

AC 2007-608: STRUCTURED READING GUIDE (SRG): A GRAPHICAL ORGANIZER FOR MATHEMATICAL, PHYSICAL AND ENGINEERING SCIENCES

Alhaji Cherif, Cornell University

Alhaji Cherif is an undergraduate student in engineering science (Independent Major Program) at Cornell University, where he is majoring in Theoretical and Applied Mechanics, and Electrical and Computer Engineering with minor concentrations in Applied Mathematics, Engineering Mechanics, and Africana Studies (Politics and Society focus). Before coming to Cornell, he tutored and served as a private tutor for high school and community college students in Riverdale and New Carrollton, MD. He held various positions as course assistant (Department of Mathematics), tutor (Math Support Center, Learning Strategic Center, Engineering Learning Initiatives). He has also served as a peer-counselor/tutor at Delaware State University (while participating in Intensive Summer Science Program, ISSP) and College Counselor/Mentor for National Society of High School Scholars (NSHSS). His research interests cover a range of science and engineering disciplines ranging from mathematics education, mathematical modeling and methods in Physical, Biological, Engineering; Political Science (Concentrating on African Continent); Nanobiotechnology with emphasis on Micro-fluidic device, biosensors, thin-film, and nano/micro-actuators; to Dynamical and theoretical modeling of Electrophoretic Deposition (EPD) of Nanoparticles and origin of cracks and fracture during EPD.

Gary Fleming, University of Maryland

Gary Fleming has 25 years experience as a mathematical engineer and currently works as a Multi-discipline Systems Engineer for the MITRE Corporation in McLean, Virginia. He was an Undergraduate Advisor for University of Maryland (College Park) Mathematics Department and continues to teach as an evening adjunct.

Siaka Kroma, Cornell University

Siaka Kroma is a visiting professor in the Department of Education at Cornell University. He is involved teaching university courses on diversity and literacy education in the teacher education program.

Structured Reading Guide: A Graphic Organizer for Mathematical, Physical and Engineering Sciences

Abstract

The public school system has been working on a set of procedures collectively called “Reading across the Curriculum.” It emphasizes the importance of reading in all classes including mathematical and physical sciences. Educators and researchers have offered several procedures, not just for mathematics context, but also for all subjects. Many of these procedures use what have come to be known as a “graphic organizer,” a tool of breaking the linear presentation style of prose text and ideas into a multidimensional matrix so that the key ideas can be readily identified and contrasted. However, no pedagogic methodology was identified as being applicable to mathematical and physical science texts and no current organizer seemed general enough to capture the highly varied information present in these texts. Reading mathematics and science textbook is not a well-liked past-time — even for mathematics, physical science and engineering students. This is primarily due to the fact that students have never been taught how to read these texts. Yet mathematics and science instructors, both secondary and collegiate, are universally aghast that students do not read the assigned text. In the paper presented herein, we have developed a tool in the form of graphic organizer (due to students’ familiarity with graphic organizer) to prevent students from reading mathematical, physical and engineering texts like any other Shakespearean novel or Harry Potter’s novel. Historically, many students entering high schools and colleges are not well equipped to read science and mathematics textbooks. As a result, many students believe that these texts can be read as rapidly as any novel. But we know that it is not feasible to read mathematical and scientific texts in the same manner one reads Victorian, Elizabethan or modern novels. Due to the taxonomic elements encountered in Mathematics and Scientific textbooks, reading them as a novel prevents students from fully understanding the concepts they read. We have developed a graphical organizer tool that takes advantage of the taxonomic elements students are more likely to encounter while reading their textbooks to facilitate the textual and conceptual understandings of the texts. The paper will include a brief description of the structure of texts encountered in mathematical, physical and engineering sciences. The Structured Reading Guide, SRG, uses the assumptions of functional cognitive processes in learning. The main focus is to guide students’

attentions towards essential concepts, to nurture the ability of connecting textual concepts within the text and to the prior knowledge by taking advantage of the structure of the textbooks, and to become self-sufficient learners.

Key Words: Mathematical exposition, graphic organizer, types of mathematics and science textbooks, named concepts, technical prose, and duality of symbols, examples and taxonomy.

Introduction

As avid mathematics textbook readers, tutors and teachers in science and mathematics courses at high school and university levels, the authors have found that many of their tutees and students lack the necessary skills for reading science and mathematics textbooks. As a result, they lack the competency to acquire complete knowledge from the textbook, and the ability to understand the concepts of the text. The most common difficulty the authors have observed in students is the inability to comprehend mathematical and scientific textbooks. This is particularly due to the fact that many of the entering students in high school and in college are not well equipped to read science and mathematics textbooks for their comprehensive, summary and encyclopedic natures. Most of these texts, especially those of high school and lower undergraduate levels, are saturated with colors, pictures, varied font types and cartoons. Most of these textbooks have little content, poor reasoning and vapid presentation. They do not treat the subject matter with the breadth and depth necessary to fully cultivate ideas and concepts¹. As a result, students do not understand the nature of taxonomic elements (e.g.; examples, pictures, graphs, terminologies, etc...), and usage of these taxonomies of the technical prose, for they do not know how to approach these texts. Many students believe that science and mathematics texts can be read as rapidly as any novel. Sure, we can begin with the first page of a section and read all the words until reaching the last page, but seriously how many of us can actually remember what content we just read? Since mathematical, physical and engineering sciences tend to contain and combine few words with symbols compare to literature, social sciences, and the like, understanding of the subject manner mostly depends on how one understands the duality of the symbols, words, terminologies, and other taxonomic elements, which is discussed in the later section of the paper.

The other problem, as the authors have observed among their tutees, students and friends,

is that a student who is proficient at solving problems often experiences difficulties in articulating methods for solving the problem. If a student knows how to read and approach, and/or can decode mathematical and scientific texts, s/he will still encounter problems if s/he is not able to distinguish between various terms, their usage, and significance in specific contexts. So the student's ability to solve problems is an indication that some information they obtain has been transferred to long-term memory where s/he can easily recall the topics, but it is obvious that not all of the information was transmitted. Most often, a clear understanding of the material in the textbook is missing.

As results of all these problems, the authors have devised a structured reading system to guide students through learning and understanding of mathematical, physical and engineering sciences (The authors have focus on mathematics for illustrative purpose and space constraint only). The goals of the “*Structured Reading Guide*,” are to facilitate the process of learning and understanding mathematics and science, and increase the competency in mathematics and science literacy. The *Structured Reading Guide*, which we abbreviate as SRG, will show various strategies of reading, decoding mathematical and scientific texts, and of breaking the duality of the linear presentation style of texts into multi-dimensional paradigm or matrix, where the identifications, comparison, linkage of ideas in various levels are facilitated. Students will be more motivated to become problem-solvers, and to demonstrate, communicate and reflect on mathematical ideas.

Mathematics Textbooks

In the proceeding section, we discuss the taxonomy of mathematics, implication of the elements, fabrication and modification of graphics organizer, and how elements in the taxonomy of the texts can be used to comprehend the texts.

Mathematics and Scientific Textbooks: Taxonomy of the Texts

Modern mathematics and science texts use colors, pictures, varied font types and even cartoons to illustrate and facilitate explanations. Understanding mathematical and scientific texts depends on decoding these elements. Most technical texts are either written in Narrative

Paradigm (which is similar to a plain writing with little or no symbolic expression) or Label Paradigms (which is structured around labeling each section and/or subsection of the text).² Narrative Paradigm as shown in figure 2A is often seen in upper undergraduate and graduate level texts. Prose in Narrative Paradigm is characterized by running in sentences and paragraphs. This narrative paradigm is used to describe a sequence of events. Ideas and concepts in mathematical and scientific registers are written in a natural language with embedded symbolic expressions. Labeled Paradigm is written in sub-sectional style with section labeled or numbered. High school texts frequently combine these two paradigms, narrative and labeled, in the hope of clarifying the contexts of the texts, and making the identification of concepts and ideas easier.

In mathematics and science, most of these elements are embedded in prose text. Prose text is a portion of text that lies between pairs of headings. Sometimes a definition or other named concept and/or other elements are embedded in the midst of prose text. Sometimes, examples and/or theorem proofs are presented in prose text. Otherwise, the textual prose usually serves to explain, amplify, motivate, or discuss the ideas presented in the named concepts or examples. These sections typically cover no more than two or three paragraphs before a new named concept or example is formally presented. In understanding mathematics and science, it is important that the reader ponder this information so as to assimilate the ideas as fully as possible.

Named Concepts consists of definitions, theorems, terminologies, specifications, proofs, propositions, and formula. They are the main ingredients of non-computational examinations. They are also features of problem solving and computational examinations. A named concept is information that has a component that must be memorized, and understood. Diagrams are often used to illustrate relationships among various mathematical entities. In this case, the diagram also qualifies as a named concept; the concept is the particular relationship being illustrated, as shown in figure 1. Another element in prose text is an *example*. An example in a mathematics text is more than just a collection of computations; it serves as an illustration of some concept the authors consider to be important.

Implications of the Textual elements

Prose text is a segment of text that lies between pairs of headings. It consists of mathematical terminologies, definitions, examples, theorems, proof, graphics (pictures and diagrams) and named concepts. Prose text is often written in mathematical register, a discourse in which most of the elements are written.² In modern mathematical texts, authors often use mathematical register and ordinary natural language* to illustrate and communicate ideas and concepts. (Figure 2B shows an example of mathematical register that includes the elements that will be discussed later). The combination of mathematical and natural registers sometimes creates ambiguity, because students have difficulties distinguishing mathematical register from thought-experiment or mental representation of the author.

In mathematical register, technical vocabularies are used to refer to definitions, theorems and facts. Terminologies or vocabularies are specialized expressions indigenous to that particular field, subject, trade, or subculture.³ For instance, *system* in Electrical and Computer Engineering differs from *system* in mathematics, physics, or even in chemistry. Definition associated with the term varies from field to field, and from author to author. Terminologies in mathematics, as in physical and engineering sciences, are the basis of comprehension essential to understanding the contexts, the ideas and theorems. Terminologies differentiate concepts and ideas. Manipulating one terminology with respect to others can create a deep understanding of the concepts.

Symbols are also embedded in mathematical, physical and engineering sciences registers. In mathematics, they are written signs used to represent operations, elements, quantities, qualities, or relations. Symbols codify information and abridge concepts and ideas. In chemistry for instance, one or two letters represented each element (e.g. Aluminum (Al), Hydrogen (H), Helium (He), and the like). In physics (e.g. Proton (p), Momentum (P), Resistance (R), Entropy (S), etc...) and in mathematics (e.g. Partial derivative in calculus or Boundary in topology (∂), there is/exist (\exists), Gradient (∇) and Summation (\sum), and so on), are employed as symbols. These symbols are used as abbreviators, operators and/or formula. Chemical, engineering and mathematical formulas are written in chemical, engineering and mathematical symbols, which

are specific or commutative to that particular or various fields. In the sense of chemical symbolic expression, symbols are used to abbreviate names, as illustrated in the above example, or to signify ideas. To illustrate the roles of symbols in mathematics, we compare the following sentence in figure 2A to its mathematical representations in figures 2B and 2C.

The sentence in figure 2 illustrates text written in narrative paradigm. For simplicity, sentences in figure 2 use mathematical expressions or register. In the example, symbols are used to both convey meaning and simplify the expressions in figures 2B and 2C. This simplified version of the statement is much easier to read and to understand compared to the elaborated and narrated version of the same statement. In mathematics, symbols have dualism. Symbols in figures 2B, for example, abbreviate mathematical register and present concepts, ideas, calculations and formula. Sentence in figure 2C shows an extreme use of symbols where every words and sentence have been replaced by symbols. Symbols in mathematical, physical and engineering sciences are utilized to compress mathematical, physical and engineering registers, and they contain more ideas, and concepts than regular and vernacular prose. As a result, comprehending mathematical registers (as a focus of this work) requires understanding the words and their relationships to symbols. One also needs to master their meanings and applications.

Guide for Graphic Organizer with Mathematical and Scientific Texts

In the sub-section, we discuss the “how” to use the Graphic Organizer we developed to facilitate readings and learning of mathematics and sciences texts. Various researchers have theorized that graphic organizers to increase achievement slightly. But what is a graphic organizer? Ausubel and others have theorized that Graphic Organizer -- “an orderly arrangement of concepts”— can help students learn concepts by helping them to incorporate new ideas with prior or existing concept.¹ As a result, we have modeled our device (*Structured Reading Guide SRG*) in the form of graphic organizer in the hope of facilitating the learning process. The authors believed that ideas incorporation occurs when new ideas are integrated with, and/or when new ideas are added to pre-existing mental networks of knowledge, experiences and information stored in long-term memory also known as schemata.

In mathematical texts, one may also encounter graphics. Graphic representations in mathematics are visual illustrations of verbal registers and are fundamental to understanding some mathematical registers depending on their purposes. Some graphics are used to either discuss the information not mentioned in the texts and/or to elaborate on already stated information in text to further illustrate concepts, and ideas. Graphics allows readers to make connections apparent. There are various types of graphic aids and are consisted of diagrams, maps, graph, tables, charts, pictures, and cartoons^{**}. In mathematics, graphics serve different purposes. They are in the text to give students other views of the concepts. For instances, diagrams in some books are used to summarize ideas mentioned in that chapter or section; others are used to show relationships among ideas and concepts in the text. Yet must students ignore these taxonomic elements when reading and studying.

Diagrams are also employed in mathematics textbook to show algorithmic approach to some calculations or problems, and concepts. They can be used to facilitate the visualization of the problems, ideas, and method of solving the problem as shown in Figure 1 below. So it is important that students pay attention to different forms of graphics because they contain information that could be relevant to the understanding of the problems and/or the concepts. To get much out of the diagram, we advise the reader/student to turn the graphic organizer into its textual representation if that simplifies the underlining meaning of the concepts or ideas being illustrated on the organizer. However, if the graphic organizer clarifies the ideas and/or the concepts better, we advice the reader use it or modify in such ways as to help the reader.

S.R.G. for Mathematical and Scientific Texts

Structured Reading Guide is a tool we have developed and modified to facilitate textual learning in mathematics, science, and engineering. It was primarily developed for students in high school and lower undergraduate technical courses in physical science, engineering and mathematical sciences, even though some extensions have been made to accommodate upper level undergraduate course that emphasize proof. In this section, we discuss parts of the device and instruction related to the usage of the tool in mathematical sciences for simplicity and illustrative reason. However, it can be used in physical and engineering courses with little or no

modification.

Text Identification Information

One purpose of taking structured notes is to provide a record for future reference. Records are best accessed when clearly identified. The first few lines of the graphic organizer provide space for identification information. The categories are self-explanatory and require no further elaboration. If the student intends to take notes from several sections of a given text, it is useful to develop a special graphic organizer for that text. SRG will have the *text title* and *subject* boxes already completed. Photocopies of an otherwise blank form will serve as well.

Engaging Prose Text

As described in the previous section, *Prose text* is a text that lies between pairs of concept heading. In the Structured Reading Guide, sections have been allocated for concepts assimilation. The graphic organizer (Structured Reading Guide) provides blocks for summarizing a block of prose text. The student will benefit most when the summary is in his or her own words. However, the beginning student may find this exercise too difficult; it is recommended that a student who is having difficulty paraphrasing the main ideas of the text be content with identifying and copying the topic sentences of the paragraphs. Summaries of main ideas can be represented in various forms. Interpretations, key words, selective diagrams, themes, topic issues, thesis, titles, and gist are different modes students can employ or write in the Summary section of the organizer. The reader may also wish to annotate the entry with a page number so that the text and graphic organizer may be compared at a later date.

Named Concepts

Named concepts are any ideas written with a commonly understood name. Examples include the definition of skew lines, the Euclidean Parallel Postulate, the Triangle Angle-Sum Theorem, Pythagorean Theorem, Fundamental Theorem of Calculus, the Quadratic Formula, and so on. These are often labeled as Definition, Theorem, Fact, etc... Named concepts are fundamental understanding the conceptual problems and sometimes to quiz and test questions. It

is advised that the student trying to master the concept memorizes the named concepts or grasps their fundamental meanings. It is also possible to use diagrams as named concepts when they illustrate a particular relationship among various mathematical entities.

When one encounters a named concept while reading a section, the concept may be recorded in the *Named Concepts* block of the graphic organizer. Various researches have shown that recording a concept aids in the memorization process. Note that the linear flow of the text is broken by providing a single space on the graphic organizer for recording *all* named concepts encountered in the section. This type of organization makes it easier to locate a definition or formula when one recalls the section in which it was introduced – all the information of this type is located in one place. Sometimes these concepts are novel and do not directly relate to prior learning; at other times named concepts are constructed using previously learned concepts. The reader is encouraged to make notes that link a new concept with previously encountered concepts or prior knowledge, as indicating the example portion of the graphic organizer.

Examples

An example in a mathematics text is more than just a collection of computations (as previously defined); it serves as an illustration of some of the concepts the author considers to be important in understanding the materials. Some authors, as illustrated in the *Taxonomy of Mathematical Text*, only use examples couple with theorems, facts, definitions and other terminologies as means of explaining concepts and showing the mechanics of the problem solving. In the graphic organizer, a space is provided, on the left hand box, so the student can reproduce the salient details of the concept. This means the reader may skip steps that are obvious or add additional steps or commentary in order to clarify the text presentation. Once the mechanics have been considered, the student is invited to delve into the higher-order concepts associated with the example.

A second boxed space is provided on the right for the purpose of investigating higher mechanism of the text. Three higher-level questions are posed for each example to facilitate the process of learning: (1) *why was the example given?* That is, why does the author provide this particular example? The stock answer is that the example illustrates some concept the author

finds important. What is that concept? This is usually answered in the paragraph that immediately precedes the example. It is important that the student articulate this reason. We suggest writing a complete sentence such as “The author provides this example to illustrate the use of the quadratic formula.” (2) *What prior principles are used in the example?* This question provides an opportunity to link the example to prior learning. Sometimes an example illustrates a new, stand-alone concept and no (non-trivial) prior learning is illustrated. In this case, the question can be ignored. Sometimes an example builds on recently learned concepts; prior knowledge in the pre-requisite courses and/or is a unifying mechanism for multiple concepts. In this case, the student should attempt to clearly identify these concepts. For example, an example that illustrates the quadratic formula will use the concept of discriminant that may have been previously defined (it depends on the text). (3) *What did you learn from the example?* It is an important question that a reader must answer. It is also very difficult to answer such a question in a nontrivial manner. In fact, the reader may not be able to answer this question immediately after studying the example.

Theorems and Proofs

Except for geometry classes, theorems and proofs are somewhat rare in secondary and lower division texts. A reader wishing to use this method with a text that devotes a significant amount of resources to proofs, we have included a section that is appropriate for proof depending on the nature of the course. However, students in a course entirely devoted to proof may use the examples section of this organizer for noting proof or may want to use graphic organizer in appendix A3 and/or A4 for proof. In a course that is partially proof oriented, students may wish to use the combination of the graphic organizer, using both the organizer in Appendix A. However, students are encouraged to find a way to adapt the existing graphic organizer to the structure of their courses, whether the course for which the organizer is being used is partially or entirely proof oriented. One way is to redesign the organizer from scratch. One easy way of doing this is to design a third page that is devoted to proofs, as we have done so for illustrative purpose. However, for courses where proof is almost nonexistent, one can just use the existing format of the organizer.

Another way where modification of the current graphic organizer can be minimized is to

use the example section of the organizer, see the appendix. Notice that a proof has much in common with an example. Both are logical sequences that need to be understood, both are presented for some reason that transcends the individual example/proof, both link new concepts to prior learning, and the reader has an opportunity to express in his or her own words some aspect of a learning experience. Thus, the reader can, in a pinch, use the existing graphic organizer without modification when confronting a rare proof. In the following subsection, we discuss technique we believe to facilitate the understanding of proofs that are often encountered in Mathematics.

Proposition/Statement Text:

In this section, we discuss the application of S.R.G. in mathematical course with emphasis placed on proof. Proposition/Statement Text block is primarily used to copy whatever is to be proved. Proposition is a sentence that is either valid or invalid. Like definition and other mathematical terminologies, proposition is an argument as to the meaning of a term. “There does not exist x in Q such that $x^2 = 2$ ” is an example of theorem that, if one encounters while reading and using SRG, should be written in this block.

Proof Analysis

Proof is a way of communicating a mathematical truth to others. It is a convincing argument in mathematical registers. While reading a proof, it is important to note some of the details that were omitted by the author, for some authors either assume that the omitted details are obvious or comprehensive to the reader. So doing proof analysis allows the reader to retrace the logics that were put into writing the condense version of the proof. It also brings the techniques the author used to prove a particular proof to come to the surface, hence making the proof more understandable. In the figure below, student should replicate the proof as seen the text or the note. A block has been designated for proof analysis as mentioned the above sentence. In Structured Reading Guide (Right hand side of the tool), we have incorporated a technique of reading mathematical texts that include proof. By doing so, students who use this tool are equipped in breaking and decomposition all the conceal details and logics. As mentioned in previous paragraph, proof analysis allows students to follow the logics, reasoning

and procedures, and decodes all the hidden details containing in the specific proof, for published proofs including those seen in the mathematics textbooks combine several steps with no mention of the techniques, references or assumptions. (See the appendix for sample proof analysis as illustration of the concept).

Proof Analysis Summary Section:

Like summary block for *Prose Text*, Proof Analysis Summary Block is used to condense the analysis that was performed to decompose the omitted details of the actual proof. We advise students to rewrite the analysis in their own words by writing main ideas of the analysis. It can be represented in various forms: Interpretations, key words, and selective diagrams. However, it much contains essential ideas from the proof that was analyzed.

Reflection

The reflection is a period of contemplation, as Fleming speculated, that occurs after the section has been read with the use the graphic organizer. Reflection is a vital part of any learning. The student may wish to take a few moments to allow the mind to “organize” the material into order to bring creativity into his/her learning. Reflection takes when the student looks at the material in the lens of prior knowledge.

When reading, the student then uses the *Reflection* portion of the organizer to summarize, in his or her own words, the main ideas of the section. The act of articulation helps with the process of understanding the material. In addition to the summary, the reader is asked to respond to these additional higher-level questions: (1) *what would you like to have explained differently?* Not everything the student reads will be crystal clear, even after a second or third reading. This is why it is critical that a student read the text *before* the material is presented in lecture. A student is prepared to pay closer attention to a lecture when he or she recognizes what is about to be discussed next and also recognizes that their understanding of the material is weak. If the student still does not understand a concept after the instructor’s presentation, that student should be very motivated to ask a clarifying question at that moment. In short, reading the text before a lecture transforms a student from being a passive participant in a lecture to being an active

participant. A student can anticipate what is coming next and be continuously checking those expectations against what actually transpires. Correct anticipations will reinforce the learning process; deviations from the expected information will stimulate discussion or further investigation. (2) *What more would you like to know about this subject?* Of course, the stock answer for most students most of the time is likely to be very little or nothing at all. However, there is always the opportunity to recognize a connection between the section material and something of interest the student has encountered elsewhere. Answering this question is a precursor to self-motivated research. It becomes a type of post-it-note the student can read at a later time and motivates the student to pursue the extended learning process. The student may also want to ask the following questions: what principle or principles are they based on; what are the significance of these facts and ideas; what could they applied to; how do they fit in with what is already known; what can I learn from these facts, ideas, theorems, definitions, etc...? All these questions are viable and would help students reflect on concepts they are learning. The proceeding section will illustrate how the student should use SRG when reading and/or after reading a mathematical and scientific text.

Engaging Surveying, Reading and Analyzing System (S.R.A.S.) in S.R.G.

In the following section we summarize all the steps involving the Structured Reading Guide for Mathematics texts. We believe that familiarizing yourself with the features of mathematical texts, as illustrated in previous sections, can reinforce your understanding of the mathematics and other technical texts.

Surveying and S.R.G.

Many Education research scholars have shown that surveying can bolster the understanding of the reader. Surveying before reading the text puts the reader in to the reading mode. When surveying a chapter or a section of a mathematical text, it is advised to go through the elements (propositions, definitions, and the like) we discussed in the previous sections. For instance, reflecting on the title of the section can give you an idea about the content of the text. For example, reflecting on a chapter title like “Limit and Continuity” might help guess that the chapter will discuss topics like limit definition, method of finding limits, concepts of continuity,

rates of changes and the like. After surveying the chapters and sections, proceed on to reading the introductions and the summaries (if there is any). Reading these introductions and summaries may help you generate a visual framework of the textual organization. While surveying the content, we advise the reader to pay close attention to the heading and subheading of the section or chapter. Also pay close attention to information that are highlighted, boxed, boldfaced, italicized, bulleted and visual materials because these texts often fall in the categories definition, proposition, axiom, theorem, facts, proof, and other taxonomical elements that were discussed in previous chapters and section of this paper. They are the elements all readers should pay attention to.

Reading, Analyzing and S.R.G.

To read a mathematical text, we believe that it is appropriate to read the text in a section-to-section mode by reading a section at a time and juggling down the taxonomical elements in the Structured Reading Guide. Writing the definitions, propositions, theorems, axioms, and proofs in their appropriate block in the Structured Reading Guide graphic organizer can simplify the revisions and recitation of your notes in the future, hence allowing the utilization of the information in the text to answer the higher level questions we pose in the section concerning example in the previous subsection or section. Higher questions such why was the example given, why does the author provide this particular example, what is concept being illustrated, what prior principles are used in the example and what did you learn from the example are all plausible questions that the reader should always keep in mind while reading. These answers should be placed in the block on the right of example (see SRG graphic organizer in Appendix A).

After reading the text and following the step underlined in the above sections, it is time for contemplation. Analyzing what the reader have been reading and trying to understand the concepts, the reader should reflect on the concept to try to find bridges or links between ideas encountered in that and other previous sections. Like example, we advise the reader to explain the ideas in different lens or light if possible. This allows the student or the reader to crystallize the concepts and incorporate creativity in the learning process.

Conclusion

In this paper, we report anecdotal results on implementation of this novel method of learning mathematics, physical and engineering sciences. In the future, we hope to report rigorous results since anecdotal results lack statistical validity of the tool in improving learning and thinking in technical field.

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Endnotes:

* Natural Language is a language that has nothing to do with mathematical expression.

Most authors write informally in order to engage students in the mathematical exploration.

Therefore, they utilize natural language in the process

** Each type of these graphic aids may have sub-types. For instance, pictographs, pie, bar and lines are all subtype of graphs. As mentioned in the objective, students often ignore these graphics. They see them as space filler, and pay little attention to the information presented in graphic forms

Error!

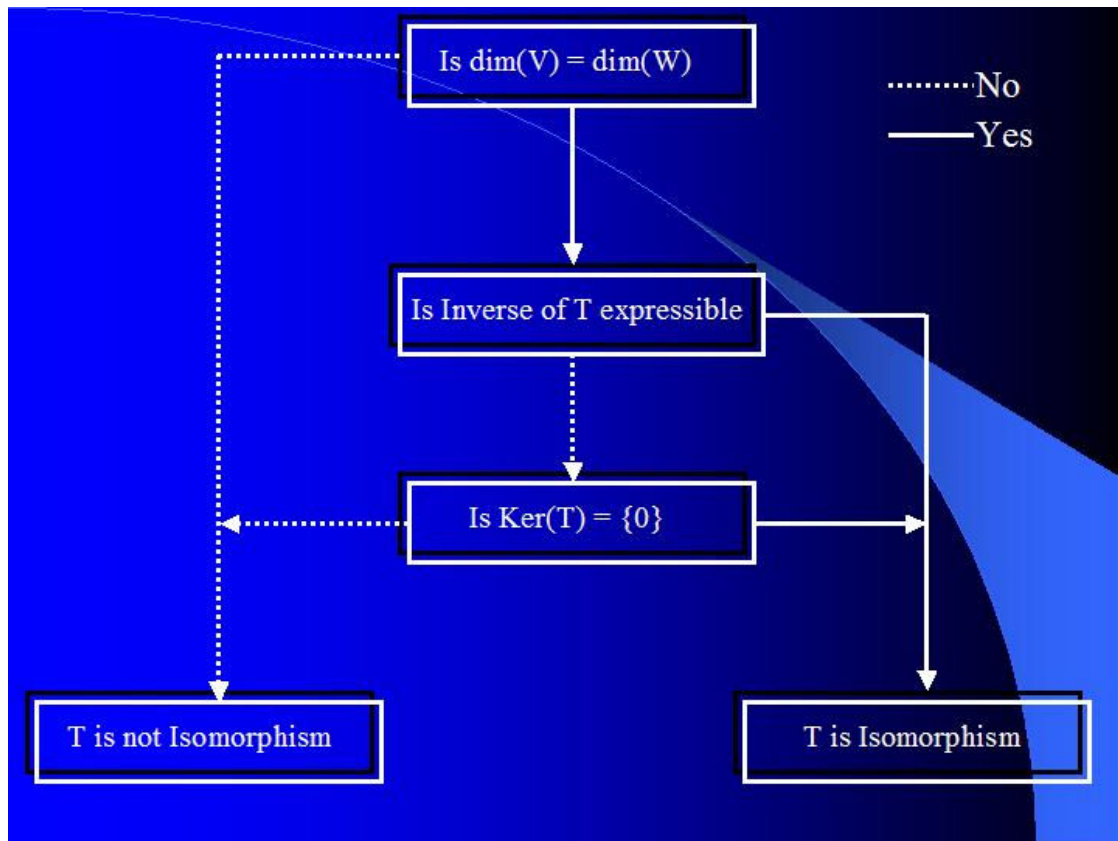


Figure 1: This illustrates how graphic organizer sometimes hints problem methodology or can be used as a guide in determining, in this illustration, whether a transformation from subspace to another is an isomorphism⁴

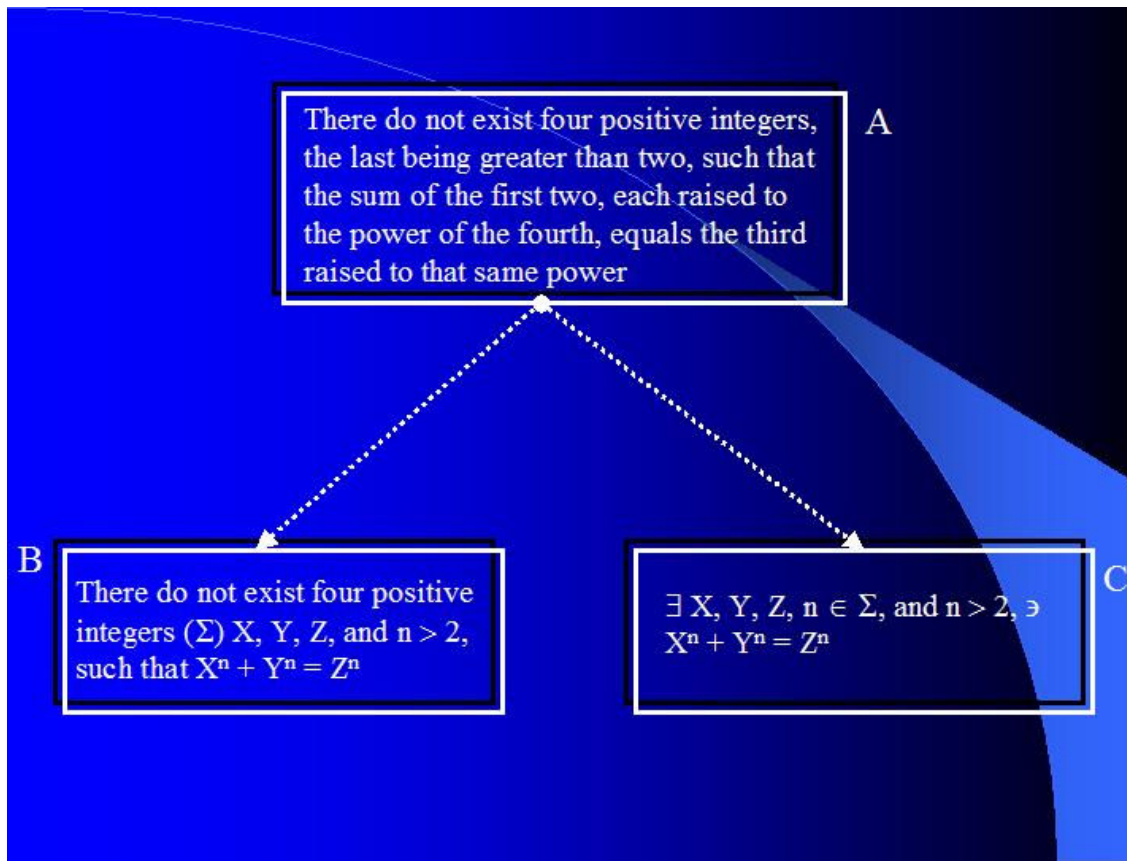


Figure 2: A narrative version of Fermat's Last Theorem (A) ⁸. A reduced version of the theorem but written with the combination mathematical registrar and natural language, and symbolic notation (B and C)

Appendix A:

Structured Reading Guide for Mathematics and Scientific Texts		
Text	Date	Subject
Section Title	Chapter, Section, Subsection Number	Pages
<p>Named Concepts: Note any vocabulary terms introduced and defined, named theorems, named postulates, and formulas. Reproduce any graphic organizers. Use additional pages, if required.</p>		
<p>Summary Any text (except examples) between two headings should receive a one-to-three sentence summary. One should be able to read the summary and have a clear picture of the substance of the text.</p>	<p>Summary Any text (except examples) between two headings should receive a one-to-three sentence summary. One should be able to read the summary and have a clear picture of the substance of the text.</p>	
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A1: Complete version of SRG (It shows the front page of SRG)

<p>Example: Use this space to reproduce the details of an example.</p>	<p>Use this space to explore the meaning of the example. Why was the example given? What prior principles are used in the example? What did you learn from the example?</p>
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<p>Reflection – Complete after reading all the text and examples. Describe the section in four or five sentences. What would you like to have explained differently? What more would you like to know about this subject?</p>	

<p>Proposition, Theorem, Axiom, Statement to Be Proven: Use this space to reproduce the details of a statement to be proven</p>	
<p>Proof: Use this space to reproduce the details of the proof as stated in the book.</p>	<p>Proof Analysis: Use this space to explore the meaning of the proof via analysis of the techniques used and by including the omitted details. Explore why the proof is given in this particular manner? Are any axiom, theorem or principle is used in the proof?</p>
<p>Summarize Proof Analysis – After completing proof analysis, write a brief short proof using the proof analysis.</p>	

A3: Addition page allocated for proof and proof analysis of theorems, statements, axioms, etc... It used to compliment the tool.

Text	Date	Subject
Section Title	Chapter, Section, Subsection Number	Pages
Proposition, Theorem, Axiom, Statement to Be Proven: Use this space to reproduce the details of a statement to be proven		
<div><div>Proof: Use this space to reproduce the details of the proof as stated in the book.</div><div>Proof Analysis: Use this space to explore the meaning of the proof via analysis of the techniques used and by including the omitted details. Explore why the proof is given in this particular manner? Are any axiom, theorem or principle is used in the proof?</div></div>		
Summarize Proof Analysis – After completing proof analysis, write a brief short proof using the proof analysis.		

A4: Figure 8: SRG for courses that are entirely proof based. Examples include Analysis courses, Theoretical Linear Algebra, Algebra, etc...