

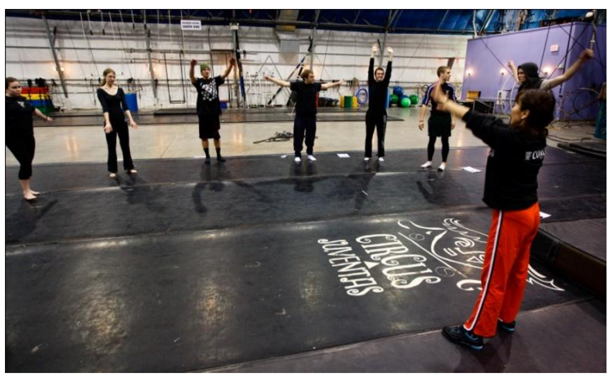

Figure 1: Students warming up with coaches

Before students launched into the formal engineering labs, an “Introduction to Circus Arts” session was held in which students were introduced to the circus school and its equipment. They were given a chance to meet the coaches that they would be working with, as well as to try out various pieces of equipment. The goal for this session was to get all of the course participants comfortable with one another and with the space they would be working in.

German Wheel and Rotating Reference Frames

While the German wheel may be readily identified as a piece of circus equipment, many would not know its name. It comprises two large hoops, oriented with a common axis, connected shoulder-width apart by several rods used to support the performer. Because the German wheel is effectively a cylinder, it tends to roll in a single plane. During a simple rolling maneuver, it is manipulated by the performer’s shifting center of mass. This lab focuses on the simplest of dynamics of a rolling wheel. The students experimented with fixed centers of mass. Since this was the first lab for the course, the students explored reference frame transformations. As with all of these labs, the topic can be altered to suit the needs of a particular curriculum. For example, here, it is easy to see how one would alter the lab to accommodate a study of the center of mass (fixed or changing during the wheel motion.)

Prelab

Before lab the students were asked to calculate the velocity and acceleration of points on a nonslipping wheel as it rolls in a straight path (see Figure 2). Their assignment was:

In an inertial reference frame "locked" to the ground, calculate the velocity and acceleration of the following points when you are rolling in the German wheel.

- *The center of the hoop*
- *A point on the outside of the hoop in line with your head*
- *The top of your head*

Assume that the wheel is nonslipping. Use ***r*** *for the radius of the hoop.* Students were asked to bring this write-up with them to lab.

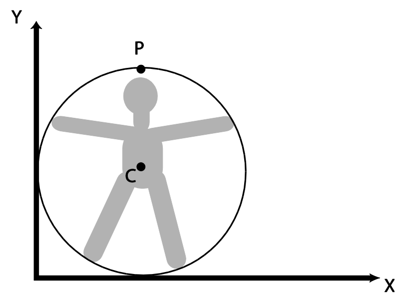

Figure 2: Simplified schematic for the German wheel

Lab

During the lab, students were taught how to perform a cartwheel in the German wheel. (see Figure 3) The students were dressed in fitted black clothing with white tape marking their shoulders, elbows, wrists, hips, knees, and ankles. The diameter of the German wheel was measured. Students were then videotaped as they performed a cartwheel in the German wheel. For scale, a calibration stick was filmed in the first few frames of each trial (see Figure 4).

Figure 3: A composite image (created using Dartfish) of a student performing a cartwheel in a German wheel

Figure 4: Reference stick being filmed before German wheel lab

Post-Lab Analysis

At the next class session, the Kinematic Analysis software KAVideo was used. Each pair of students chose one video to analyze together. The students digitized three points: the top of the head, the top of the wheel, and a third point of their choice. The third point could be the right shoulder, left knee, right wrist, etc. Digitizing specific points on the body allows the software to recognize specific XY coordinates of these points over the course of the video.

After the video segment was digitized, the students used a second program, KA2D, to view and analyze data from the recorded segment. For this lab, the students were specifically interested in the velocities. The XY velocities of each of the three points chosen above were exported into a spreadsheet. With this data, the students created plots of velocities and accelerations as a function of time.

Students were required to do a lab report in which they explained their process and compared their measured data to the theoretical velocity and acceleration plots created using the equations that they derived in their prelab. Figure 5 shows a comparison of the actual data to the predicted for the y-component of the velocities (for one student pair). It should be noted that, as expected, most students found a strong correlation between the predicted and actual velocities and accelerations.

The German wheel lab took approximately 2 hours at the circus, and 3 hours to complete the digital analysis in the computer lab. Students worked in pairs for both the lab and the lab write-up.

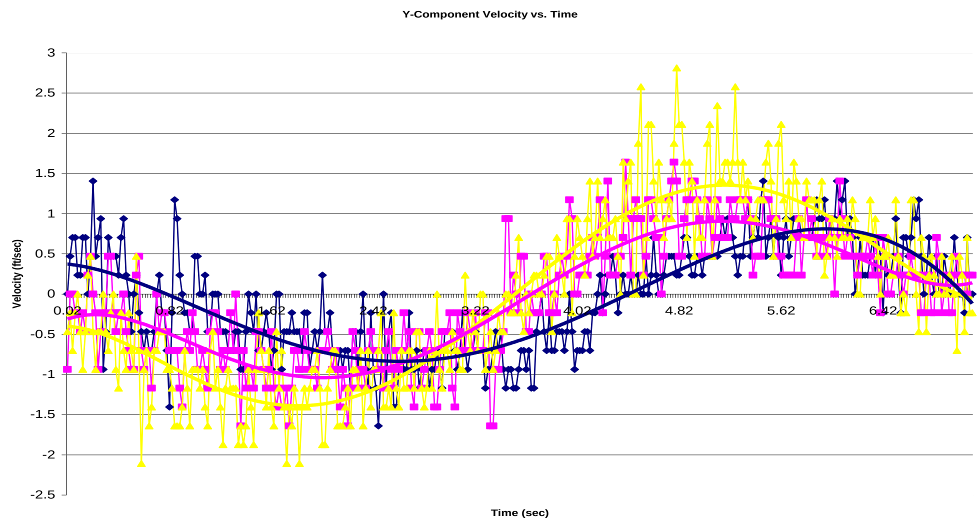

Figure 5: Comparison of Actual and Predicted Y-Velocities of a point on the German wheel.(Actual: jagged with markers; Predicted: smooth, no markers)

Flying Trapeze and Low-casting: Pendulum

Perhaps the most recognizable piece of circus equipment is the flying trapeze. Surprisingly, however, there is very little scientific literature on the dynamics of the flying trapeze. This lab focused on equations of motion for pendulums. Students explored:

- Is a simple pendulum a good model for the flying trapeze?
- Would a double pendulum be a better model?
- How does active swinging effect the motion of a flying trapeze artist**? (**In terms of momentum and energy, what purpose does "beating" serve when swinging on the flying trapeze?)

It should be noted that this lab involved both flying trapeze and low-casting. A low-casting rig is essentially a small-scale flying trapeze rig. This smaller setup allowed us to more easily place the cameras needed for the motion capture exercise. Alternately, the flying trapeze worked well for the capture of acceleration and velocity data using the data-logger vests.

Prelab

Students were asked to complete the following problems before the lab session.

Single pendulum:

- Derive the equation of motion for the single pendulum, shown in Figure 6, using the method of your choice. (It can be done in a straightforward fashion using the direct approach, the principle of angular momentum, or Lagrange's equation.)

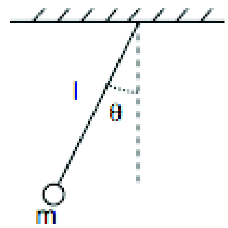

Figure 6: Single Pendulum

- Rewrite your solution using the small angle approximation.
    - Solve this differential equation for θ(t).
    - Solve for the period of oscillation
- Derive the equations of motion for the double pendulum shown in Figure 7 .

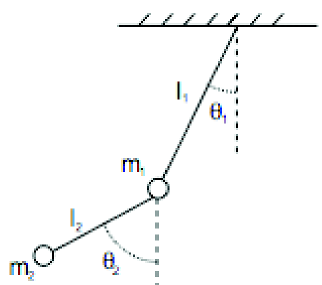

Figure 7: Double Pendulum

Lab

The first task for the students in this lab was to acquire measurements for the following variables for each apparatus:

- Weight of the trapeze bar
- Length of the trapeze cables
- Height of the trapeze platform
- Horizontal distance from trapeze platform to the trapeze pivot point

The students made the measurements for the low-casting rig. Given the difficulty of taking measurements of the larger flying trapeze rig, the necessary values for that equipment were provided for the students.

The class was split into two groups, with one group starting at the low-casting rig and one group starting at the flying trapeze. Both groups were able to try each experiment.

Low Casting

The initial portion of this lab consisted of motion capture for a low-casting trapeze swinging freely with no human on it. A piece of reflective tape was placed on the side of the trapeze and video was taken of it using the same process that was used for the German wheel lab.

Students were then asked to model a human on the trapeze as a double pendulum by hanging weights . To do this, students first needed to find the center of gravity for one member of their group. Note that the CG needs to be calculated with the student's hands extended overhead.

Weights (approximating the weight of this student) were hung from the trapeze using a strap with a length that was approximately the same as the distance between this student's CG and the trapeze.
Next, the student who was used for the center of gravity measurement was then marked with tape on their elbow, shoulder, hip, and knee. Video was then taken of this student swinging on the trapeze both passively (i.e., hanging) and while actively "beating." Beating is a method of kicking and positioning one's body so as to maximize the amplitude of their swing.

Flying Trapeze
Students donned PASCO 3-axis accelerometer vests and climbed up to the trapeze platform. Each student, in turn, mounted and swung on the trapeze (see Figure 8). The instructions to the students were as follows:

- For your first (passive) swing, do ***not*** beat (kick) or press. Simply step off of the trapeze platform (while holding onto the trapeze) and focus on trying to keep your body in a straight line.
- For subsequent (active) swings, work hard to follow your coaches' instructions for beating and pressing out.
- After every run, jot down your coaches' comments in your notebook and download the accelerometer data into the laptop computer.

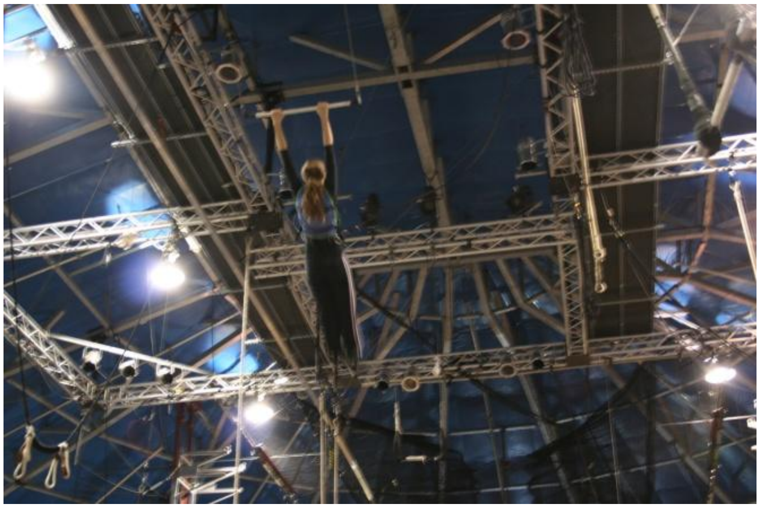

Figure 8: Student swinging on the flying trapeze wearing a data-logger

Post-lab analysis
The low-casting video was downloaded to KA Video. The students then digitized the video from their low-casting experiments. In their lab write-up students were asked to include the following:

- A comparison of the actual position of the trapeze swinging by itself and the predicted position for a simple pendulum
- A comparison of the actual position of the trapeze with a passive person hanging from it and the predicted position for a simple pendulum

- A comparison of the actual position of the trapeze with an active swinging person hanging from it and the predicted position for a simple pendulum
- The above three plots done with predictions based on a double pendulum (rather than a single pendulum)
- A discussion of the data above, and a discussion on what a better model might be

For the flying trapeze, students were asked to submit:

- Calibrated altitude data for their initial passive swing on the flying trapeze
- Calibrated altitude data for their first active swinging run
- Calibrated altitude data for their best (however they chose to define "best") swinging run
- A comparison of this data with a single pendulum model
- A comparison of the peak altitude for the passive and "best" swings. How much higher did beating allow the flyer to swing?
- An annotated set of data showing key points in the swing

The presentation of the data from the various students/groups varied, especially in the annotation of the data. Figures 9 and 10 are from the low-casting portion of the lab and illustrate data digitized from the video footage. Figure 9 is a plot comparing the measurements and predicted values for the single pendulum.

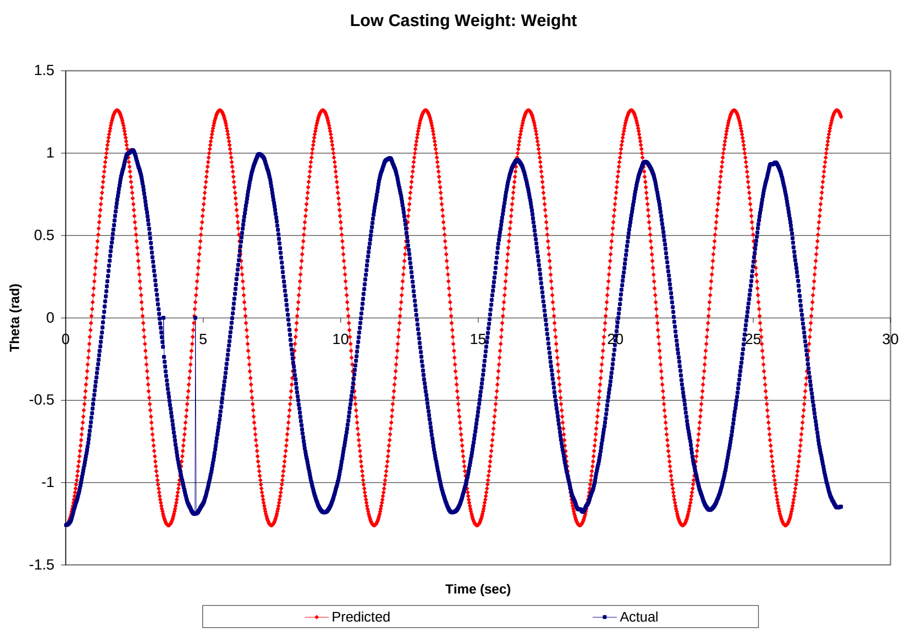

Figure 9: Single pendulum angles (radians) vs. time (seconds)
Predicted (red); Actual (blue)

The example low-casting plots in Figure 10 show the y velocities from one group for a student's passive and active swings as well as for a set of weights configured as a double pendulum.

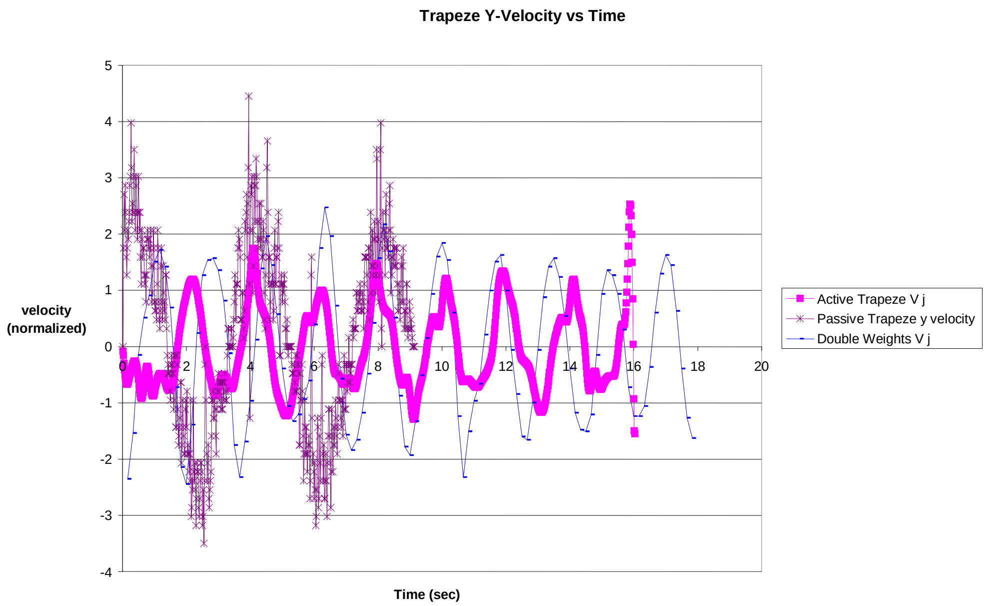

Figure 10: Y velocities for low-casting experiment.

The data from the flying trapeze was gathered using data collection vests. This data was periodically downloaded into a laptop vests and then analyzed by the students. Figure 11 is a plot comparing swings on the flying trapeze. Note how the labeling on the plots helps to understand how the data corresponds to the events. Each student begins their swing on the platform and after a number of swings drops into the safety net. The maximum amplitude of the passive swing decreases at a much greater rate than that of either of the active swings.

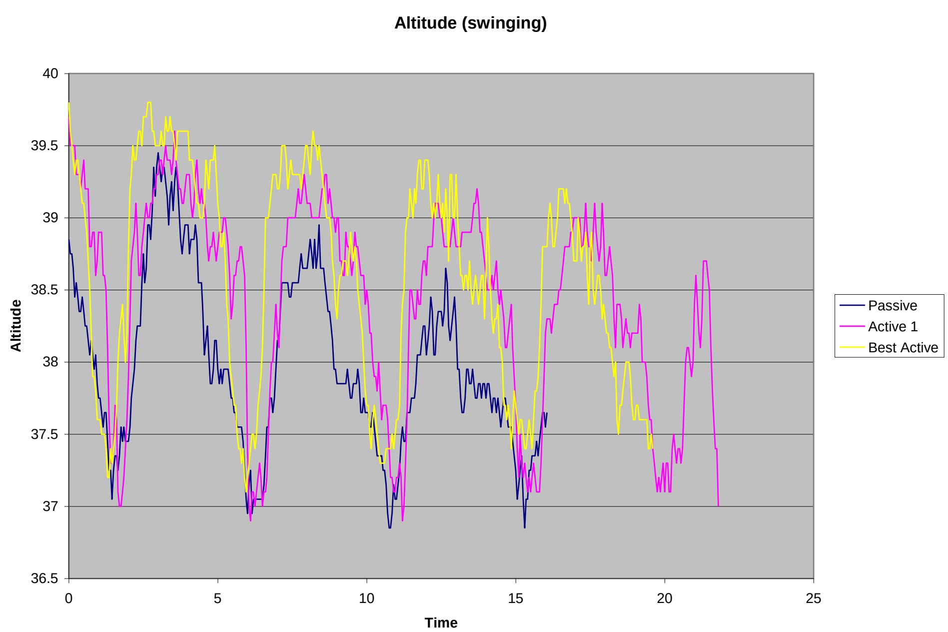

Figure 11: Altitude plot for active swinging on the Flying Trapeze. Passive (blue); Active 1 (magenta); Best Active (yellow).

Bungee Trapeze: Damped Oscillation

The Bungee Trapeze equipment includes a static trapeze, used as a platform, and a set of bungee cords attached to the performer. Maneuvers typically involve a drop from and a return to the trapeze with various aerobatic tricks in between. The performer is supported by the bungee cords while performing these tricks and may use a single or multiple "bounces" on the bungee cords. For some tricks, the performer is able to immediately regrasp the trapeze but, for most tricks, several bounces are used and the bungee cords must be repeatedly stretched to lift the performer high enough to mount the trapeze. Figure 12 illustrates the student mounted in a standing position on the trapeze with slack bungee cords (left) and suspended by the bungee cords (right).

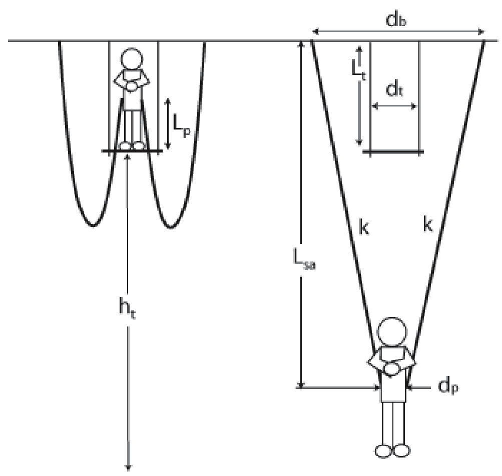

Figure 12: Illustration of the bungee trapeze lab

This lab focuses on the damped oscillations that come about as a result of the action of the bungee cords on the performer (without any tricks or action on the part of the performer/student).

- Derive a theoretical equation for the motion of a person stepping off of a trapeze while wearing bungee cords.
- Compare this theoretical model with actual data.
- Discuss the shortcomings of the theoretical model.
- How does the performer overcome the losses (damping)?

Figure 13 shows (on the left) three students measuring the length of a bungee cord stretched by a known weight. This information is used to calculate the spring constant of a length of bungee cord. Also shown (on the right) is a student suspended more than 20 feet off the ground by two sets of bungee cords connected at his waist. The student is wearing a data collection vest (light blue) that holds the sensors over the sternum.

Prelab

Prior to the lab the students were asked to derive the equations of motion for the system.

In-Lab Assignment

- Find the spring constant ***k*** for the bungee (this step is pictured in Figure 13)
    - Hang a single bungee from a fastening point
    - Measure the distance from the end of the bungee to the ground
    - Attach weights from the end of the bungee
    - Measure the new distance from the ground

- Repeat this process for at least 3 different values of weights

- Weigh each member of your lab group
- Record altitude data, using the sensor vests, of each person in the lab group stepping off the bungee trapeze, and passively bouncing until they are at rest (Figure 14).

Figure 13: (left) Students hanging weights from a bungee cord to estimate its spring constant, (right) student taking data on the bungee trapeze

Lab Discussion and Write Up

The students were asked to complete a laboratory writeup, as follows.

- Write a formal description of what was done in this lab.
- Derive, and include, the equation of motion for a person stepping off of the bungee trapeze.
- Find the value of ***k*** using the data taken by hanging weights from a single side of the bungee cables.
- Produce a plot of your height versus time, using your sensor data.
- Find ***k*** using your knowledge of the unstretched length of the bungees, the geometry of this system, and the position data for each of your lab members. Use L_0 for the unstretched length of the bungee cords, and L_{sa} for the vertical length of the angled, cord when the jumper is at rest (see figure below)[included here as Figure 12].
    - Did you find a different ***k*** value using this method? If so, discuss the possible reasons.
- Plot all three lab members' data on the same plot. As you know the weight of all three lab members, does this data look reasonable? Discuss.
- Pick one person's data and plot it against your theoretical model (using what you know of the person's height and weight.) [See Figure 14 for a sample of students' data.] Does it match? Why not? Are there some places where it matches better than others?
- For this model to be more accurate we will need to incorporate damping into the model. Based on the above plots, is this an underdamped, overdamped, or critically damped system?

An example plot from the bungee lab is shown in Figure 14. Both the measured and calculated/theoretical results are shown. The initial oscillations match very well but begin to differ as the oscillations are damped.

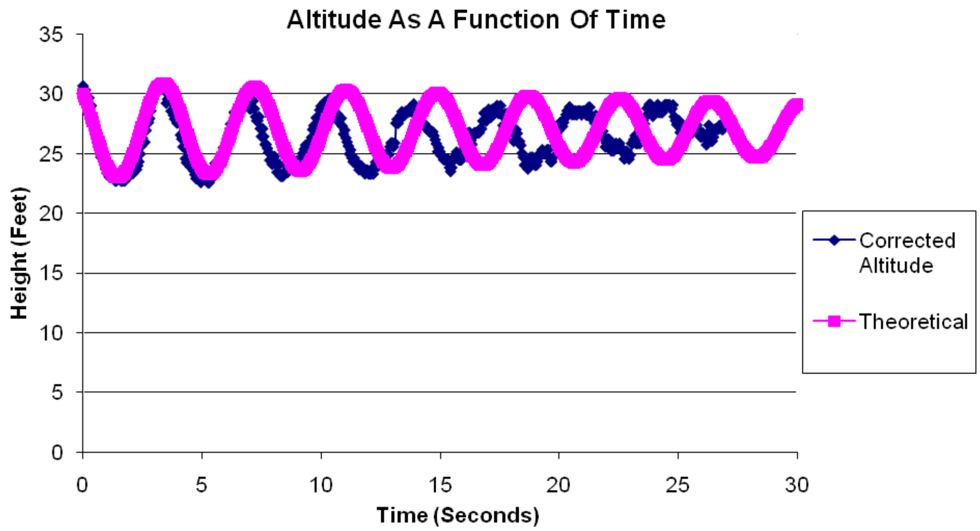

Figure 14: Sample of student data from the bungee lab

Final Project

As this class was an elective, it was decided that rather than a final, the students would be given a somewhat unusual project. Working in small groups, the students were asked to put together a "circus of dynamics" that would be appropriate for an audience of 6th graders. The final day of the course culminated in students from Farnsworth Aerospace School taking a field trip to Circus Juventas where the "Dynamics with Circus Laboratory" students performed their presentations (Figure 15).

- One group used the German wheel to demonstrate gravity
- One group used the low-casting rig to demonstrate how potential and kinetic energy can be converted back and forth (with a bonus discussion of how one's breakfast gives them the energy to perform such feats)
- One group used the bungee trapeze to discuss harmonic motion and damping

The presentation was deemed so successful that the group was asked to perform again for the general public, to an audience of hundreds of children and adults.

Figure 15: The finale of the class' presentation of "an engineering circus" to a class of 6th grade students

For the lead instructor one highlight of the course came during the presentation to the middle school students. While performing on the German wheel, one student slipped and fell. Uninjured, she stood up and looked at the kids and said, roughly, "This is what engineering is also about. Sometimes we make mistakes and fall down, but we pick ourselves up and try again." She then did just that, completing the maneuver successfully.

Challenges

A class like this brings with it its own unique set of challenges. First, a circus needed to be found which would be willing to collaborate. While this may seem like a unique situation, it should be noted that there are circuses and circus schools throughout the country. Alternately, one could imagine a similar course created around a gymnastics studio rather than a circus school. Transportation requirements to a possibly remote facility may need to be addressed.
The specific measurement and analysis tools used here are not required. You may find that additional or alternative equipment would be helpful for these or other experiments. If you do not already have the needed equipment, you may have access to a colleague's, if it can be located. Otherwise, they will have to be acquired.

We worked closely with our university to assess what, if any, safety waivers were needed. As we were working with a professional circus school, they provided their own liability forms which were filled out by all students in the class. It should be stressed that students were always given the option of observing the labs, or taking a role that did not involve performing any of the circus feats. For this iteration of the course, all students chose to participate.

Feedback

The feedback for this class was generally positive. Students were enthusiastic about the class, and put a lot of energy and time into their work. It was not unusual for the instructor to come in on weekends and find students working together on lab reports. The intensive nature of the course seemed to pull the students together as a team. The class will be offered again, and it is the authors' intent to do formal effectiveness assessment for the next iteration of the course.

Students did comment that they would have liked to have had the course stretched over an entire semester, instead of just a few weeks. On a related note, students did say that the course workload was too much at times. However, this did not seem to diminish their enthusiasm. One student did drop the class after approximately a week, while the remaining students completed all of the work successfully.

Acknowledgements:

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