AC 2011-2240: MATHEMATICS AND ARCHITECTURE OF THE INCAS IN PERU

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I am a professor of mathematics at the University of St. Thomas in St. Paul, Minnesota, where I have been a faculty member since 1983. I received my Ph.D. in 1979 from Brown University in Formal Calculus of Variations. My recent area of research is mostly in computer vision, with applications to object recognition. My publications are in diverse areas of mathematics and engineering. I love to work with undergraduate students, in particular, underrepresented students, to get them involved in doing research in mathematics and encourage them to give conference presentations/posters and submit their work for publication. In addition to teaching regular math courses, I also like to create and teach innovative courses such as "Mathematical symmetry of Southern Spain" and "Mathematics and Architecture of the Incas in Peru", which I have taught as study abroad courses several times.

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Michael P. Hennessey (Mike) joined the full-time faculty as an Assistant Professor fall semester 2000. He is an expert in machine design, computer-aided-engineering, and in the kinematics, dynamics, and control of mechanical systems, along with related areas of applied mathematics. Presently, he has published 41 technical papers (published or accepted), in journals (9), conferences (31), or magazines (1). In 2006 he was tenured and promoted to the rank of Associate Professor. Mike gained 10 years of industrial and academic research lab experience at 3M, FMC, and the University of Minnesota prior to embarking on an academic career at Rochester Institute of Technology (3 years) and Minnesota State University, Mankato (2 years). Mike holds a Bachelor of Mathematics from the University of Minnesota (with distinction), an MS in Mechanical Engineering from MIT, and a Ph.D. in Mechanical Engineering from the University of Minnesota. He is also a member of ASME, SIAM, and ASEE.

Mathematics and Architecture of the Incas in Peru

Abstract

This novel study-abroad January (J-term) course co-taught by the authors (from mathematics and mechanical engineering) is described in detail. It evolves as a journey, both academically and geographically, with most topics being directly related to the Incas and / or symmetry. Students learned about quipus (a complex system of knot-tying for recording numerical and other data), la chakana symbol, la yupana (an abacus-like calculating device), geometrical symmetry of frieze and wallpaper patterns commonly depicted on buildings and in textiles, and the stability of structures, important because of the common occurrence of earthquakes. In addition, the famous enigmatic Nazca lines and the steamship, Yavari, on the highest elevation commercial lake in the world, (Lake Titicaca) were studied onsite. This adventurous course, whether by land, air, or water, traversed through much of Peru (0-14,000 ft), including the cities of Lima, Nazca, Arequipa, Puno and Lake Titicaca (islands of Uros and Taquile), Cusco, Aquas Calientes, and the beach community of Huanchaco Trujillo. The highlight of the course was visiting the famous lost city of the Incas, Machu Picchu. Samples of student work, assessment, and lessons learned are provided that offer practical advice for others considering offering similar courses.

Keywords: Peru, Incas, study-abroad, mathematics, architecture, engineering

1. Introduction

Study-abroad courses at many US universities and colleges are commonplace these days, especially on traditional liberal arts topics like languages or history, in disciplines where field work is highly valued such as geology or biology, and for courses that by definition possess attributes with an international flavor to them (e.g. international business). That's great, but what about mathematics or engineering? Historically speaking, there haven't been that many study-abroad courses offered in mathematics or engineering and participation rates have been low.[1,2] Clearly, one major source of motivation (beyond an individual faculty member's professional interests) for offering our course described below is to appeal to the needs of this underserved population of mathematics and engineering students, as well as students from other technical disciplines that value analytical thinking (e.g. physical sciences and economics), with regard to meaningful, discipline specific study-abroad course offerings.

One well-established, popular, and effective mechanism for offering study-abroad J-term courses at the University of St. Thomas is through a regional 5 school consortium, UMAIE[3] (Upper Midwest Association for Intercultural Education). On a yearly basis UMAIE typically offers between 20-30 courses. Each school contributes faculty and students to create an annual January curriculum, with the support of a coordinating organization for enrollment, marketing, logistics and financial management. In our case, no other courses (of the courses offered in J-term 2010) were mathematics or engineering related. The niche market for this course is for students (26 total in our case) who have completed 3 semesters of Calculus (a strict requirement), which narrowed it down to mostly upper classman in engineering and / or mathematics, although other technical majors, such as chemistry were represented. The course can be used as either a 4 semester-credit mathematics or engineering course, appealing to many who are seeking a minor, typically in mathematics.[4]

Course description (from J-term 2010 UMAIE brochure)

The ancient Incas had elaborate mathematics, astronomy, architecture, and communication systems. The purpose of this course is to study their mathematics and see how they incorporated it in their art, architecture, calendar and communication systems. They devised an amazing tool, the quipu, to record numerical data for accounting and for transmitting complex messages with astonishing mathematics which is very similar to the binary codes used in the early versions of computers. Another aspect of their culture filled with mathematics is their architecture that was also enhanced with use of symmetry in their art form. The frieze patterns that occurred on buildings and found on their pottery represent all seven basic types of mathematical frieze patterns. We will visit many of the buildings that were built by the Incas all throughout Peru including the Inca city of Machu Picchu. The gigantic stones used in this city were carved and put together with such precision and mathematical accuracy that to this day one cannot fit a thin knife between stones. The stones are held together not due to mortar but because of pure craftsmanship. We will also visit and study the geometry of the mysterious Nazca Lines, which are a set of zoomorphic, phytomorphic, and geometric figures that appear engraved in the surface of the Nazca desert, a high arid plateau that stretches for 37 miles. They are the most outstanding group of drawings on the ground in the world.

From a pedagogy point of view, the course is structured such that there are different elements that work together synergistically, including on-site visits (possibly with accompanying instruction), travel, readings, lectures, student presentations (both technical & tourist oriented) to promote ownership and engagement in the course, and traditional homework in a 300+ page custom bound booklet & workbook with over 90 traditional mathematics problems[5] (also posted to Blackboard, **Bb**). The itinerary is shown below in Table 1.

Table 1 Abbreviated itinerary

Date (1 / X / 2010)	City	Sites (excluding class and tourism)
Saturday, 2	Twin Cities / Lima	
Sunday, 3	Lima	Pachacamac, Archaeological and Anthropological Museum
Monday, 4	Lima	
Tuesday, 5	Lima / Nazca	
Wednesday, 6	Nazca	Nazca lines
Thursday, 7	Nazca / Arequipa	
Friday, 8	Arequipa	Monastery of Santa Catalina
Saturday, 9	Arequipa	
Sunday, 10	Arequipa / Puno	
Monday, 11	Puno	Chulpas of Sillustany, Yavari steamship
Tuesday, 12	Puno	Full-day motor boat to floating islands of Uros and Taquile
Wednesday, 13	Puno / Cusco	Puka Pukara (Red Fortress), La Raya, Raqchi (Temple of Wiracocha God), and church of Andahuaylillas
Thursday, 14	Cusco	Sacsayguaman, Kenko, Pucapucara and Tom-Bomachay
Saturday, 16	Cusco	Sacred Valley of the Incas, Pisac Indian market, Urubamba, Ollantaytambo fortress, and Chincheros
Sunday, 17	Cusco / Aguas Calientes	Ollanataytambo train station
Monday, 18	Aguas Calientes / Cusco	Machu Picchu
Tuesday, 19	Cusco / Lima	Cusco airport
Wednesday, 20	Lima / Huanchaco Trujillo	
Thursday, 21	Huanchaco Trujillo	Temple of the Moon and the Sun, Chan Chan with visits to the Temple del Arco Iris, the site museum, and the Tshudi Palace
Friday, 22	Huanchaco Trujillo	
Saturday, 23	Huanchaco Trujillo / Lima	
Sunday, 24	Lima / Twin Cities	

The lead author and course originator (Shakiban, Mathematics Professor) is an expert on the topic of mathematical symmetry and fractals, in addition to being a seasoned international traveler (having travelled to over 50 countries), often to sites that exhibit interesting symmetry or fractal attributes. Symmetry and fractals appear in many places including manmade forms, such as textiles and impressively in architecture, which, to maximize appreciation, makes sense to be physically on-site. The second author (Hennessey, Mechanical Engineering Associate Professor) specializes in kinematics and dynamics of mechanical systems and this is his first involvement with the course. Because of the emphasis on, and fairly ubiquitous nature of symmetry, fractals, mechanics, and architecture, the course offers some flexibility with regard to its venue. That said, regional opportunities that had a mathematical theme to them would naturally be included in the course. Prior to the January 2010 offering, the course had been taught once before in Peru (2008), and a similar course twice in Spain (2006, 2004).

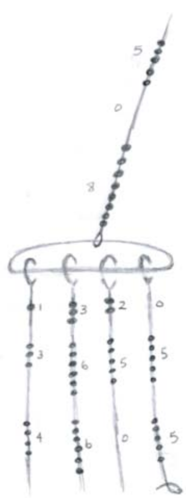
2. Major Academic Topics

Major academic topics pertain to: quipus, la chakana, la yupana, Nazca lines, Yavari steamship, geometric symmetry (general theory), frieze patterns (1D symmetry), wallpaper symmetry (2D), Fibonacci numbers, Golden ratio, self-similarity & fractals, and structures. Collectively, these topics represent a blend of fairly generic, venue independent topics intertwined with the best of what the Incas, and Peru more generally, have to offer from a mathematics point of view. In each case, an overview of the topic is presented, possibly with sample student work.

Quipus:[6] A Quechua word that means "knot" refers to a complex record keeping system based on threads and knot tying that was used by the Incas to record various types of data, often of a numerical nature, such as for accounting and crop records (Table 2). As can be seen below, the Incas used a decimal, or base 10 system, with knots arranged in order of units, tens, and so-on, up to hundred thousands, apparently large enough to accommodate their needs. Quipus have been studied extensively by mathematicians and different theories have emerged related to their structure, the common presence of prime numbers (especially the number 7), astronomical numbers of significance (e.g. number of synodic revolutions of major planets), and even binary coding.

Table 2 Quipus, the Incan system for record keeping

An actual quipu on display at the Archaeological and Anthropological Museum in Lima.



Example conceptual quipu used to record numerical data indicated through knots (shown as dots) and threads.

Student teams presenting their fabricated quipus to the rest of the class. In this case, the quipus represented the nominal elevation (in ft) at each significant location that we visited (1 color-coded cord per location).

La Chakana:[7] The chakana (or 3-level cross) is one of the most prominent Andean symbols, dating back more than a 1,000 years, and present in numerous ancient civilizations including the Incas, Paracas, Chavin, Tihuana, and Nazca peoples (Fig. 1). Its purpose is not known for certain. To some it represents the Southern Cross constellation, the "stairway to the cosmos." This symbol also suggests that the Incas possessed knowledge of π, as the symbol's geometry exudes the famous "squaring the circle" problem. To the Incas, π (called *Katari*) was what we know today to be the square root of 10, or about 3.16 (vs. 3.14…).

La Yupana:[8] The ancient calculating device of the Incas is much like an abacus. Different operational theories have been proposed with one of the more plausible theories being put forward by Glynn. Common arithmetic operations, including addition (Figs. 2-3), subtraction, multiplication, and division, can be performed with one or more yupanas.

Fig. 1 *The chakana 3-level cross-symbol from our visit to Chan Chan.*

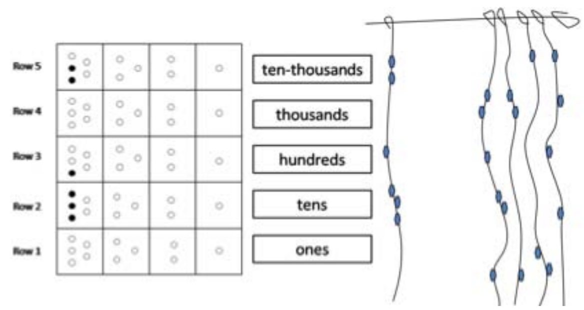


Fig. 2 *How recorded numerical quipu data is tabulated on the yupana.*

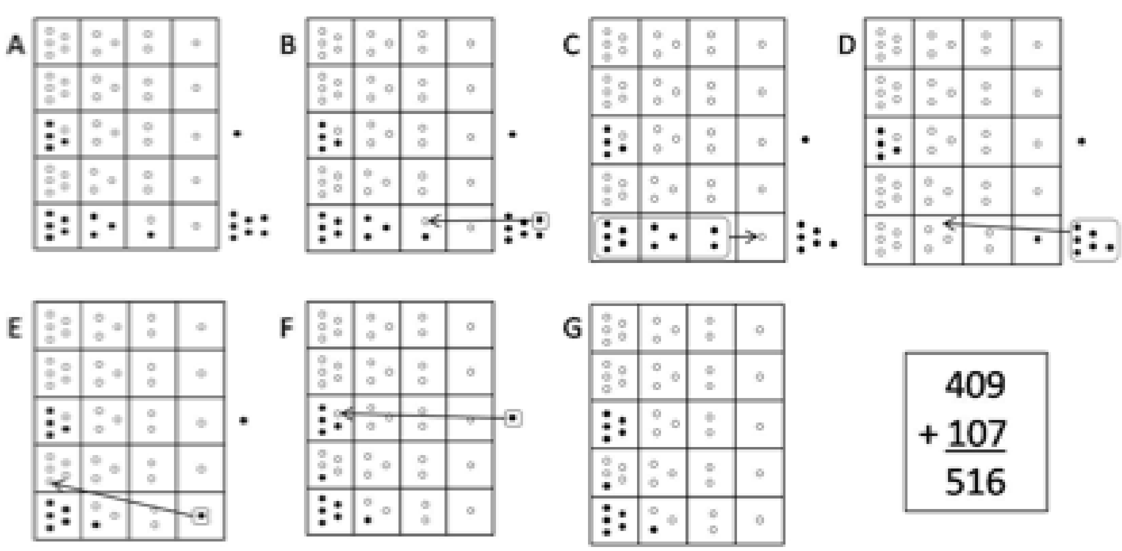
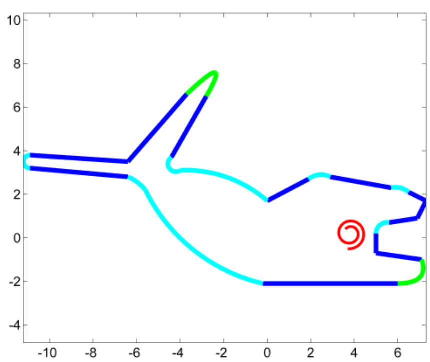


Fig. 3 *Step by step illustration of how the yupana is used to perform the arithmetic operation of addition, e.g. 409 + 107 (= 516), with the 4th column used as temporary memory.*

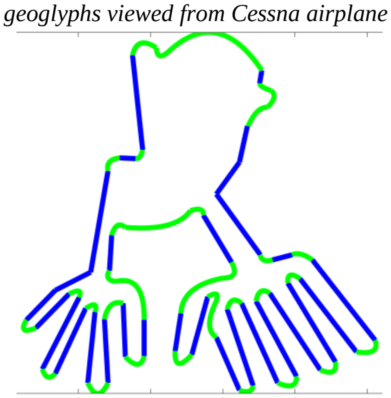
Nazca Lines:[9-13] The Nazca lines (and geoglyphs) are giant etchings in the desert, created by removing rocks from the sand and piling them up to create vast shapes when viewed from the sky. They were "discovered" back in the 1930's when viewed from an airplane and researched / preserved by Dr. Paul Kosok and his assistant, German mathematician Maria Reiche, who, because of her research over subsequent decades, has become singularly famous as *the* Nazca line researcher. Many theories abound as to how and why they were constructed. Some deal with ancient aliens, racetracks for competitions, giant astronomical calendars, maps of the Tihuanaco Empire, or to appease the gods. In addition to viewing them in spectacular fashion from Cessna aircraft, our focus was on creating mathematical representations of them as a concatenation of different curve segments using MATLAB[TM,14] (Table 3), and when viewed from different vantage points using projective geometry based on rotation matrices (Table 4).

Table 3 The enigmatic Nazca lines (and geoglyphs)

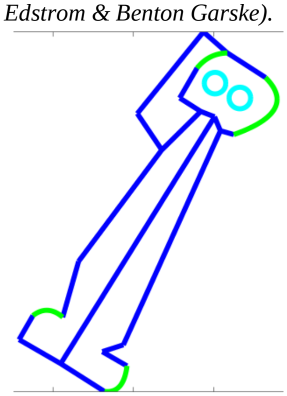
The whale, hands, and astronaut Nazca geoglyphs viewed from Cessna airplane (Tyler Edstrom & Benton Garske).



The whale geoglyph when viewed "straight on" and created in MATLAB™ as a concatenation of line segments, circular arcs, a spiral, and parametric polynomials; a paradigm for students to follow.



The hands geoglyph created in MATLAB™ based on Cessna airplane photo (Jacob McAlpine).



The astronaut geoglyph created in MATLAB™ based on Cessna airplane photo (Benton Garske & Natasha Wright).

Yavari Steamship:[15] The Yavari steamship is famous for being transported piece by piece (2,766 total) in the 1860s timeframe by mule-train over the Andes to Lake Titicaca, the highest elevation commercial lake in the world (at over 12,000 ft). While obviously not related to the Incas, because of its notoriety, and being that we were in Puno, it made sense to visit and study, and it was a good excuse to teach students about the dynamic modeling of steam engines using Simulink[16], a block-diagramming software environment that works with MATLAB™ (Table 5).

Table 4 The whale geoglyph viewed from different directions (perspective @ ∞)

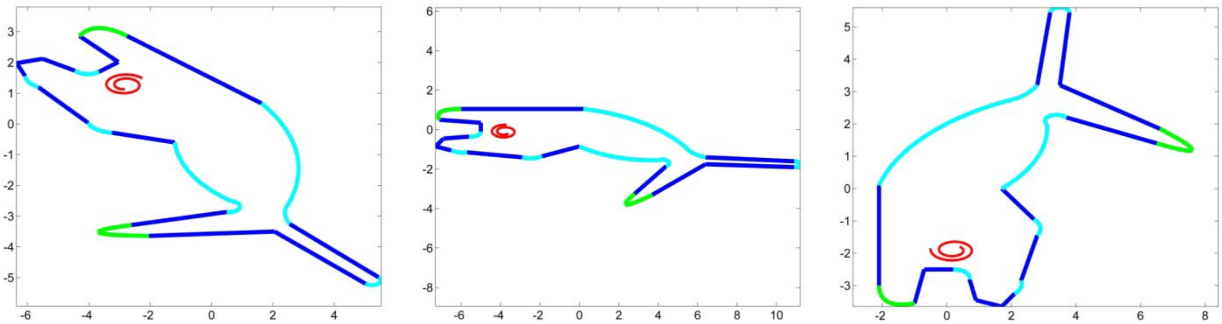
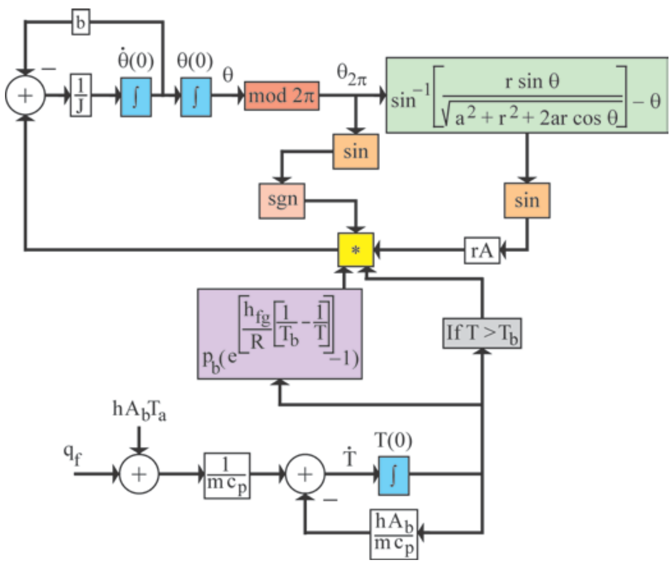


Table 5 The Yavari steamship in Puno Harbor



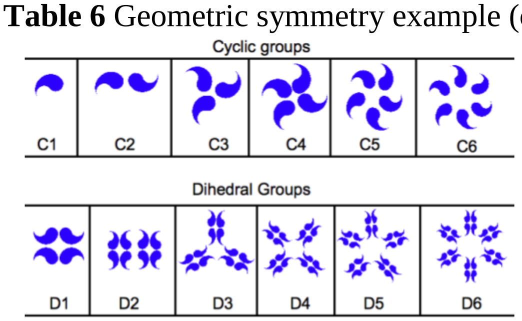
Conceptual dynamic simulation diagram of a steam engine model.

Students boarding the 150 ft Yavari ship in Puno harbor on Lake Titicaca.

Yavari engine (now converted to a diesel).

Geometric Symmetry (General Theory): Group theory, an abstract algebra notion is at the heart of truly understanding what symmetry is all about. Our focus is on 1D and 2D symmetries, which practically can be thought of as relating to frieze patterns (1D) or wallpaper patterns (2D). Geometric groups of interest are cyclic and dihedral groups, often depicted geometrically as shown below in Table 6. As part of their symmetry exercises, all students were encouraged to find as many examples of symmetry as possible and document, either with a sketch or photo for inclusion in their workbooks.

Table 6 Geometric symmetry example (cyclic, dihedral, and with cyclic group C_4)



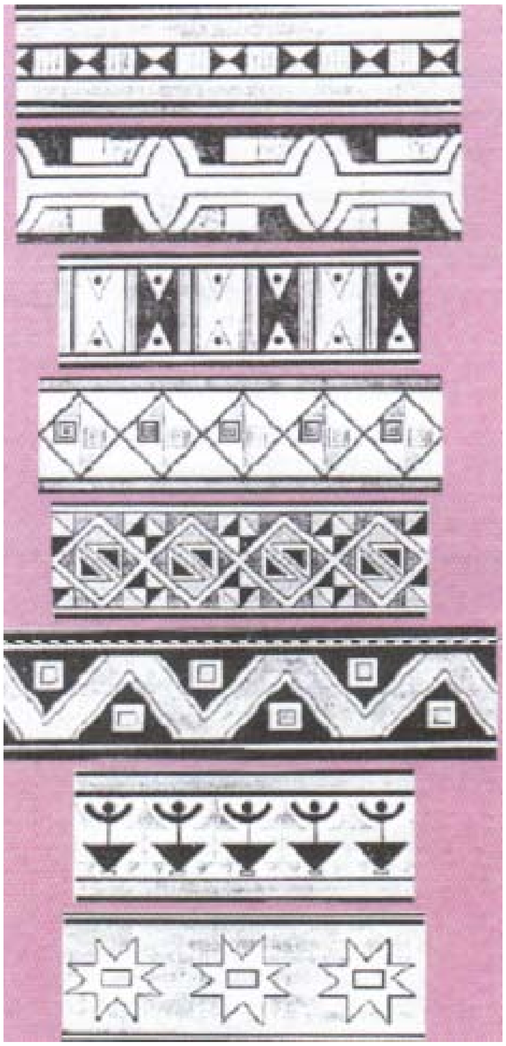
Groups of finite figures, cyclic (C) groups and dihedral (D) groups.

	r_0	r_{90}	r_{180}	r_{270}
r_0	r_0	r_{90}	r_{180}	r_{270}
r_{90}	r_{90}	r_{180}	r_{270}	r_0
r_{180}	r_{180}	r_{270}	r_0	r_{90}
r_{270}	r_{270}	r_0	r_{90}	r_{180}

The cyclic group C_4, with 4 elements.

Frieze patterns (1D Symmetry):[17] It is well-known that there are only 7 different types of so-called "line groups," that visually correspond to what are commonly known as frieze patterns. Translations, half-turns, vertical reflections, horizontal reflections, vertical & horizontal reflections, glide reflections, and vertical reflections & glide reflections with half-turns constitute a practical visual manner in which to identify them (Table 7). Throughout our travels in Peru, students were on the lookout for examples of all 7 types.

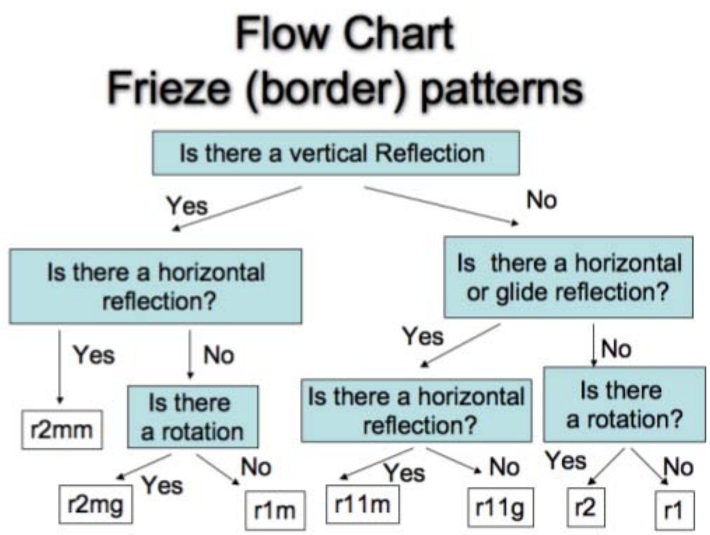
Table 7 Frieze patterns and their categorization



Eight different Incan frieze patterns (top-bottom using standard notation: r2mm, r2, r2mm, r2, r2, r2mg, r1m, and r2mm).

No rotations or reflections	r1	LLLL
Half-turns	r2	NNNN
Vertical reflections	r1m	VVVV
Horizontal reflections	r11m	DDDD
Vertical and horizontal reflections	r2mm	HHHH
Glide reflections	r11g	LΓLΓ
Vertical reflections and glide reflections with half-turns	r2mg	VΛVΛ

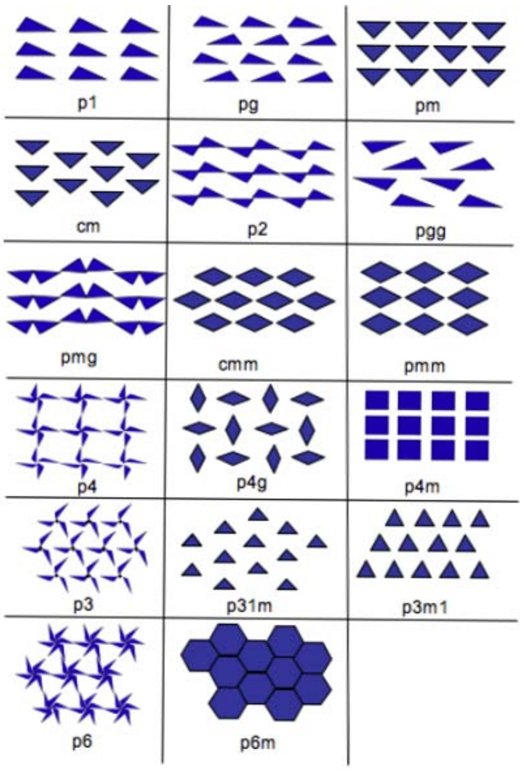
Basic visual coding of all 7 types of frieze patterns using letters of the alphabet.



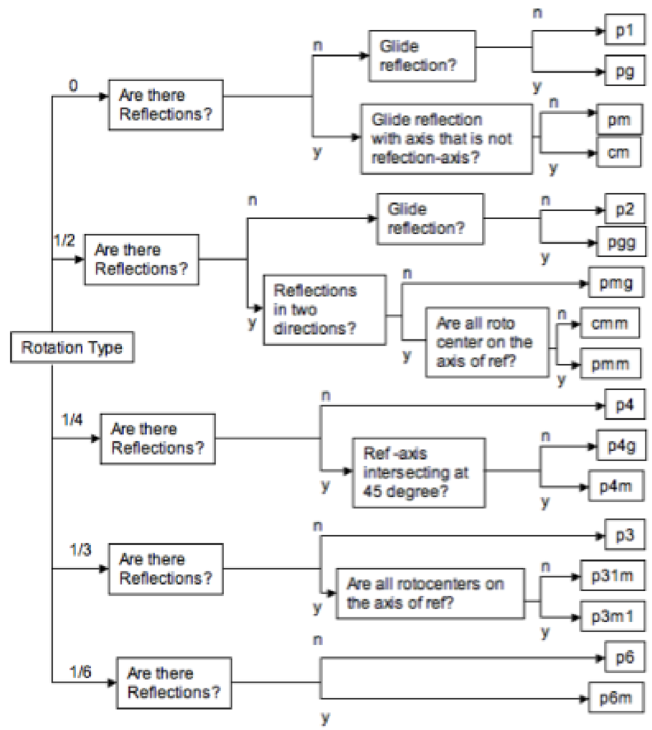
Flow chart for categorizing the 7 different frieze patterns.

Wallpaper Symmetry (2D):[18] It can be shown that there are exactly 17 different types of wallpaper patterns. Based on "rotation type," the existence (or lack) of reflections, existence (or lack) of glide reflections, and responses to several other questions one can ascertain the unique identity of the specific symmetry (Table 8). Finding as many types of symmetry as possible throughout our Peruvian travels proved challenging, although many of them were observed.

Table 8 Wallpaper symmetry summary



Examples of all 17 different types of wallpaper patterns.



Flow chart for classifying the 17 different types of wallpaper.

Fibonacci Numbers: Fibonacci numbers (1, 1, 2, 3, 5, 8, 13, …, or $f_n = f_{n-1} + f_{n-2}$ in general with f_0, f_1 = 1) have intrigued mathematicians for centuries, in part because they arise in seemingly unexpected places in nature and geometry. Common examples often discussed include rabbit population growth, as originally mentioned in Fibonacci's problem in 1202, and flowers (sunflowers in particular). In Peru, they play a key role in efficient use of the yupana.

The Golden Ratio: The Golden Ratio of $\phi = (1+\sqrt{5})/2$ = 1.618… is related to the Fibonacci sequence and appears in many places, both in the natural and man-made world. Figure 4 illustrates the geometric interpretation of the Golden Ratio, which also has the reputation of being pleasing to the eye. Its usage in architecture, certainly in medieval times in Europe is well known. Is it present in Peru? One student noted that the ratio of window height to window width for a window at Machu Picchu was 43 in / 25.5 in, or 1.69, which is close to ϕ.

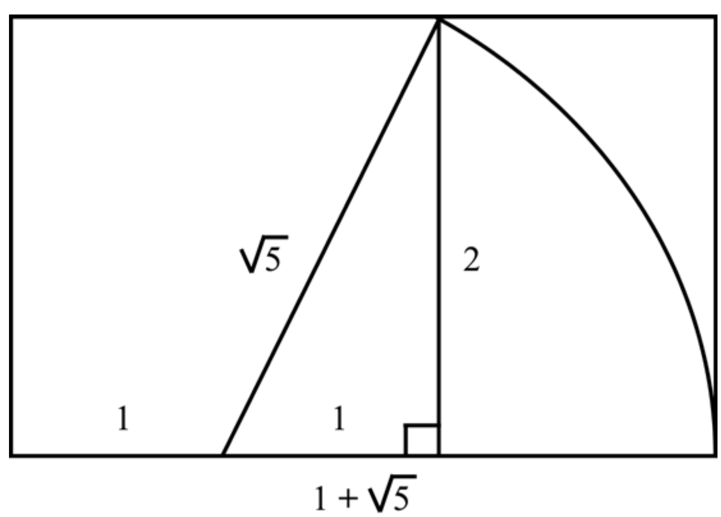


Fig. 4 *The Golden Ratio implied from the length and height of the Golden Rectangle.*

Self-Similarity & Fractals: The Koch snowflake, first constructed by Swedish mathematician Heldge Von Koch in 1904, provides a good introduction to the topics of self-similarity and what are now called fractals (Fig. 5). The fact that fractals typically take on non-natural number dimensions is an interesting notion and of educational value. In real life, fractals don't exist, but there many examples of fractal-like behavior, such as fractals that have been truncated at a finite stage number (often small).

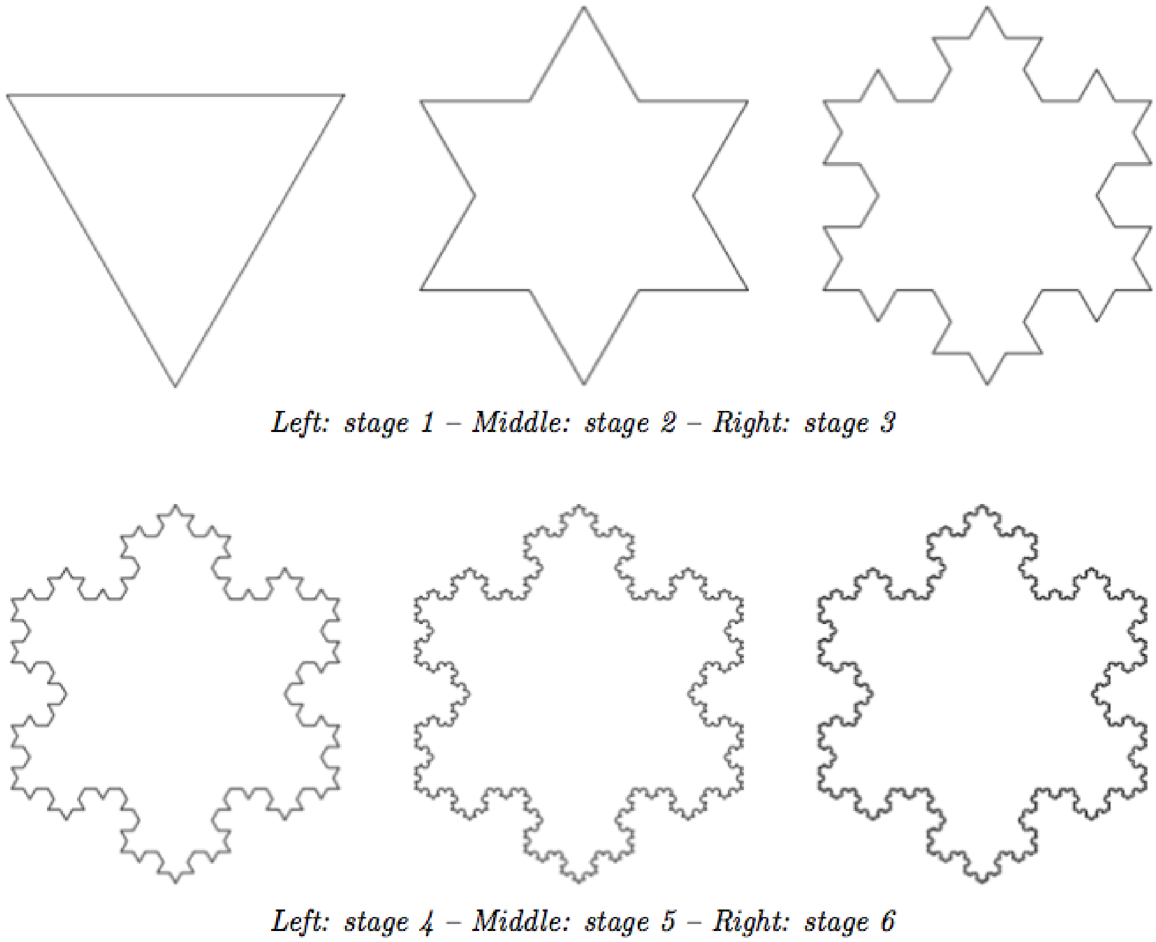
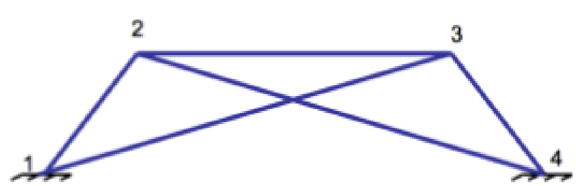


Fig. 5 *Evolution of the Koch snowflake fractal (stages 1-6).*

Structures:[19] The topic of structures (Fig. 6) is best understood with the aid of concepts from linear algebra, including matrices. Basic matrix algebra ideas were covered (largely for the benefit of students who haven't had the course, *Introduction to Linear Algebra and Differential Equations* yet) in addition to delving into various subspaces such as the column space and null space (or kernel) of a linear operator. The null space of the appropriate structural matrix can be interpreted as describing internal motions (such as in a 4-bar linkage). A procedure was introduced that allows one to determine the stiffness matrix for a simple structure.



Doubly Reinforced Planar Structure

Fig. 6 *Example 2D structure.*

3. Photo Gallery in Approximate Geographic and Chronological Order

Palace of Tauri Chumpi Incan ruin at Pachacamac (partial descriptive signage).

Palace of Tauri Chumpi Incan ruin at Pachacamac (corridor within complex).

Ornate, feature rich (e.g. arches, columns, and domes) Catholic church-front in downtown Lima.

Rich displays of symmetry expressed through Incan textiles (rug / mat from Lima's Archaeological and Anthropological Museum).

Students on a 3D spiral helix track leading into a fresh water well in the Nazca desert.

Students presenting their findings to the class on la chakana in an old Arequipa hotel.

Volcano Mt. El Misti (summit 5,822 m or 19,101 ft above sea level) day-mountain climbers who ascended from 3,415 to 3,800 m into the mist.

Chulpas of Sillustany inverted frustum (finished side) – notice masonry craftsmanship such as smooth surface finish and very small cracks at stone-stone interfaces.

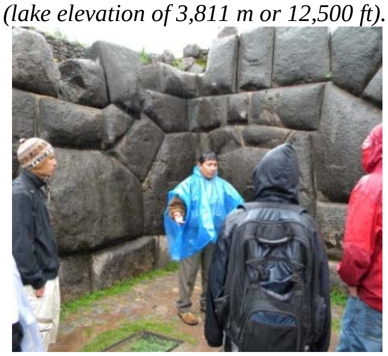
Chulpas of Sillustany inverted frustum ("ruin" side) – notice increasing radius with height.

Cross-sectional model of how the floating grass islands of Uros are constructed.

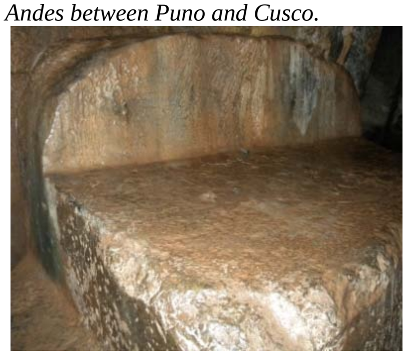
Free-standing stone arch on Lake Titicaca's remote island of Taquile (lake elevation of 3,811 m or 12,500 ft).

The high elevation saddle point of La Raya Pass (4,335 m or 14,222 ft) in the Andes between Puno and Cusco.

Famous 12-sided polygonal stone embedded in a wall in the heart of Cusco that showcases Incan masonry and associated craftsmanship at its finest.



Onsite local tour guide describing attributes of some of the massively impressive and mysterious masonry work at Sacsayhuaman, outside of Cusco.



Human sacrificial altar used during religious ceremonies conducted by the Incans, at Kenko (also spelled "Qenqo"), in the mountains overlooking Cusco.

An ornate, metallic souvenir facade of the instrument (called a Tumi) used by the Incas to perform human sacrifice.

The ruins of Raqchi (Temple of Wiracocha God), known for its symmetric trapezoidal, earthquake resistant windows.

Terraced ruins of the Ollantaytambo fortress between Cusco and Aquas Calientes.

Frieze pattern on a wall at the Catholic Church of Andahuaylillas.

A stone wall at Puka Pakara (the Red Fortress), with a trapezoidal window / doorway embedded in it.

Prominent bath at Tam-Bomachay.

The valley of the Incas.

The class and instructors (28 total) assembled in front of Machu Picchu, the famous lost city of the Incas.[20]

The Intihuatana, a large stone "solar calendar" at Machu Picchu.

A foggy view of Machu Picchu's terracing from neighboring mountain Wayna Picchu (opposite side from main entrance).

Machu Picchu's terraced arrays of stacked stone structures – at least a 7^{th}-order fractal, when viewed as "nested open rectangular boxes."

Close-up of individual building unit at Machu Picchu fabricated out of stacked stone walls with symmetric trapezoidal windows.

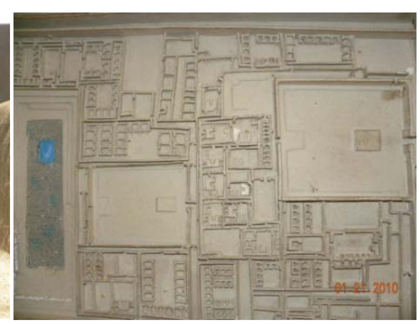
Machu Picchu 3^{rd}-order "flattened mountain peak" fractal (red, yellow, green).

Ruins of the Temple of the Sun, a large pyramid structure.

Recently unearthed wall with symmetric planar art form at the Temple of the Moon.

Fish frieze pattern at the Chan Chan complex (Tshudi Palace).[21]

Large walls at the Chan Chan complex (Temple del Arco Iris).



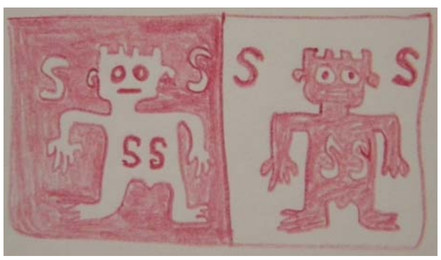
Plan view of the Chan Chan complex, at least an 8^{th}-order fractal of "nested rectangles."

4. Additional Noteworthy Student Work

After grading all of the student work, including their site journals, final papers, and collections of symmetry (excluding that from the internet), there are several noteworthy pieces of work worth sharing. In most cases, the work is tied directly to both visiting Peru and emphasized one of more key ideas that possess either a mathematical or architectural / engineering theme to it. Table 9 depicts frieze and wallpaper symmetries observed by students and Fig. 7 shows Sierpinski's version of Pascal's triangle. Several comments from the final papers are insightful, including:

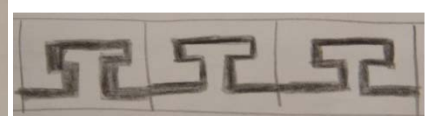
- Reasonable pace and balance between traditional academics and site visits, appreciated visiting sites first-hand and really liked Machu Picchu
- Impressed that the Incas were able to perform complex mathematical calculations without a written language, enjoyed the quipu, yupana, chakana, and making a quipu
- Best experience that I have lived, dipped my feet in the water of abstract algebra, encourage other students to study-abroad, highlight was Machu Picchu

Table 9 Example hand-drawn sketches (12 of 24 possible varieties) of frieze and wallpaper patterns observed by students

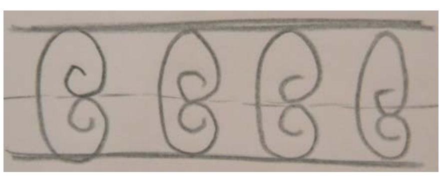


r1 frieze (Elizabeth Langer)

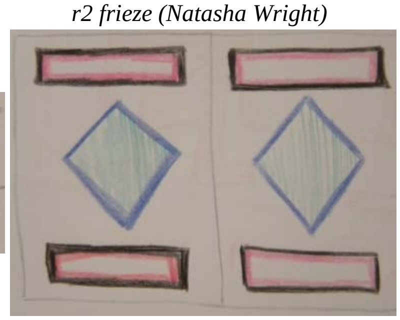
r2 frieze (Natasha Wright)



r1m frieze (Natasha Wright)



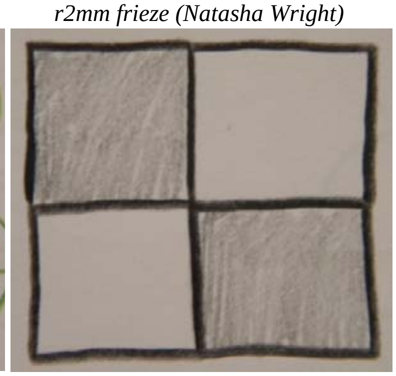
r11m frieze (Elizabeth Langer)



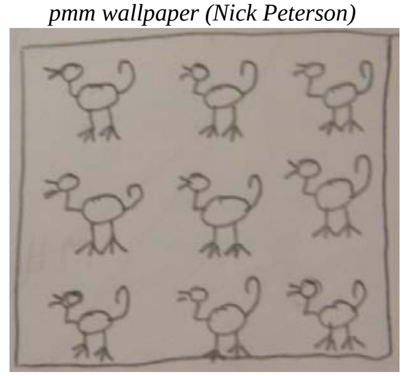
r2mm frieze (Natasha Wright)

pmm wallpaper (Nick Peterson)

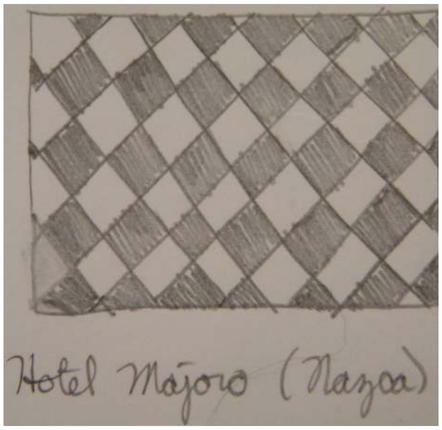
p4m wallpaper (Nick Peterson)



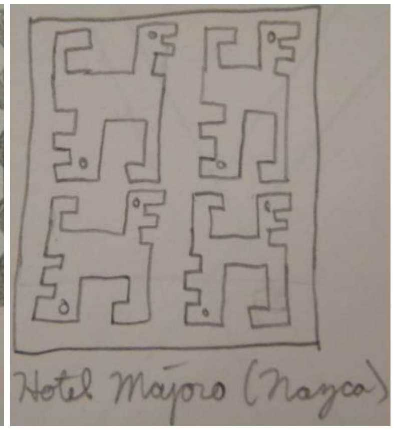
pmg wallpaper (Nick Peterson)



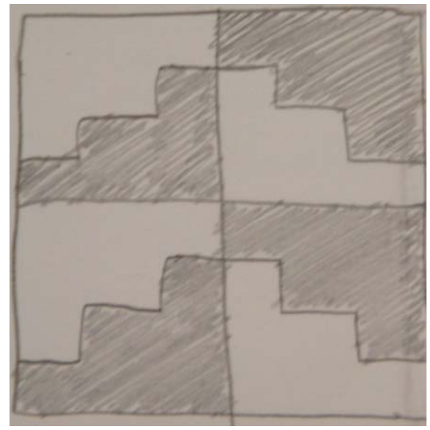
p1 wallpaper (Joanna Thielen)



pmm wallpaper (Joanna Thielen)



p2 wallpaper (Joanna Thielen)



pg wallpaper (Nicholas Greczyna)

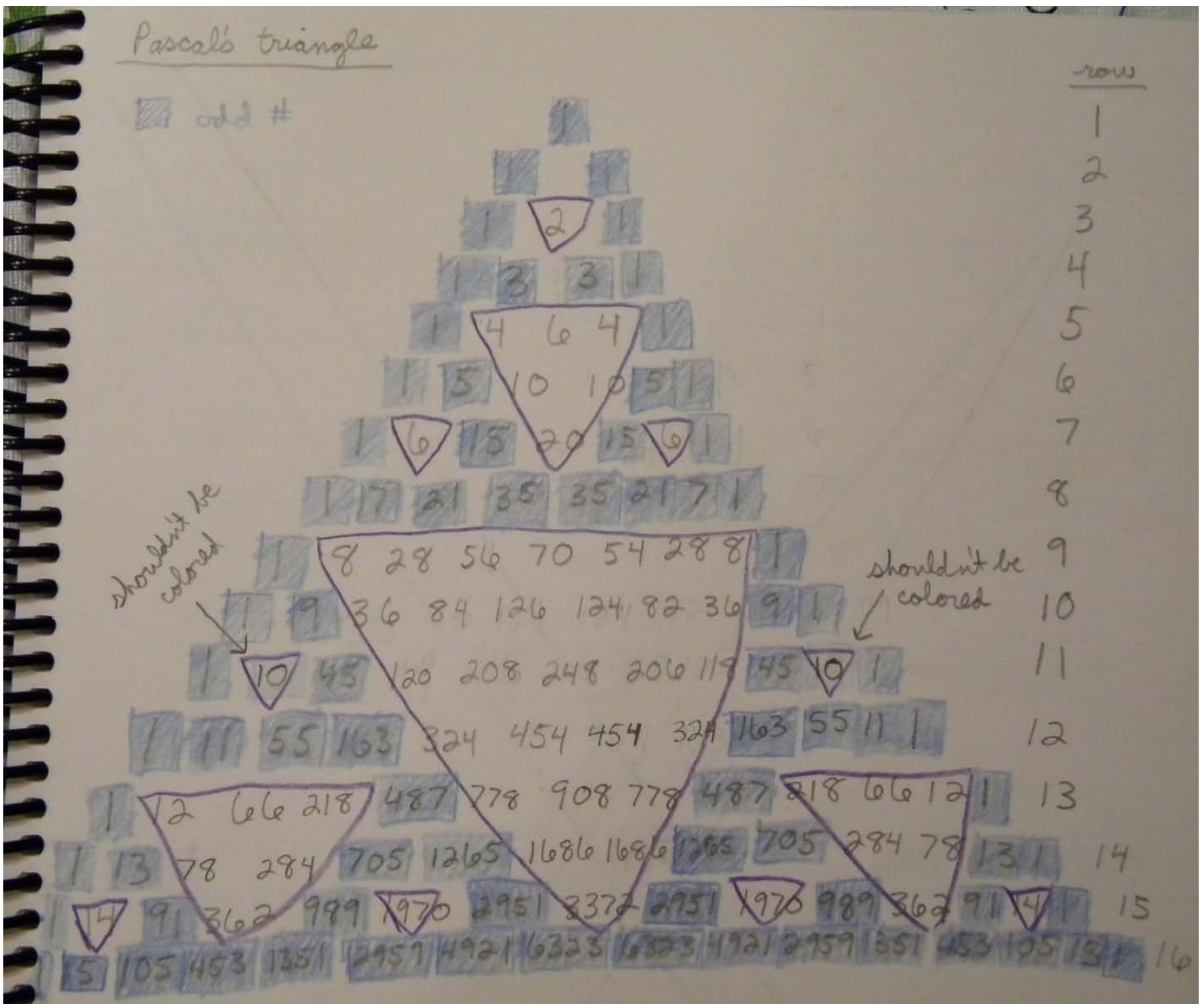


Fig. 7 *Sierpinski's version of Pascal's triangle with odd / even number partitioning (Joanna Thielen).*

5. Assessment and Lessons Learned

Assessment took on several forms, the official UMAIE teaching evaluation forms, final paper written by students which often highlighted useful summative information, and both of the instructor's personal assessment. Overall, the course went well. Favorite academic topics pertained to quipus and the yupana. Students enjoyed traveling to the sites selected, especially seeing impressive and mysterious structures, not just at Machu Picchu, but at Ollantayambo and Sacsayhuaman as well. It was common for them to comment on cultural issues that made a big impression on them, be it the friendliness of the people, the food, or the nightlife. One student, admittedly one of the best in the class, even commented on how "the guides sometimes seemed surprised at our knowledge of the motifs!" (due to our coverage of the topic of symmetry). There was only one negative comment, written by a student who felt that at times the mathematical material was too theoretical.

6. Conclusions

Generally speaking, teaching a course like the one described is a lot of work and requires a high energy level. Proposals must be written, administrative approvals obtained, pre-visits are needed to scope out sites, course materials must be prepared, an advertising campaign launched, in addition to teaching the course itself, which entails almost daily travel in a foreign country with at times difficult terrain, weather, language / cultural issues, and logistics, with quite of number students who understandably have certain needs and expectations. That said, life is an adventure and overall the work is very rewarding, both for the faculty involved as well as the students. It's worth mentioning again that study-abroad courses for mathematics and engineering are not that common and complementing a student's slate of traditional lecture / lab courses (at their university) with this type of course adds cultural context, an international perspective to their studies, and emphasizes application of concepts, all of which are noble objectives. These notions play especially well at liberal arts universities, such as the University of St. Thomas.

In this particular case, many of the sites in Peru have the reputation of being mysterious and intriguing in nature, which has the effect of getting people's instant attention and there is a clear benefit to examining these sites through the lens of mathematics and engineering, because the phenomena studied exude these discipline specific underpinnings. Being able to apply "in-class" and "in-the-textbook" concepts to the real world was very engaging for the students. Because of the available student population, we were able to set the prerequisites "high" (i.e. 3 semesters of Calculus) and this was beneficial – it's always more fun to work with very bright students as they are generally very capable and can engage the discipline at a higher level and they also enjoy working with each other.

Another conclusion is that while the course took place in Peru, the underlying ideas are fairly generic (e.g. symmetry and mechanics) and can be adapted to other venues. In fact, architecturally rich eastern-Europe (specifically Turkey, Greece, and Italy) will be the site of our next adventurous course in January 2012 (course title: *Mathematics and Mechanics of Byzantine, Greek, Roman, and Islamic Architecture*).

Lastly, here are several practical conclusions and useful pieces of advice for those considering teaching such a course:

- At higher elevations (such as over 5,000 ft) bring a separate fan to cool your portable projector. The air is very thin and typical projectors can overheat in just a few minutes.
- As an unfortunate J-term Gustavus Adolphus College class found out at Machu Picchu (about 9 days after our visit), the weather can be problematic due to rain and mudslides.[22]

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The authors acknowledge Mollie Leonard[23] (St. Thomas, BA Mathematics, 2009) for writing booklet & workbook sections on quipus, the yupana, and the chakana. Additionally, we acknowledge the support of the Department of Mathematics, School of Engineering, ARCIE (Academic Review Committee for International Education), UMAIE, and all 26 students who actively participated in the course:

Table 10 Students in the course

Annoni, Jennifer	*Barazesh, Ellyar*	*Bensen, Katherine*	*Berger, Jason*	*Coss, Kelly*
Edstrom, Tyler	*Garske, Benton*	*Giancola, James*	*Greczyna, Nicholas*	*Helgeson, Justyn*
Hendrickson, Sven	*Korte, James*	*Lais, Sara*	*Langer, Elizabeth*	*Maciej, Ryan*
Mcalpine, Jacob	*Munoz Pineda, Israel*	*Peterson, Nicholas*	*Plooster, Scott*	*Rodriguez, Aaron*
Scardigli, Matthew	*Sosinski, Andrew*	*Stemig, Amanda*	*Uecker, Nathan*	*Victor, Christopher*
		Wright, Natasha		

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