AC 2011-1526: EXCEL ADD-INS FOR GAS DYNAMICS COURSES

Robert P Taylor, University of Alabama

Dr. Robert Taylor is professor of mechanical engineering at The University of Alabama and associate director of the Alabama Industrial Assessment Center. Before joining the UA faculty in 2004, he served for 25 years at Mississippi State University as an ME professor, associate dean of engineering, and interim dean of engineering. Bob has a BS and PhD from MSU and a master's degree from Purdue University. He also worked as an engineer for Texaco, Inc., early in his career. Bob teaches courses in heat transfer, energy systems design, and gas dynamics. His research interests are in the areas of heat transfer and fluid mechanics and engineering experimentation and uncertainty analysis. He is the coauthor of the textbook "Analysis and Design of Energy Systems" and the author and coauthor of approximately 150 journal articles and published technical papers.

Keith A. Woodbury, The University of Alabama

Excel Add-ins for Gas Dynamics Courses

Abstract

This paper discusses the development of an Excel add-in tool kit for basic gas dynamics. The tool kit includes functions for isentropic flow, normal and oblique shocks, expansion waves, flow with friction—Fanno flow, and flow with heat transfer—Rayleigh flow. The scope and availability of the tool kit are discussed, and examples are provided. The tool kit has been used for two classes of advanced undergraduate/beginning graduate student sections of gas dynamics in a mechanical engineering program. Classroom experiences and student viewpoints are discussed.

Introduction

With partial support of an NSF CCLI grant, the authors and their colleagues at The University of Alabama have developed and made available to the public a suite of Visual Basic modules in the form of "Add-in" macros for Microsoft Excel spreadsheets that provide the basis for computations in the mechanical engineering thermal science course sequence. Macros have been developed for thermodynamics, heat transfer, and energy systems. This paper discusses in detail the thermodynamics subset of functions for gas dynamics calculations.

The thermodynamics suite contains functions to compute steam properties, properties of refrigerants R22, R134a, R407c, and R410a, properties of calorically imperfect ideal gases, psychrometrics, and the gas dynamics relationships that are the subject of the present paper. The heat transfer suite contains functions for transient 1-dimensional transient conduction, fin efficiencies, heat exchanger effectiveness-NTU relations, convection heat transfer correlations, radiation view factors, and blackbody functions. The energy systems suite adds piping analysis modules and viscous pump corrections. Components and various stages of the development of these packages have been reported in references [1, 2, 3, 4, 5, and 6]

The theme is to take properties or relationships that were traditionally found in tables, charts, or nomograms, e.g., the steam tables, the Heisler charts for 1-D transient conduction, the Hydraulic Institute nomograms for pump performance corrections for viscous fluids, and replace them with readily available public-domain function calls in Excel. Furthermore, we did not want to merely replace the tables, charts, and nomograms with spreadsheet based calculators or spreadsheet templates. We wanted to provide the students and ultimately the practicing engineer with a problem solving worksheet that could adapt to the problem at hand instead of adapting the problem to the tool. Using Excel in the 4-column problem-solving format discussed below in the body of this paper and the Add-ins fits this theme well.

Microsoft Excel was selected to be the computational platform. Firstly, Excel is very powerful and flexible. It combines spreadsheet flexibility, programmability through Visual Basic, a broad suite of other engineering applications, visual analysis tools, and wide availability. The visual basic editor allows the programming tools needed to develop problem-solving algorithms. The spreadsheet format is very familiar to the students and allows data to be stored and displayed in a logical order. The charting and graphing capabilities provide good functionality to present results. Most importantly, Excel is ubiquitous. A recent survey of mechanical engineering alumni from our institution revealed that no one used software tools that were packaged with textbooks, only a small fraction used the higher-level computational tools they learned in the engineering curriculum such as Matlab and Ansys. Everyone used Excel on almost a daily basis. In addition, Excel is on almost every personal computer in the world. Every computer in the classrooms, in the labs, in the library, at work, at home will likely have Excel already installed.

The Add-in modules are all available in the public domain and can be downloaded from the website www.me.ua.edu/excel. The suites are packaged around the three areas, thermodynamics, heat transfer, and thermal systems. All in one downloads have been developed in Microsoft installer that install the add-in files and help files in the proper directories on your computer to make them available with the standard Microsoft Add-ins such as Solver and Analysis ToolPak. Many of the functions have "help on this function" screens that give the user information on the basis for the calculations and instructions on how to use the function. The website also contains tutorials on Excel basics, use of Add-ins, and use of Excel as an engineering problem solving platform.

The remainder of this article discusses the gas dynamic functions, the verification of the functions, the 4-column format to organize and document engineering problem solutions, example solutions of gas dynamic problems, and student reactions to the spreadsheet paradigm in a gas dynamics class. Finally brief conclusions are presented.

Description of Gas Dynamic Functions

Functions are given for isentropic flow with area change, normal shock waves, oblique shock waves, Prandlt-Meyer flow, Fanno-line flow, and Rayleigh-line flow. The functions are discussed below.

Isentropic Flow with Area Change

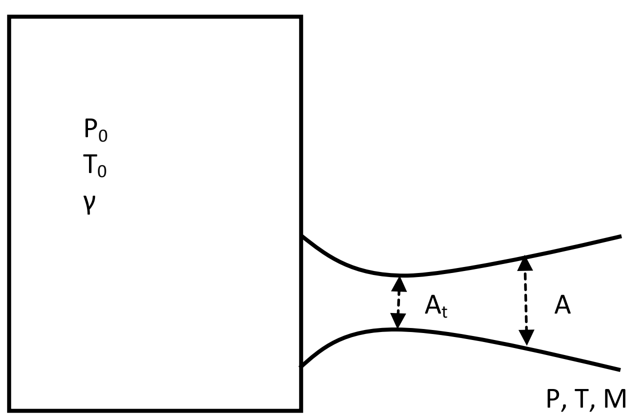


Figure 1. Schematic for isentropic flow nomenclature.

Figure 1 gives the basic nomenclature for isentropic flow. A large reservoir contains a perfect gas, ideal gas with constant specific heats, at essentially zero velocity. Therefore, the pressure and temperature in the reservoir are equal to the stagnation temperature and pressure. The flow expands through a nozzle. At some location area A, the flow has Mach number M, static pressure P, and static temperature T. The following relationships are well known. Those here were taken from the textbook by John and Keith [7]. The ratio of constant pressure specific heat to constant volume specific heat is noted as γ. The first entry in the equation shows the Excel function call format as it would appear in the spreadsheet.

$$P\_P0(M,\gamma) = \frac{P}{P_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}} \quad (1)$$

$$T\_T0(M,\gamma) = \frac{T}{T_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1} \quad (2)$$

$$den\_den0(M,\gamma) = \frac{\rho}{\rho_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{-1}{\gamma-1}} \quad (3)$$

$$rho\_V(P_0,T_0,\gamma,M,R) = \rho V = P_0 M\sqrt{\frac{\gamma}{RT_0}}\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \text{ [has units]} \quad (4)$$

The parameter A* is throat area where a flow at area A with Mach number M would obtain M_t = 1. For supersonic flows, A* is equal to the throat area, A* = A_t, of a converging-diverging nozzle. For subsonic flows, A* and the physical throat area may be different.

$$A\_Astar(M,\gamma) = \frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (5)$$

Normal Shocks

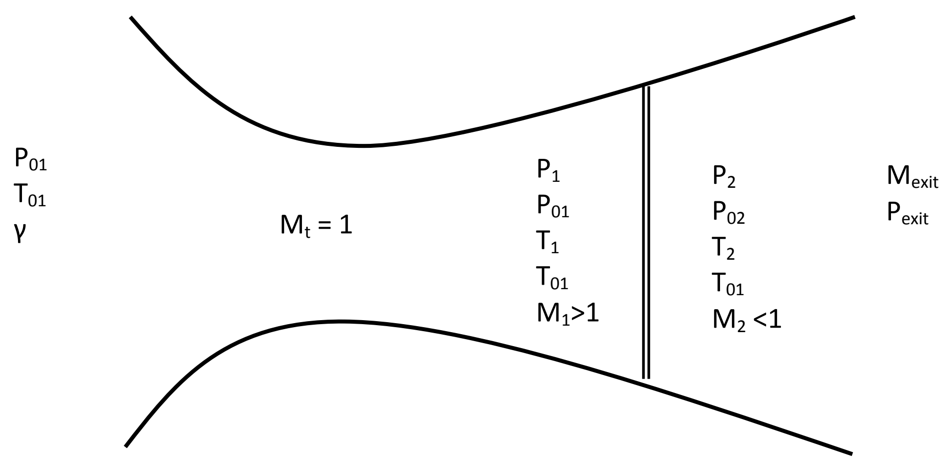


Figure 2. Schematic for normal shock nomenclature

As shown in Figure 2, a normal shock will appear in the diverging section of a converging-diverging adiabatic, frictionless nozzle in a certain range of pressure ratios P_{exit}/P_0. A shock is a discontinuity where the flow abruptly changes from a supersonic flow, $M_1 > 1$, to a subsonic flow, $M_2 < 1$. John and Keith [7] give the following relationships for property changes across the normal shock. The "SNS" prefix in the Excel function indicates a Standing Normal Shock function.

$$SNS\_M2(M1,\gamma) = M_2 = \sqrt{\frac{M_1^2+\frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2-1}} \quad \text{.......(6)}$$

$$SNS\_P2\_P1(M1,\gamma) = \frac{P_2}{P_1} = \frac{2\gamma M_1^2}{\gamma+1} + \frac{\gamma-1}{\gamma+1} \quad \text{.......(7)}$$

$$SNS\_P02\_P01(M1,\gamma) = \frac{P_{02}}{P_{01}} = \left(\frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{-1}{\gamma-1}} \quad \text{.......(8)}$$

$$SNS\_T2\_T1(M1,\gamma) = \frac{T_2}{T_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)\left(\frac{(\gamma+1)^2}{2(\gamma-1)}M_1^2\right)^{-1} \quad \text{.......(9)}$$

$$SNS\_den2\_den1(M1,\gamma) = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \quad \text{.......(10)}$$

Since the flow is adiabatic across the shock, the stagnation temperature is constant.

Normal shocks can also occur in constant area ducts with friction and/or heat transfer.

<u>Oblique Shock Functions</u>

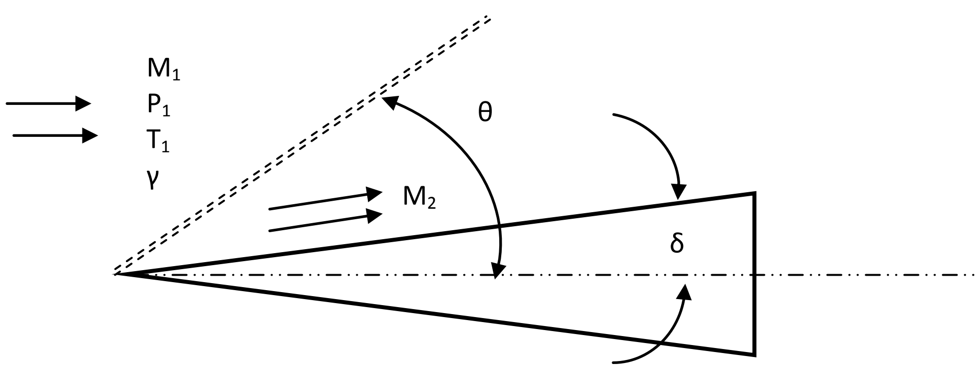


Figure 3. Schematic for oblique shock nomenclature.

Figure 3 shows the arrangement for oblique shocks. For a given turning angle δ, an oblique shock may attach to the object. For two-dimensional flows such as flow over a wedge, a relationship can be derived between the turning angle and the shock angle θ. For a given Mach number, M_1, not all turning angles will support an attached shock. If the turning angle is larger than a critical value, δ_{max}, the shock will detach and become a bow shock. John and Keith [7] give formulas for δ, δ_{max}, and M_2 in terms of M_1, θ, and γ. The "OBS" prefix in the Excel function call indicates an Oblique Shock function.

$$OBS\_delta(M1, \theta, \gamma) = \delta = tan^{-1}\left(\cot(\theta)\frac{M_1^2 sin^2(\theta)-1}{\frac{\gamma+1}{2}M_1^2 - M_1^2 sin^2(\theta)+1}\right) \text{ [has units]..............(11)}$$

$$OBS\_M2(M1, \theta, \gamma) = M_2 = \sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 sin^2(\theta)-\frac{\gamma-1}{2}} + \frac{M_1^2 cos^2(\theta)}{1+\frac{\gamma-1}{2}M_1^2 sin^2(\theta)}} \text{ ....(12)}$$

To compute δ_{max}, first θ_{max} is computed with equation (13) then δ_{max} is computed using equation (11)

$$\theta_{max} = sin^{-1}(\sqrt{\frac{1}{\gamma M_1^2}\left[\frac{\gamma+1}{4}M_1^2 - 1 + \sqrt{(\gamma+1)\left(\frac{\gamma+1}{16}M_1^2 + \frac{\gamma-1}{2}M_1^2 + 1\right)}\right]} \text{ [has units]...(13)}$$

$$OBS\_deltamax(M1, \gamma) = OBS\_delta(M1, \theta_{max}, \gamma) \text{ [has units]................................(14)}$$

The usual known parameters are M_1 and δ, and the shock angle, θ, must be found from equation (11) by iteration. These solutions are double valued; there are two values of θ that satisfy equation (11) for a given value of δ. The smaller of these is the so called weak shock, and the larger is the so called strong shock. Once θ is known, the component of the incident velocity normal to the shock can be computed using $M_{1n} = M_1 \sin(\theta)$, and the property ratios across the shock computed using the normal shock relationships with a Mach number of M_{1n}.

Prandlt-Meyer Flow

Figure 4 shows the nomenclature for Prandlt-Meyer flow. Prandlt-Meyer flow is a supersonic expansion around a convex corner. As the supersonic flow accommodates the turn, it expands through an expansion fan as demonstrated in the figure. Unlike shocks, these expansions are isentropic, and the isentropic relations can be used to find the property variations. The main task is to determine the Mach numbers after the expansion. Notice that the result of this supersonic expansion is an increase in Mach number.

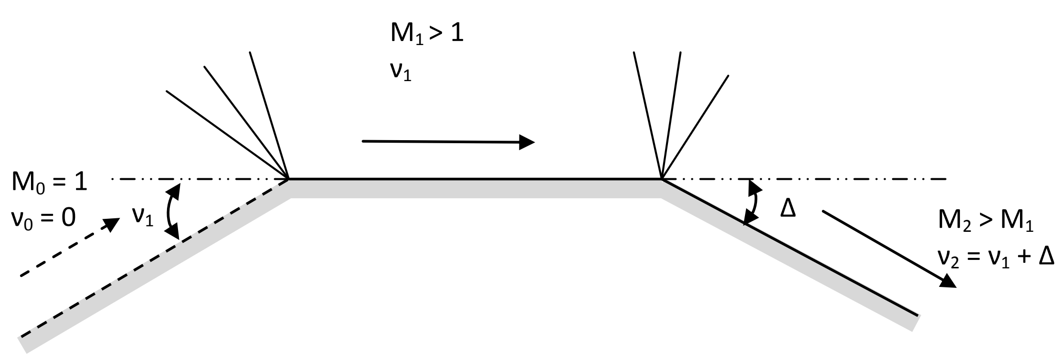


Figure 4. Schematic for Prandlt-Meyer nomenclature

The Mach number and the turning angle are related through the Prandlt-Meyer function ν, where ν is the angle through which a sonic flow ($M_0 = 1$) must be turned to obtain Mach number M. John and Keith [7] give the following relationship. The "PMF" prefix in the Excel function call refers to Prandlt-Meyer Function

$$PMF\_nu(M,\gamma) = \nu = \sqrt{\frac{\gamma+1}{\gamma-1}}\,tan^{-1}\left[\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right] - tan^{-1}\left(\sqrt{M^2-1}\right)\text{[has units]}.(15)$$

Usually the approaching Mach number M_1 and the turning angle Δ are known and the Mach number after expansion, M_2, is computed. The procedure is to compute ν_1 using equation (15), compute $\nu_2 = \nu_1 + \Delta$, and then compute M_2 from equation (15) by iteration.

Fanno-Line Flow

Fanno-line flow is flow in a constant cross-section area adiabatic duct with friction. Figure 5 shows the nomenclature.

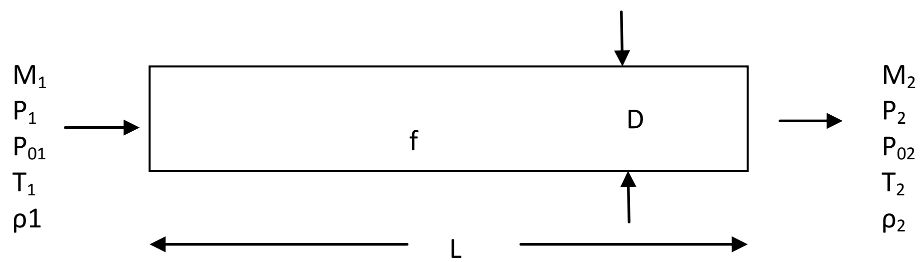


Figure 5. Schematic for Fanno-line flow

John and Keith [7] give the relationship for the relative friction length, fL/D, and the inlet and exit Mach numbers as follows. The "Fan" prefix in the Excel function call refers to Fanno flow.

$$Fan\_fL\_D(M1,M2,\gamma) = \frac{fL}{D} = \frac{\gamma+1}{2\gamma}\ln\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right) - \frac{1}{\gamma}\left(\frac{1}{M_2^2}-\frac{1}{M_1^2}\right) - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M_2^2}{M_1^2}\right) \text{..........(16)}$$

Similarly they give for the property ratios [7]

$$Fan\_T1\_T2(M1,M2,\gamma) = \frac{T_1}{T_2} = \frac{2+(\gamma-1)M_2^2}{2+(\gamma-1)M_1^2} \text{..(17)}$$

$$Fan\_P1\_P2(M1,M2,\gamma) = \frac{P_1}{P_2} = \frac{M_2}{M_1}\sqrt{\frac{T_1}{T_2}} \text{..(18)}$$

$$Fan\_P01\_P02(M1,M2,\gamma) = \frac{P_{01}}{P_{02}} = \frac{M_2}{M_1}\left(\frac{T_1}{T_2}\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}} \text{..(19)}$$

$$Fan\_rho1\_rho2(M1,M2,\gamma) = \frac{\rho_1}{\rho_2} = \frac{P_1}{P_2}\Big/\frac{T_1}{T_2} \text{..(20)}$$

The normal computation procedure is to be given f, L, D (or the friction relative length, fL/D) and one of the Mach numbers. The other Mach number is computed from equation (16) by iteration. Once both Mach numbers are known, the property ratios may be computed. Tables for Fanno-line flow usually set M_2 =1.

<u>Rayleigh-Line Flow</u>

Rayleigh-line flow is flow in a constant cross-section frictionless duct with heat addition. Figure 6 gives the nomenclature.

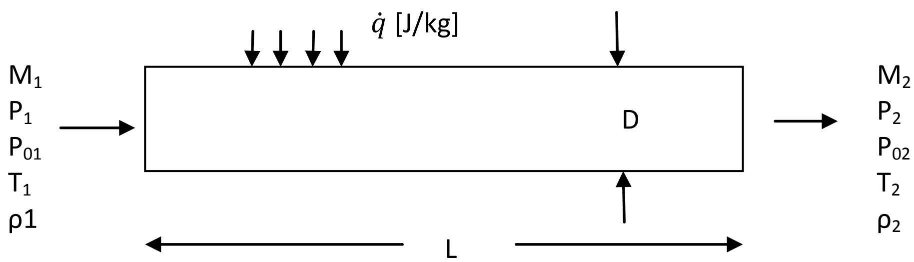


Figure 6. Schematic for Rayleigh-line flow nomenclature.

The property ratios are [7], where the "Ray" prefix in the Excel call refers to Rayleigh flow.

$$Ray\_T1\_T2(M1, M2, \gamma) = \frac{T_1}{T_2} = \frac{M_1^2}{M_2^2}\frac{\left(1+\gamma M_2^2\right)^2}{\left(1+\gamma M_1^2\right)^2} \quad (21)$$

$$Ray\_P1\_P2(M1, M2, \gamma) = \frac{P_1}{P_2} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2} \quad (22)$$

$$Ray\_P01\_P02(M1, M2, \gamma) = \frac{P_{01}}{P_{02}} = \frac{P_1}{P_2}\left(\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right)^{\frac{\gamma}{\gamma-1}} \quad (23)$$

$$Ray\_T01\_T02(M1, M2, \gamma) = \frac{T_{01}}{T_{02}} = \frac{T_1}{T_2}\left(\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right) \quad (24)$$

$$Ray\_rho1\_rho2(M1, M2, \gamma) = \frac{\rho_1}{\rho_2} = \frac{P_1}{P_2}\Big/\frac{T_1}{T_2} \quad (25)$$

Tables for Rayleigh-line flow usually set M_2 = 1.The specific heat transfer rate enters through the relation with the total temperatures.

$$\dot{q} = C_p(T_{02} - T_{01}) \text{ [has units]} \quad (26)$$

Verification

The formulae above have been programmed as Visual Basic Macros in Microsoft Excel and packaged as Excel Add-ins as discussed above. Since all are algebraic functions, verification at only one point is needed to verify that the formulae have been recorded and coded correctly. To do this, values are compared with tables in Dennard and Spencer [8] for the oblique shock values and John and Keith [7] for the others. Table 1 shows the comparisons. For values such as T/T* in the Fanno-line case, the starred values correspond to $M_2 = 1$. The agreement is seen to vary between perfect and excellent.

Problem Solution Organization and Documentation

A spreadsheet can appear to be a random collection of numbers and a dizzying chain of arcane cell references in the formulae, making them difficult to read and almost impossible to debug when errors in the formulae happen.

Table 1. Verification Comparisons for the Gas Dynamics Excel Add-in Functions

Isentropic, M = .9, γ = 1.4			
Value	Excel	Tables	% Diff
T/T_0	0.8606	0.8606	0.0000
P/P_0	0.5913	0.5913	0.0000
ρ/ρ_0	0.6870	0.6870	0.0000
A/A*	1.0089	1.0089	0.0000

Normal Shocks, M_1 = 2.0, γ = 1.4			
Value	Excel	Tables	% Diff
M_2	0.5774	0.5774	0.0000
P_2/P_1	4.5000	4.5000	0.0000
T_2/T_1	1.6875	1.6875	0.0000
P_{02}/P_{01}	0.7209	0.7209	0.0000
ρ_2/ρ_1	2.6667	2.6667	0.0000

Oblique Shock, M = 2, γ = 1.4			
θ	Excel-δ	Table-δ	% Diff
32.510	3.005	3.000	0.169322
40.420	10.997	11.000	-0.02492
45.340	14.997	15.000	-0.01798
51.510	19.002	19.000	0.010821
64.669	22.974	22.973	0.002399

Prandlt-Meyer, γ = 1.4			
Mach	ν-Excel	ν-Table	% Diff
1	0.0000	0.0000	0.0000
1.5	11.9052	11.9052	0.0000
2	26.3798	26.3798	0.0000
3	49.7574	49.7573	0.0002
5	76.9203	76.9202	0.0001

Fanno-Line Flow, M = 1.5, γ = 1.4			
Value	Excel	Table	% Diff
T/T*	0.8276	0.8276	0.0000
P/P*	0.6065	0.6065	0.0000
P_0/P_0*	1.1762	1.1762	0.0000
ρ/ρ*	0.7328	0.7328	0.0000
f L_{max}/D	0.1361	0.1361	0.0000

Rayleigh-Line Flow, M = 0.5, γ = 1.4			
Value	Excel	Table	% Diff
T_0/T_0*	0.6914	0.6914	0.0000
T/T*	0.7901	0.7901	0.0000
P_0/P_0*	1.1141	0.1141	0.0000
P/P*	1.7778	1.7778	0.0000

However, with a little effort and use of some of the features of Excel, an organized, self-documenting engineering problem solution document can be created. Figure 7 in Example 1 below can serve as an example. All values are named and values assigned to the names using the "Defined Names" feature under the formulas tab. The first column, column A in Excel, is reserved for the parameter name. In Figure 7, the reader will see descriptive variable names such as "R_air," "T_0," and "M_2." The second column is reserved for values for the constants or formulae where dependent variables are being computed. All formulae are written in terms of the assigned names, "= P_b/P_P0(M_2,gam)." Generic cell references are never used, ~~"= B13/B10."~~ The third column contains the units. Units are emphasized, since neglecting units is a primary source of errors in engineering problems. Notes or cut-and-paste copies of the formulae are displayed in column D and beyond. Having visible copies of the formulae is very important for readability and to debug the spreadsheet when an error occurs.

Screen captures, sketches using the drawing tools, and text boxes can be added to further document the spreadsheet solution. As the figure shows, the result can be a readable, well-documented problem solution.

Tutorials in the form of PowerPoint presentations have been developed to guide the students and other users through this 4-column formatting paradigm. The tutorials can be found on the internet at www.me.ua.edu/excel.

Examples

In this section four examples are given that show the usage of the gas dynamics Add-ins.

Example 1, Problem 3.14 from John and Keith. "A converging-diverging frictionless nozzle is used to accelerate an airstream emanating from a large chamber. The nozzle has an exit area of 30 cm^2 and a throat area of 15 cm^2. If the ambient pressure surrounding the nozzle is 101 kPa and the chamber temperature is 500 K, calculate the following: a) the minimum chamber pressure to choke the nozzle, b) the mass-flow rate for a chamber pressure of 400 kPa, and c) the mass-flow rate for a chamber pressure of 200 kPa."

A partial screen shot of the excel solution is seen in Figure 7. The minimum chamber pressure to choke the flow corresponds to the case of isentropic choked flow with subsonic flow at the exit. The exit Mach number is found using the Excel Goal Seek data tool to find the Mach number that gives A/A* equal to the exit to throat area ratio. Once this is found, the isentropic relationships are used to find the stagnation pressure when the exit pressure is 101,000 Pa. The flow will be choked for all chamber pressures above the computed value of 107,768 Pa. Since both 400 kPa and 200 kPa are above this critical chamber pressure, the nozzle will be choked in both cases. The flow rates are computed using the throat area, the fact that the throat $M_t = 1$ for choked flow and the mass flux relationship equation (4).

Example 2. A reservoir containing a perfect gas with $\gamma = 1.3$ discharges through a converging-diverging nozzle with an exit to throat area ratio of 1.75 into a duct with friction. The duct is 1 cm in diameter, 300 cm long and has a friction factor of 0.025. The back pressure is slowly lowered from the reservoir pressure. Will the flow choke at the nozzle throat or the duct exit? Support your answer with appropriate calculations.

This problem is solved by taking the two possibilities in turn. First the flow is assumed to choke at the nozzle throat and the flow in the nozzle is approximated with an isentropic flow. A Goal Seek loop is set up to compute the subsonic nozzle exit Mach number, M_i1, for isentropic flow in the nozzle. The second Goal Seek loop computes the subsonic duct inlet Mach number, M_i2, for choked flow at the duct exit. If M_i2 is the lower of the two values, the flow will choke first at the duct exit. Continuing to lower the back pressure will have no effect on the flow in the nozzle and duct. If M_i1 is the lower value, the flow will choke first at the nozzle throat. Continuing to lower the back pressure will result in a series of cases involving normal shocks first in the diverging section of the nozzle and later in the duct. For this case, the flow chokes first at the duct exit.

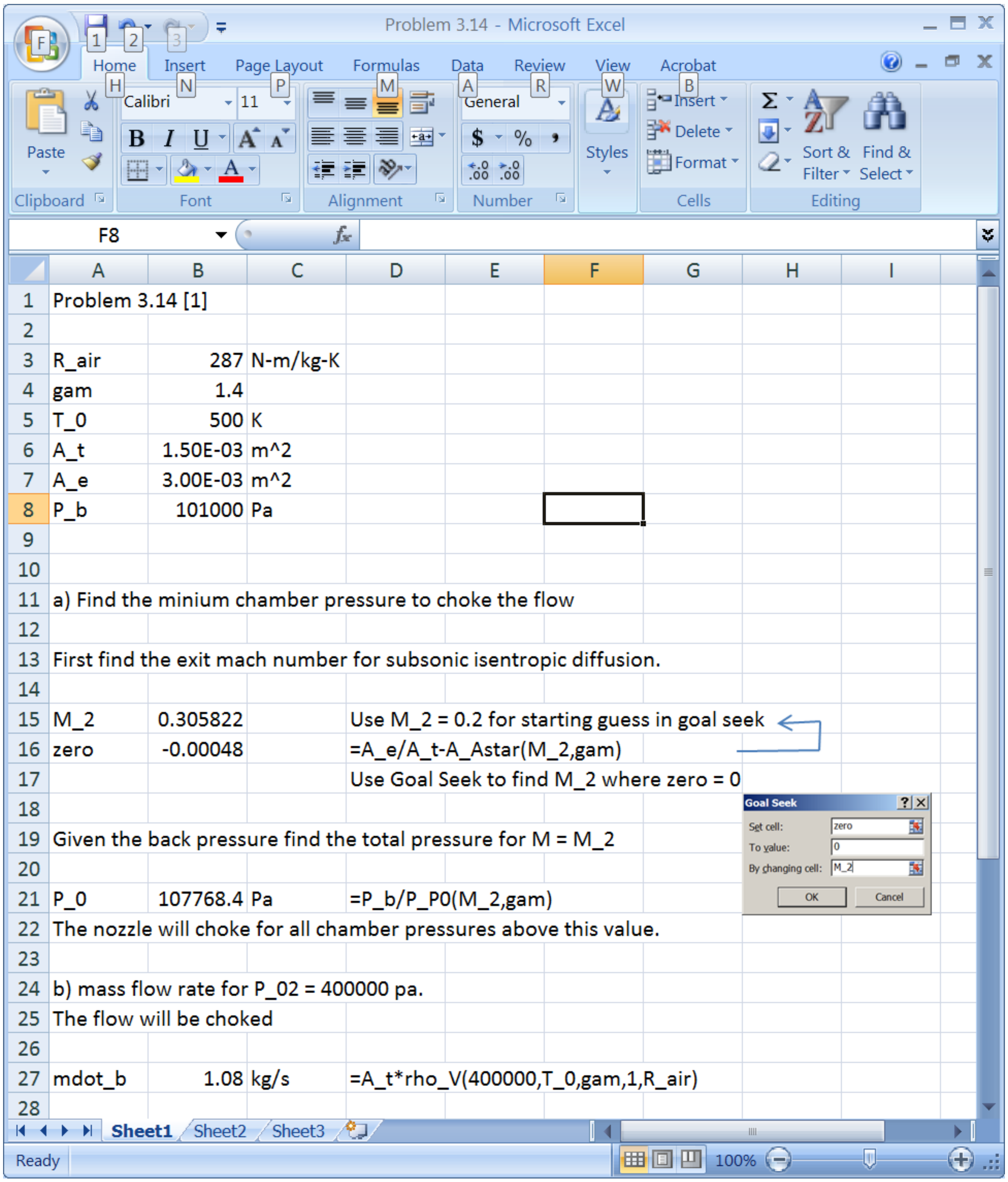


Figure 7. Excel solution for Example 1.

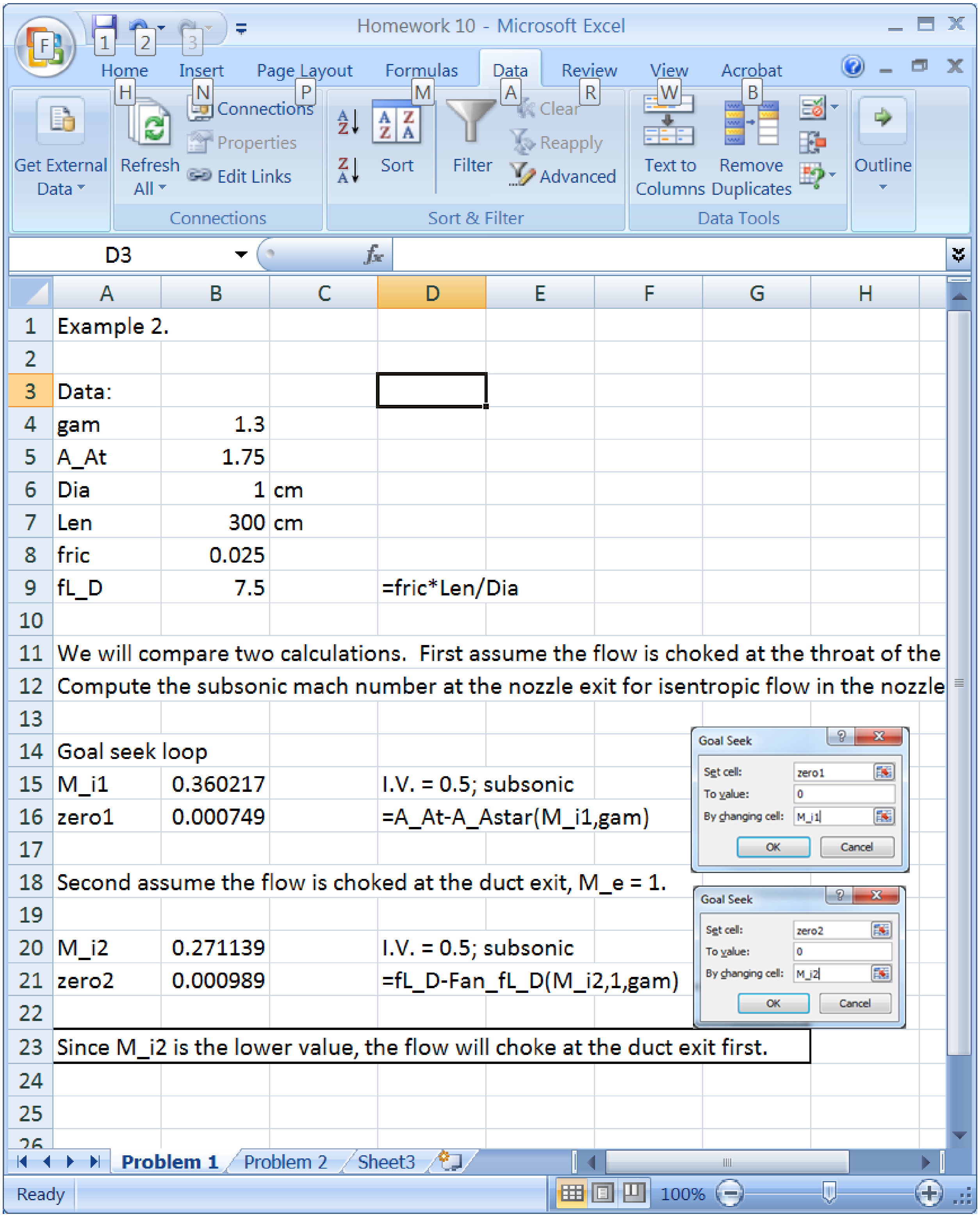


Figure 8. Excel solution for Example 2.

The first two examples could be solved using standard gas dynamics tables and calculators; although, Example 2 would be a little tedious. The next example would be very tedious to solve by hand with tables; it may be so tedious as to be unpractical.

Example 3. Adapted from Example 9.4 in John and Keith [7]. A converging-diverging nozzle with exit to throat area ratio of 2 to 1 is supplied by a reservoir containing air at 500 kPa. The nozzle exhausts into a constant area duct with length to diameter ratio, L/D, of 10 and a friction coefficient of 0.02. Previous calculations show that a normal shock will occur in the nozzle for back pressures in the range 238,000 Pa to 461,000 Pa. Find the shock location as area ratio A_{shock}/A_t for a back pressure of 350,000 Pa.

Figure 9 shows a schematic of the situation and gives the nomenclature.

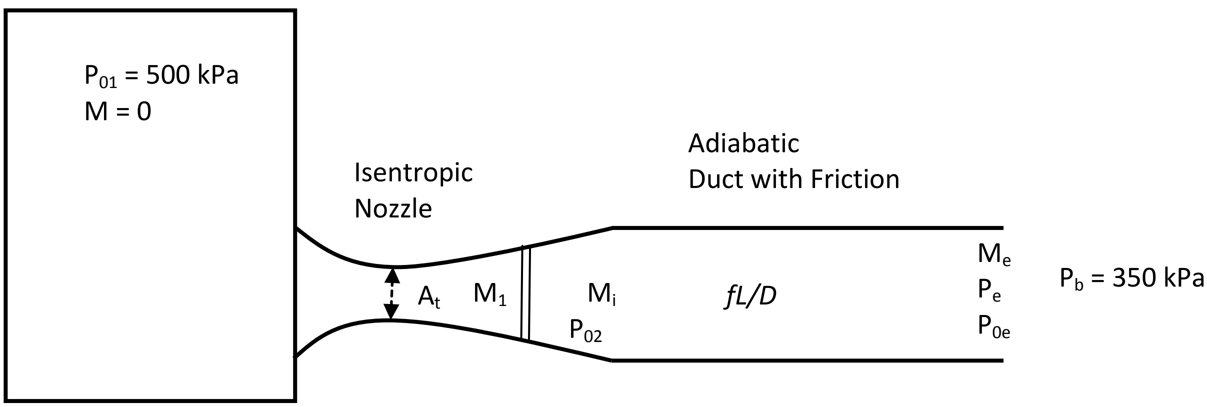


Figure 9. Schematic for Example 3.

For this case, the flow in the diverging section of the nozzle will be supersonic upstream of the shock. The flow downstream of the shock in the nozzle and the duct will be subsonic. None of the Mach numbers, M_1, M_i, or M_e is known a priori. We must develop a set of three equations in the three unknown Mach numbers that can be solved simultaneously. The first two equations are rather straight forward. For subsonic flow, the exit pressure must equal the back pressure. The friction relative length is related to the duct inlet and exit Mach numbers through equation (16).

Upstream of the shock, the stagnation pressure is constant at P_{01}. The stagnation pressure abruptly decreases across the shock to P_{02}, which remains constant in the remainder of the isentropic nozzle. The stagnation pressure changes continuously in the duct with friction. The equation $P_b = P_e$ can be written using series of pressure ratios as

$$\frac{P_b}{P_{01}} - \frac{P_{02}}{P_{01}}\frac{P_{0e}}{P_{02}}\frac{P_e}{P_{0e}} = F(M1, Mi, Me, \gamma) = 0 \quad \text{...(27)}$$

where

$$\frac{P_{02}}{P_{01}} = SNS\_P02\_P01(M1, \gamma) \quad \text{...(28)}$$

$$\frac{P_{0e}}{P_{02}} = 1/Fan\_P01\_P02(Mi, Me, \gamma) \quad (29)$$

$$\frac{P_e}{P_{0e}} = P\_P0(Me, \gamma) \quad (30)$$

The equation for friction relative length is

$$\frac{fL}{D} - Fan\_fL\_D(Mi, Me, \gamma) = 0 \quad (31)$$

The final equation must involve the area ratios for the nozzle, since area change drives the flow conditions within the nozzle. If A_1^* is the critical area for flow upstream of the shock and A_2^* is the critical area downstream of the shock, the area ratios can be formed into the equation, since $A_1^* = A_t$

$$\frac{A_e}{A_t} - \frac{A_2^*}{A_1^*}\frac{A_e}{A_2^*} = F(M1, Mi, \gamma) = 0 \quad (32)$$

where

$$\frac{A_2^*}{A_1^*} = 1/SNS\_P02\_P01(M1, \gamma) \quad (33)$$

$$\frac{A_e}{A_2^*} = A\_Astar(Mi, \gamma) \quad (34)$$

Equations (27), (31), and (32) form a set of three equations in terms of the unknowns M_1, M_i, and M_e. These equations can be readily solved for the unknown Mach numbers using the Solver Add-in in Excel. Solver is an optimization Add-in that comes packaged with Excel. All Excel programs have Solver; however, it must be "turned on" manually. When a solution to the equations is found all equations will equal zero at the same time. Solver solves the equations by minimizing the sum of the squared residuals, $R_2 = eq1^2 + eq2^2 + eq3^2$. Assuming a solution exists, the only way R_2 can be at a minimum is for each and all equations to have values of 0. Figure 10 shows a screen shot of the excel spreadsheet for this example.

Example 4. Adapted from Problem 10.12 from John and Keith [7]. Air flows through a constant area 0.02 m diameter duct connected to a reservoir at a temperature of 500 C and a pressure of 500 kPa by a converging nozzle. Heat is added at the rate of 250 kJ/kg. Determine the mass flow rate if the back pressure is 0 Pa. Repeat for a back pressure of 100 kPa.

Figure 11 shows a screen shot of the Excel solution. With a back pressure of 0 Pa, the flow will have to be choked. Since we are adding heat, the flow will choke at the duct exit and the flow in the duct will be otherwise subsonic. A Goal Seek loop is set up to determine the duct inlet Mach number that is consistent with the given stagnation temperature ratio. The flow upstream of the heat addition is taken to be isentropic; so, the mass flow rate can be computed using the isentropic mass flux equation (4). The exit pressure corresponding to choked flow is computed to be 241 kPa; hence the flow will still be choked when the back pressure is 100 kPa, and the mass flow rate will remain unchanged.

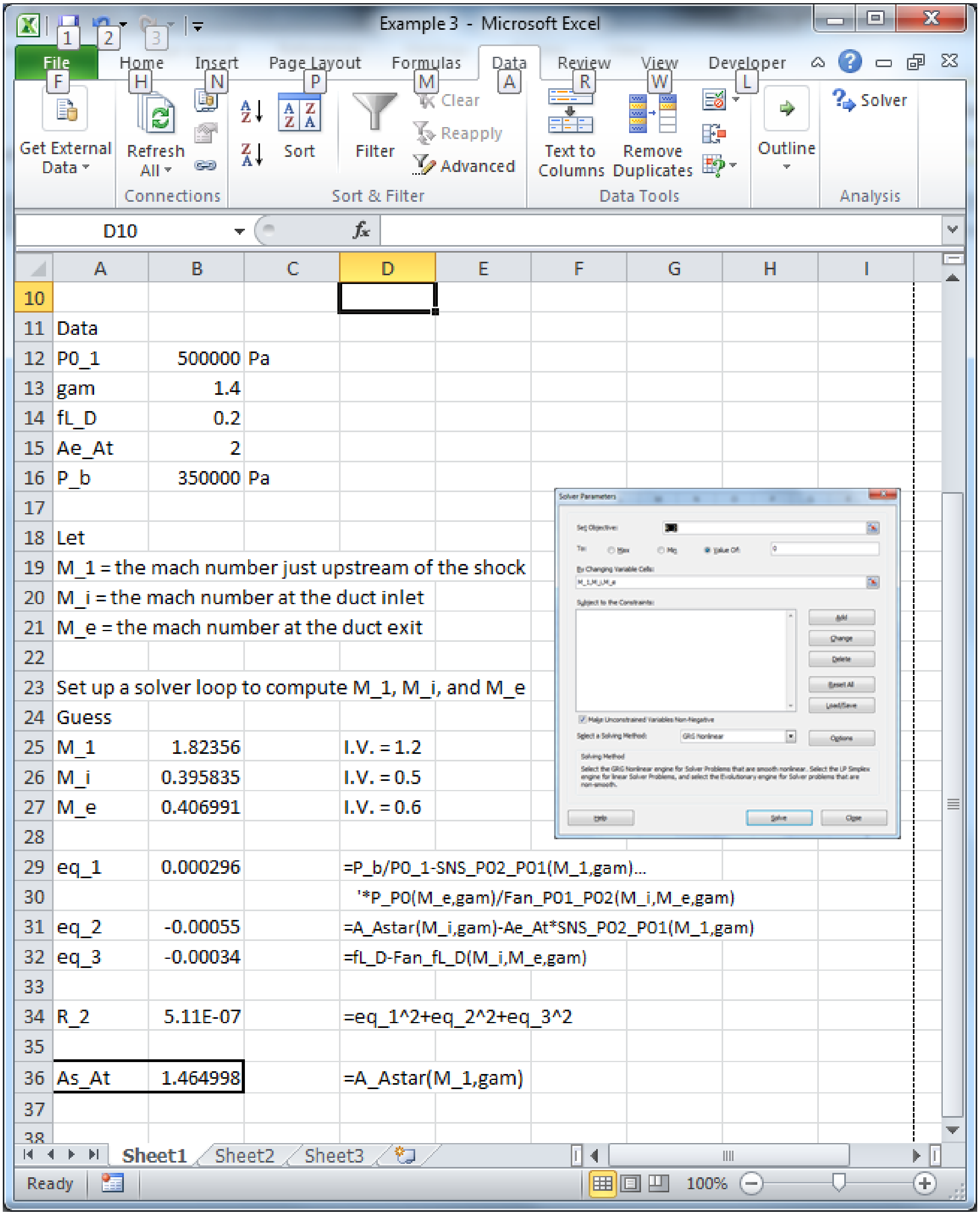
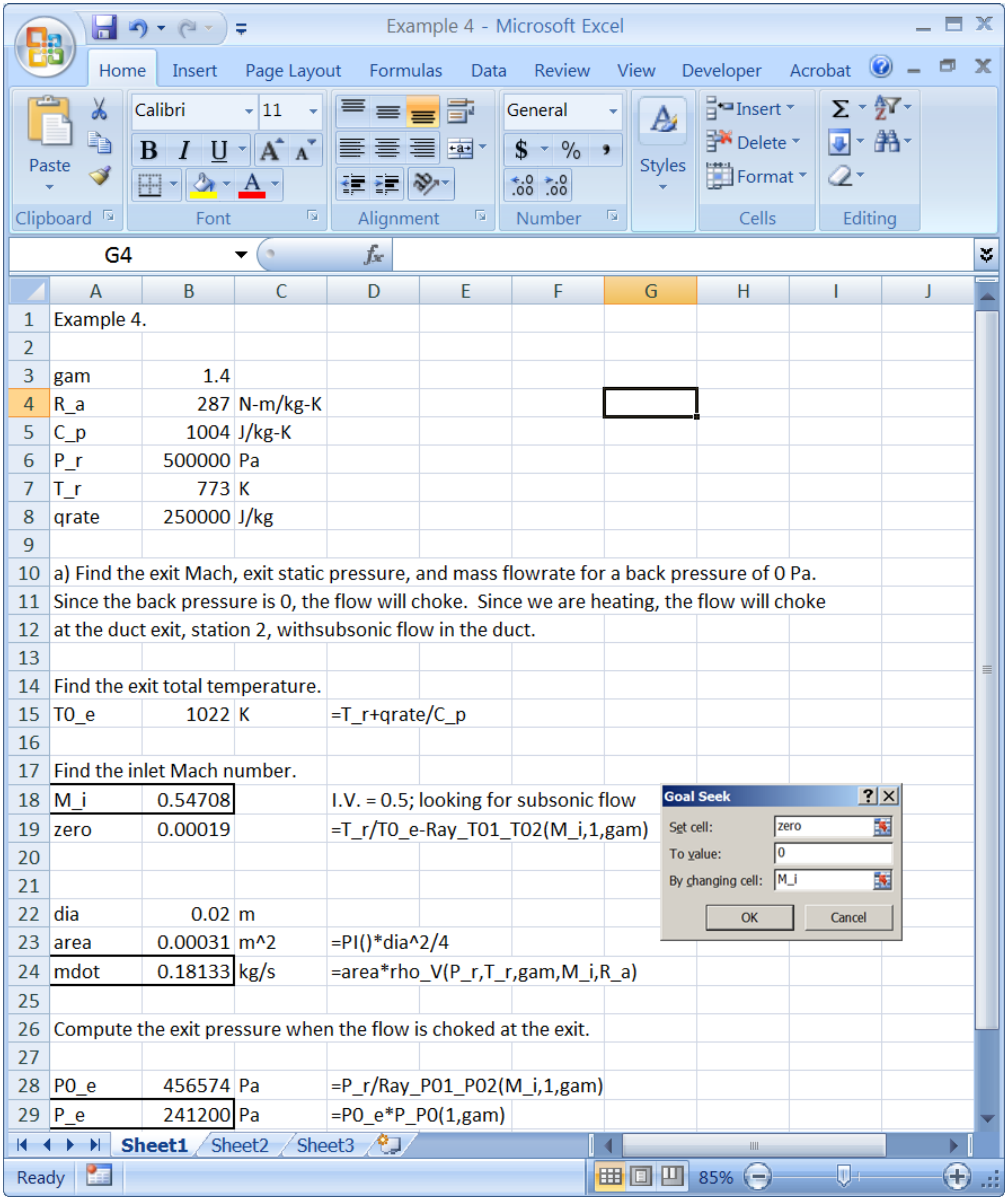


Figure 10. Excel solution for Example 3.



	A	B	C	D
1	Example 4.			
2				
3	gam	1.4		
4	R_a	287	N-m/kg-K	
5	C_p	1004	J/kg-K	
6	P_r	500000	Pa	
7	T_r	773	K	
8	qrate	250000	J/kg	
9				
10	a) Find the exit Mach, exit static pressure, and mass flowrate for a back pressure of 0 Pa.			
11	Since the back pressure is 0, the flow will choke. Since we are heating, the flow will choke			
12	at the duct exit, station 2, withsubsonic flow in the duct.			
13				
14	Find the exit total temperature.			
15	T0_e	1022	K	=T_r+qrate/C_p
16				
17	Find the inlet Mach number.			
18	M_i	0.54708		I.V. = 0.5; looking for subsonic flow
19	zero	0.00019		=T_r/T0_e-Ray_T01_T02(M_i,1,gam)
20				
21				
22	dia	0.02	m	
23	area	0.00031	m^2	=PI()*dia^2/4
24	mdot	0.18133	kg/s	=area*rho_V(P_r,T_r,gam,M_i,R_a)
25				
26	Compute the exit pressure when the flow is choked at the exit.			
27				
28	P0_e	456574	Pa	=P_r/Ray_P01_P02(M_i,1,gam)
29	P_e	241200	Pa	=P0_e*P_P0(1,gam)

Figure 11. Excel solution for Example 4.

Classroom Experiences

These Excel Add-in functions have been introduced in an upper division undergraduate-beginning graduate level course in gas dynamics at The University of Alabama in both the Fall of 2008 and the Fall of 2010. In 2008, about 20 students enrolled, and in 2010, 14 students enrolled. In 2008, about 25% of the students had completed a course in Intermediate Fluid Mechanics that included an introduction to gas dynamics and used the traditional tables. In 2010, three of the students had a background in aerospace engineering and had experience using the gas dynamics tables. In 2008, only a few of the students had had experience with the Excel 4-column format of problem solving in the thermal sciences. In 2010, about half of the students had prior experience with the current paradigm in previous course in thermodynamics and heat transfer. Some of the graduate students who came from other universities had relatively little experience with Excel as an engineering tool; although, they were familiar with Excel as a general spreadsheet and especially as a way to graph data.

In both years, the students were free to choose to use the tables, the spreadsheet, or other computational tool. About 60% of the problems could be worked with tables with about the same level of effort as using the spreadsheet with gas dynamic Add-ins, and the other 40% would have been somewhat to very tedious to work with only the tables and a calculator. The textbook used in the class was John and Keith [7] *Gas Dynamics* 3rd edition. The textbook encourages spreadsheet use as a computational tool; however, the students would have to develop their own functions. The examples in the lectures were split about 30% using the tables and 70% using the spreadsheet with Add-ins.

The students took easily to the Excel format. This was not surprising, since Excel is used widely in high schools and at the University. All freshmen engineering students at The University of Alabama have substantial exposure to Excel as and engineering tool. In 2008, two of the students always worked the problems using paper and pencil and the tables unless the solution was practically impossible without higher-level computations. The remainder of the students used Excel with the Add-ins exclusively; even though, they were free to use pencil and paper and the tables. The students who had experience with the tables in the intermediate fluid mechanics course seemed to be particularly grateful to have the spreadsheet Add-ins.

In 2010, the three students with background in aerospace engineering and one international student who had apparently never used Excel seemed to want to stick to familiar ground and use the tables for about the first one third of the semester. By the end of the term all of these students had abandoned the tables in favor of the spreadsheet with Add-ins. The other students appeared to have never considered the tables as a viable option. Perhaps the instructor's preference for the spreadsheet solutions influenced them.

In 2010, one student, who was reluctant to attend lectures, demonstrated the convenience and power of the preprogrammed Add-ins. He elected to follow the roll-your-own spreadsheet paradigm presented in John and Keith's text book. He typed the formulas directly into the Excel cells using B13, B12 type generic cell references. He found it extremely difficult to produce an error-free spreadsheet. The students using the Add-ins almost never had a typo-type bug and readily produced their solutions once they understood the theory and problem applications.

On the end of semester Student Opinion of Instruction survey, many of the students raved about the ease of use and relief from tedium that the spreadsheet paradigm provided. No students complained about the spreadsheets. One student was undecided.

From an instructor stand point, the typical student could attack a larger number of problems within the time limits of a 3 credit course. He or She appeared to be more confident to attempt problems that appeared to involve tedious calculations. Numerical accuracy was improved. The students start the solution with a blank spreadsheet and still have to produce all of the steps of the solution. They are still solving problems, not plugging numbers in to an already prepared template. So, problem solving skills improved through the opportunity to attack a larger number of problems and more sophisticated problems.

Conclusions

A suite of gas dynamics functions have been programmed and packaged for use in Microsoft Excel as Add-ins to extend the extensive set of thermal sciences Add-in modules developed at The University of Alabama These functions replace the standard tables and charts that have traditionally been used for introductory or gas dynamics courses. The functions have been verified by comparing with accepted tabular versions of the data. The spreadsheet paradigm has been introduced in two gas dynamics courses with good success and widespread student acceptance. The learning environment appears to be improved through the ability to work a large number and more sophisticated applications.

Acknowledgements

This material is based in part upon work supported by the National Science Foundation under Grant No. DUDE 0633330. The authors gratefully acknowledge support from the NSF award. The contributions of student assistants Joseph Chappell and Kenneth Mahan for help developing the help screens, packaging the Add-ins, and other assistance is gratefully acknowledged.

Disclaimer

All opinions, findings, and conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Bibliography

1. Dent, T., Woodbury, K.A., and Taylor, R., 2008, “Microsoft Excel Heat Transfer Add-in for Engineering Courses,” AC 2008-933, 2008 ASEE Annual Conference and Exposition, June 22-25, Pittsburgh PA.
2. Chappell, J., Taylor, R., and Woodbury, K.A., 2008, “Introducing Excel Based Steam Table Calculations into Thermodynamics Curriculum,” AC 2008-1398, 2008 ASEE Annual Conference and Exposition, June 22-25, Pittsburgh PA.

3. Woodbury, K.A., Taylor, R., Huget, J., Chappell, J., and Mahan, K., 2008, “Vertical Integration of Excel in the Thermal Mechanical Engineering Curriculum,” IMECE 2008-69165, 2008 ASME International Mechanical Engineering Congress and Exposition, Oct 31-Nov 6, Boston MA.
4. Huget, J., Woodbury, K. A. and Taylor, R.P., 2008, “Development of Excel Add-in Modules for Use in Thermodynamics Curriculum: Steam and Ideal Gas Properties,” AC 2008-1751, 2008 ASEE Annual Conference and Exposition, June 22-25, Pittsburgh PA
5. Chappell, J., Woodbury, K., and Taylor, R., 2009, “Excel in ME: Packaging Add-ins and Providing On-line Help,” AC 2009-2297, .2009 ASEE Annual Conference and Exposition, June 14-17, Austin Tx.
6. Mahan, K., Huget, J., Chappell, J., Woodbury, K. and Taylor, R., 2009, “Excel in ME: Extending and Refining Ubiquitous Software Tools,” AC 2009-2295, 2009 ASEE Annual Conference and Exposition, June 14-17, Austin Tx.
7. John, James E. and Keith, Theo G., 2006, *Gas Dynamics*, 3rd edition, Pearson Prentice Hall, Upper Saddle River, New Jersey.
8. Dennard, John S. and Spencer, Patricia B., 1964, *Ideal-Gas Tables for Oblique-Shock Flow Parameters in Air at Mach Numbers from 1.05 to 12.0*, NASA TN D-2221, National Aeronautics and Space Administration, Washington, D.C.