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# **AC 2011-2205: THE DEVELOPMENT OF AN INSTRUCTIONAL AND ASSESSMENT TOOL FROM STUDENT WORK ON A MODEL-ELICITING ACTIVITY**

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# **The Development of an Instructional and Assessment Tool From Student Work on a Model-Eliciting Activity**

## **Abstract**

Model-Eliciting Activities (MEAs) are interdisciplinary, engineering based problems set in a realistic context with a client. MEAs allow researchers and teachers to observe students' development of conceptual models as they go through the cycle of express, test, and revise with their solutions. MEAs are being used increasingly in K-College level classes. Research tools that can be used for instruction and assessment with MEAs are needed. This paper will describe the development of such a research tool. Cognitive Task Analysis was used to create a task model that details the subtasks necessary to complete the MEA successfully. The task model can identify the knowledge, thought processes, and goals that underlie a task. High school students' work from an MEA was coded on each of the subtasks based on three categories of naïve, routine, or sophisticated. The development of the task model and its subsequent use for the analysis of student work on the MEA provides information relevant for researchers and teachers. Benefits of developing a task model for an MEA for teachers include having a tool for assessing student work, as well as being able to provide timely feedback to students when they are working on the MEA. The benefits for researchers include having a better understanding of students' problem solving procedures and being able to identify student misconceptions and different STEM (Science, Technology, Engineering, and Mathematics) constructs.

## **I. Introduction**

How to best prepare engineers to be successful from the start in the workforce is an important question. One important way to prepare students to be engineers is with real world engineering problems. This is vital because there is a need for students to become more interested in STEM (Science, Technology, Engineering, and Mathematics) fields in part because there has been a constant amount of students finishing degrees in STEM fields in the last fifteen years but the number of jobs in STEM fields has grown.<sup>1</sup> Keeping students interested in STEM throughout K-16 is important to ensure an adequate amount of STEM graduates. Mathematics and science classes that do not focus on applications can lead to decreased motivation and interest for students.

Model-Eliciting Activities (MEAs) are being used increasingly in K-16 level classes for students to focus on applications of math and science in an engineering structure. MEAs are engineering based, interdisciplinary problems set in a realistic context with a client. MEAs allow students to work through a form of the engineering design process that is the hallmark of understanding engineering.<sup>2</sup> To be used effectively and to maximize the impact that they have on students, tools that can be used for instruction and assessment with MEAs are needed. Cognitive Task Analysis (CTA) is a method for developing a model of subtasks that are important for completing a task. CTA used together with MEAs can be useful for developing research and instructional tools. This paper will describe how CTA was used for the development of such a tool.

## **II. Cognitive Task Analysis (CTA)**

Cognitive Task Analysis (CTA) is a method for analyzing data that are related to “knowledge, thought processes, and goal structures that underlie observable task performance” (p. 3).<sup>3</sup> Researchers “are frequently interested in studying how people in a particular field make decisions in complex, dynamic, real-time environments” (p.131).<sup>4</sup> CTA can be used for this purpose and can also be used to develop ideas of experts’ decision making. It can be difficult to capture an experts’ body of knowledge. For this reason, “most methods rely on some type of analysis in situated, task-specific conditions with experts actually solving problems or making decisions” (p.133).<sup>4</sup>

While much of the work using CTA has looked at individuals, it is documented that an increasing number of studies are being done with teams as the unit of analysis<sup>5</sup>, which is the case for this study. CTA requires the development of a task model that describes subtasks necessary to complete a task successfully. CTA is appropriate if a task requires the use of a complex conceptual knowledge base or pattern recognition.<sup>4</sup> Therefore, CTA is useful for interpreting the understandings and the ways of thinking of teams in the performance of a problem solving task.<sup>6</sup>

## **III. Models and Modeling Perspectives (MMP)**

Models and Modeling Perspectives (MMP) provide researchers with a lens to view the nature of mathematics but are useful in STEM education research as well. A main tenant of MMP is that modeling is an iterative design-based process that is essential in mathematics and other STEM disciplines. MMP can be used as a framework that focuses on effective ways of helping teachers and researchers see, interpret, and help guide the development of students’ thinking. Interpretation can include the ways in which students might learn in an engineering context, pedagogical development, and possible ways of proceeding with a sequence of learning activities.<sup>7</sup>

## **IV. Model-Eliciting Activities (MEAs)**

Model-Eliciting Activities (MEAs) are designed within the MMP framework and focus on problem solving that involves differentiating, integrating, reorganizing, adapting, or extending interpretation systems. Modeling activities involve problematic situations that “are defined to be goal directed activities in which adaptations need to be made in existing ways of thinking about givens, goals, and possible solution steps”(p. 319).<sup>8</sup>

MEAs are realistic, interdisciplinary, team-based, non-routine problems.<sup>9</sup> Similar to the work of engineers, the student work in MEAs is balanced on process and product. The goal of MEAs is to have students develop models that are powerful, sharable, and reusable. In engineering, it is becoming increasingly necessary to work in teams of diverse specialists. MEAs attempt to highlight the importance of continually adapting and socially constructing integrated subject matter.<sup>10</sup>

MEAs allow researchers and teachers to observe students’ development of conceptual models as they go through the cycle of express, test, and revise with their solutions. They have also become

a tool that could help both instructors and researchers become more observant and sensitive to the design of situations that engage learners in productive mathematical thinking.<sup>7</sup> For example, Moore & Diefes-Dux<sup>11</sup> found that while developing and implementing an undergraduate engineering MEA that they went through similar developmental cycles that students go through to create models for the problem.

### *Paper Airplane MEA*

An instructional and assessment tool in the form of a task model was developed for the Paper Airplane MEA. This MEA was initially designed to have students understand the concept of drag in Purdue University's graduate program for aeronautical engineering.<sup>10</sup> For the work of this paper the MEA was adapted to be used for high school classes.

The Paper Airplane MEA begins with an article for students to understand the context and the client for whom they will work. The article describes several problems that occurred last year at a paper airplane contest. Some flight characteristics that were tested were: (a) how far the planes flew, and (b) how long the planes stayed in the air. But, it was difficult to judge some of these characteristics because the planes performance depended on which "pilots" tossed them. So, next year, the organizers of the paper airplane contest have decided that three "pilots" should fly each plane, and that the same three pilots should fly all of the planes. The problem statement of the paper airplane MEA asked students to write a letter to the judges of a paper airplane contest. The letter needed to provide a procedure which would allow the judges to decide which airplane is: (a) the most accurate flier, and (b) the best floater. Teams of three to four students then worked together to solve the MEA. The teams were given Table 1, a sample of data from a trial contest to develop and test their procedure.

Table 1. Information about Four Paper Airplanes Flown by Three Different Pilots

	Flight	Pilot F				Pilot G				Pilot H			
		Distance from Start	Time in Flight	Distance To Target	Angle from Target	Distance from Start	Time in Flight	Distance To Target	Angle from Target	Distance from Start	Time in Flight	Distance To Target	Angle from Target
Plane A	1	22.4	1.7	15.2	16	30.6	1.6	14.5	23	39	1.8	7.5	-10
	2	26.3	1.7	16.7	26	31.1	1.6	11.9	19	36.3	1.7	4.3	-6
	3	31.6	1.7	7.1	10	26.7	2.2	8.9	-4	35.9	2.2	9	-14
Plane B	1	32.1	1.9	7.6	-11	35.9	1.9	14.3	-23	43.7	2.0	9.5	6
	2	42.2	2.0	9.2	-9	39	2.1	11.1	16	29	2.0	7.6	7
	3	27.2	2.1	10.2	-11	25.6	2.0	11.7	12	36.9	1.9	12.4	19
Plane C	1	19.2	1.8	16.6	-8	42.9	2.0	9.8	9	35.1	1.6	2.8	4
	2	28.7	1.9	9.3	11	44.6	2.0	9.3	-1	37.2	2.2	2	-1
	3	23.6	2.1	17.3	-25	35.7	2.2	3.2	-5	42	2.1	9.8	10
Plane D	1	28.1	1.5	8.9	9	37.2	2.1	20.2	-32	41.7	2.2	10.1	11
	2	31.6	1.6	14.8	-24	46.6	2.0	11.4	-2	48	1.9	14.1	-8
	3	39.3	2.3	9.1	12	34.7	1.8	22.2	-36	44.7	1.7	11.5	-9

## V. Development of the CTA Tool

A multi-tiered research design was used to develop the Cognitive Task Analysis (CTA) tool. Multi-tiered research designs have proven effective in research driven by the Models and Modeling Perspectives. While students are developing conceptual models teachers and researchers can develop conceptual tools and revise their ways of thinking as well.<sup>7</sup>

### *Teachers and Students*

The Paper Airplane Contest MEA was implemented in two high schools during the 2008-2009 school year. Four different teachers implemented the problem in five classes across three disciplines: physics, aerospace engineering, and Algebra II. The different disciplines allowed for a more robust task model to be developed through the multiple perspectives of students in different classes. A future study will look at the differences in team's solutions based on the class type. Both high schools were public, suburban 9-12 schools in a large metropolitan area in a Midwestern state. See Table 2 for information regarding the schools, teachers, courses, and student teams for this study.

Table 2. Participants and settings of MEA implementations

Implementation #	School	Teacher	Class	Team #
1	A	1	Aerospace Engineering	1-5
2	A	2	Algebra II	6-13
3	A	2	Algebra II	14-21
4	A	3	Algebra II	22-29
5	B	4	Physics	30-34

All implementations of the MEA were videotaped and audio recorders were placed at student tables to record the team conversations throughout the MEA. In addition, several researchers sat with selected student teams in each class and took field notes about the team interactions and experiences with the MEA. Teachers collected final written work (i.e., team letters to the judges) from the students describing their solution to the MEA. All teams' letters were the primary source for data analysis in this study. MEAs are as much about the final written product as the process. For models to be reusable they have to be clearly communicated so that others can understand the process. Audio and video recordings were used to inform the coding and to triangulate findings when needed.

Applied Cognitive Task Analysis (ACTA) was used to create the task model for the Paper Airplane MEA. ACTA is a streamlined version of CTA that can involve the use of a knowledge audit to develop a task model. The knowledge audit is a method to determine the aspects of expertise that are required for a specific task or subtask. The knowledge audit can lead to information that can be organized around knowledge categories that have been found to characterize expertise. As each aspect of expertise is uncovered through concrete examples in the context of the problem then cues and strategies that were used can be identified, as well as misconceptions, and potential errors that a novice might make in the task.<sup>12</sup>

In order to create the task model, the researchers first developed expert solutions based on discussions of the overall quality of teams' solutions. The teams' letters were scored for their overall quality by using the Quality Assurance Guide (QAG), which has 5 levels (See Table 3). The QAG has been used in previous research to assess the overall quality of student letters to MEAs.<sup>10,13</sup>

Table 3. The Quality Assurance Guide

Score	Performance Level	How useful is the product?
1	Requires redirection	The product is on the wrong track. Working longer or harder won't work. The students may require some additional feedback from the teacher
2	Requires major extensions or revisions	The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.
3	Requires editing and revisions	The product is on a good track to be used. It still needs modification, additions or refinements.
4	Useful for this specific data given, but not shareable and reusable OR Almost shareable and reusable but requires minor revisions	No changes will be needed to meet the immediate needs of the client for this set of data, but not generalized OR small changes needed to meet the generalized needs of the client.
5	Sharable and reusable	The tool not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.

Consensus was reached on the scorings and through the discussions, multiple expert solutions were developed using different pathways that would meet (level 4) or exceed (level 5) the needs of the client. The use of expert solutions does imply that there are only certain solutions that are correct, but were used as a starting point for the task model.

The discussions lead to the formulation of strategies that were used by teams, misconceptions, and potential errors that a novice might make that would lead to the solution not meeting the needs of the client. Other themes that emerged from the discussions were students' limited use of definitions, specific misconceptions, mathematical concepts used, selection of variables, and unclear explanations or rationales. Then, the researchers created a table with separate information for the best floater and most accurate competitions by describing variables that teams used, combinations that were used, concepts used, errors and misconceptions, and extra information that did not fit the other categories.

Aspects of grounded theory as described by Strauss and Corbin<sup>14</sup> were then used to further analyze the teams' letters. Grounded theory begins with *basic description*, moves to *conceptual ordering* (organizing data into discrete categories "according to their properties and dimensions and then using description to elucidate those categories," p.19), and then *theorizing* ("conceiving or intuiting ideas-concepts- then also formulating them into a logical systematic, and explanatory scheme," p.21). Each researcher wrote a narrative for two teams' letters to describe their solution ideas by focusing on misconceptions and solution strategies. The researchers discussed the narratives that were written and further developed categories for the processes that teams went through. The categories were compared to expert solutions for any differences.

Each researcher then selected two audio files that they thought would be interesting based on the previous analysis and wrote up vignettes<sup>15</sup> that explained the process that students went through focusing on what they chose as variables, discussions that happened, how their procedures were developed, and anything interesting about how they ended up with the final solution. The idea of sampling as a subtask came out of these descriptions.

Next the researchers looked for a way to code student teams' letters for each subtask. Similarly to the study conducted by Muir, Beswick, & Williamson<sup>16</sup> of students' problem solving strategies two separate categories of novice and expert were not enough to describe the range of strategies that students used. The use of naïve, routine, and sophisticated was used to more adequately describe the quality of teams' strategies. Finally, negative case analysis was used to refine the subtask descriptions.<sup>17</sup>

The cognitive task model has six parts: (1) variable selection, (2) mathematics or statistics concepts used, (3) definition identification, (4) sampling strategies, (5) combination of statistical or mathematical analysis into a procedure, and (6) communication of the procedure to decide a winner. Descriptions of the subtasks can be seen in Table 4. The first two parts of the task model, variable selection and mathematics or statistics used, are categorical. The other four subtasks were classified based on if a team's procedure within a subtask was a naïve, routine, or sophisticated strategy.<sup>16</sup> The task model includes space for the scorer to explain the ratings used if clarification is necessary. The entire task model can be seen in Table 5.

Table 4. CTA subtasks and descriptions.

Subtasks	Descriptions
Definition identification	<p>Having a clear definition of problem tasks is a good starting point in a complex problem-solving task. It may increase the possibility that students will evolve their problem-solving procedure effectively and efficiently.</p> <p>This subtask directly influences the following problem-solving processes because it works as a foundation. Based on their definitions of problem tasks, students select variables and determine how to combine them. In this subtask students bring their prior knowledge and experiences into the problem-solving tasks. Thus it also provides teachers and researchers with a starting point to track students' misconceptions.</p>
Sampling strategies	<p>This subtask includes variable selection and determining how much of the data are used. Students need to choose reasonable variables to reflect their definitions of problem tasks. They determine what data is appropriate to describe or explain the given problem contexts based on their definitions. Beyond variable selection, students need to explore the nature of data so that the amount of data used is representative of the sample. A critical look at the data is required.</p>
Combination of statistical or mathematical analysis into a procedure	<p>Students need to determine how to make a mathematical and statistical combination of data in order to describe or explain the given problem contexts. This is essential to create a model. Multiple attempts are made to combine the data that they selected and sorted. Students are required to consider all possibilities such as counterexamples because the models that they will develop should be reusable, sharable, and transportable. Students need to make clear justifications of their strategies. Students need to bring their existing knowledge and experiences in mathematics and statistics to this subtask. Various misconceptions across several disciplines are revealed through student solutions within this subtask.</p>
Communication of the procedure to decide a winner	<p>Students need to clearly communicate how to choose a winner for each contest in their solutions to this MEA. In order to encourage good communication, multiple perspectives from team members should be welcomed, which increases the possibility that a good model will be created and communicated. The results of this subtask have strong influences on all preceding subtasks. Students also need to clearly communicate their ideas to the client. The models that they will develop should be reusable, sharable, and transportable by anyone who wants to use them as well as the client. Thus clear communication skills through writing (i.e., coherent and logical) are required.</p>



Table 5. Cognitive Task Model for solutions to the Paper Airplane Contest MEA.

<b>Cognitive Task Model</b>				
<b>Team name:</b>		<b>Scorer:</b>		
<b>Competition:</b>	<b>Best floater</b>		<b>Most accurate</b>	
<b>Variable Selection:</b>	Distance from start	Time in flight	Distance to target	Angle from target
<b>Math/stats used:</b>				
<b>Subtask</b>	<b>Naïve Strategies</b>	<b>Routine Strategies</b>	<b>Sophisticated Strategies</b>	<b>Cognitive factors that may cause difficulties</b>
Definition	<u>No definition:</u> <ul style="list-style-type: none"> <li>Start without definition or no written definition</li> </ul>	<u>Loose definition:</u> <ul style="list-style-type: none"> <li>Definition of problem tasks is loosely or unclearly defined and partially represented in the procedure.</li> <li>Concrete definition but not appropriate prior knowledge used to interpret the problem</li> </ul>	<u>Concrete definition:</u> <ul style="list-style-type: none"> <li>Using appropriate prior knowledge to interpret problem tasks and definition is clear.</li> <li>Definition may include appropriate new variables or constraints to the problem.</li> </ul>	<ul style="list-style-type: none"> <li>Whether or not to make the competition two separate contests affects the definition of each task.</li> <li>Students not using a definition of problem tasks or an unclear definition</li> </ul>
Sampling	<ul style="list-style-type: none"> <li>Shows little understanding of sampling with no rationale for data used or sampling methods that are not productive or not clear.</li> </ul>	<ul style="list-style-type: none"> <li>Shows some understanding of sampling with some rationale for data used. The method of data selection needs more work to be clearly refined and stated.</li> <li>Chose reasonable variables and then used all of the data.</li> </ul>	<ul style="list-style-type: none"> <li>Shows understanding in determining what data is appropriate and gives a fair representation of a definition of each task.</li> <li>May include removal of outliers or may be incorporated in a tiebreaker procedure.</li> </ul>	<ul style="list-style-type: none"> <li>Absence of understanding of the representativeness of data</li> <li>The use of only certain parts of the data without justification</li> <li>The idea that all of the data given must be used</li> <li>The organization or presentation of the table may affect students use of the data</li> </ul>
Combination of statistical or mathematical analysis into a procedure	<ul style="list-style-type: none"> <li>Procedures show student misconceptions and procedures are not appropriate for this problem thus significant revisions are required.</li> <li>Lack of consideration of counterexamples.</li> </ul>	<ul style="list-style-type: none"> <li>The procedure is somewhat clear or minimally justifiable with little to no misconceptions.</li> <li>There are multiple plausible justifications of varying quality of the procedure.</li> </ul>	<ul style="list-style-type: none"> <li>Clear instructions for the procedure and the math or statistics are justifiable.</li> <li>Counterexamples of the procedure have been considered.</li> </ul>	<ul style="list-style-type: none"> <li>Misconceptions and no consideration of all possibilities.</li> <li>Idea that a complex method is better than a method that is simple</li> </ul>
Communication of the procedure to decide a winner	<ul style="list-style-type: none"> <li>Unclear or missing decision schemes.</li> <li>Wrong causality.</li> </ul>	<ul style="list-style-type: none"> <li>Relatively well done, but lack of coherence. This might mean bad sentence structure or a decision scheme that is not completely clear.</li> </ul>	<ul style="list-style-type: none"> <li>Has coherence in a logical way so that it easily makes sense to someone new to the situation.</li> </ul>	<ul style="list-style-type: none"> <li>Absence of opportunity to learn or develop writing skills in a high quality expert form.</li> <li>The difficulty of communication with mathematical and statistical terms, concepts, and symbols.</li> </ul>
<b>Additional Explanation of scoring:</b> (Space removed for publication)				

Three researchers coded each team's letter according to the task model (once for each contest - best floater and most accurate). Adjustments and refinements of the descriptions for what constituted a naïve, routine, or sophisticated strategy were iterative and were revisited often during the coding process. Once the task model was finalized, all solutions were coded with the final task model. Inter-rater reliability for the coding was rated at 0.63 using Fleiss' Kappa for multiple raters, which is considered *substantial strength of agreement*.<sup>18</sup> However, to resolve issues of coding agreement, all researchers discussed the codes and discrepancies were resolved via discussion and consensus. To triangulate the findings from the CTA, researchers used audio

recordings from team discussions to validate study ideas that developed from team letters to the judges.

### *Samples of Coded Student Team Responses*

Here, we present two teams' letter to the judges (Team 26 and Team 5) complete with how the coding of the subtasks from the CTA model was performed. Figure 1 shows a diagramed version of the letter from Team 26.

Figure 1. Diagram of the letter from Team 26 broken down by competition and subtasks.

Introduction	Dear Judges,  In response to your need for an adequate equation to judge the paper plane entrees in the categories of the most accurate flyer and the best floater, we have created and tested equations for each. Each equation went through a period of trial and error and thus proved to be the most fair for the required criterion.	
The Most Accurate Competition	For the first category of the most accurate flyer, we determined that <u>the distance and the angle from the target</u> should be factored into the equation and we realized that if those two factors went into a shape, a triangle, the hypotenuse was missing. Because of this missing component, we determined that the equation for a hypotenuse in a triangle would be most effective for finding accuracy. The equation: $\text{distance from the target}^2 + \text{angle from the target}^2 = \text{accuracy}^2$ , was tested against sample data and it proved to be most effective. The equation proved that the Golden Flyer with the plane Hornet would be the most accurate overall.	No definition Sampling not enough info to tell – so assumed all data  <u>Variable selection</u>  <u>Combination</u>
The Best Floater Competition	For the second category of the best floater, we determined that the average <u>time in the air</u> for each plane would work as the equation needed. The equation: $(a+b+c)/3$ was tested and proved to work because the time in the air is the only component needed for floating. The equation proved that Hornet was the best floater and Pacific Blue was the best pilot.	No definition Sampling not enough info to tell – so assumed all data  <u>Variable selection</u>  <u>Combination</u>
Ending	Thank you for considering our equations to help better the judging and fairness of this prestigious competition.  Sincerely, Team 26	

For both the best floater and most accurate competitions, Team 26 was coded naïve for their definition, combination of statistical or mathematical analysis into a procedure, and communication of the procedure to decide a winner. They were coded routine for sampling for both competitions. This team did not provide a definition for what they meant by accurate or floating, thus the code of naïve. Teams that provided a definition usually did so at the beginning of their letter or at the beginning of their section on best floater or most accurate.

Variable selection for this team involved using the variable time in flight for best floater. They used distance from target and angle from target for their variables for most accurate. Variable selection for most teams was either in their procedure or stated before their procedure. Sampling for this team involved picking variables and then using all of the data from those variables. Some

teams restricted the data used in some way but many teams chose variables and then used all of the data from those variables.

After the variable selection and sampling most teams described their procedure next. This team used mean and showed a misconception of the Pythagorean theorem. Their combination of statistical or mathematical analysis into a procedure involved an equation for both contests.

The last subtask was the communication of the procedure to decide a winner. The final decision on how to decide a winner usually came at the end of a teams section on best floater or most accurate. However, elements of communication of the procedure were necessary throughout the letter. This team never states how it is possible to decide on a winner after using their equations. There should have been some written communication that a minimum score would win or a maximum score would win the competition but that is not included. Some teams also gave written communication for a tiebreaker as well.

Team 5 is an example of a team that provided more of a rationale and explanation for their decisions. Figure 2 shows a diagrammed version of the letter from Team 5.

Figure 2. Diagram of the letter from Team 5 broken down by competition and subtasks.

Introduction	<p>Dear Judges,</p> <p>We believe that certain measurements obtained during this competition should be brought into account in the judging, however, we believe some areas hold more value than others. Since we are looking to find the best floater and the most accurate plane, we have divided the measurements to suit the requirements of the category. For the floating competition the planes will be judged on <u>time in flight and the distance from the start</u>. For the accuracy competition the planes will be judged based on the <u>distance from the target and the angle from the target</u>. We will take <u>the average of all the measurements</u> to keep the planes from winning from one good toss.</p>	<p><u>Variable selection</u></p> <p>Sampling clearly stated as using all data</p>
The Best Floater Competition	<p>For the floating competition, we feel that the planes should be mainly judged on the time spent in the air. For this reason, we have decided on the equation <math>(\text{Average Time})^2 \times (\text{Average Distance}) = \text{Score}</math>. We incorporated distance to <u>avoid people making planes that launch straight up</u>, and we feel that <u>a floater should travel a distance and not dive straight down. It should resemble the path of a glider</u>. We decided to square the average time to put more emphasis on the time in the air. In this competition, the highest score wins.</p>	<p><u>Definition</u></p> <p>Combination</p>
The Most Accurate Competition	<p>For the accuracy competition, we decided the planes should be judged on the distance from the target and the angle. For this reason we have developed the formula <math>2(\text{Average Distance from Target}) +  \text{Average Angle from Target}  = \text{Score}</math>. We will take the absolute value of the angle so the score will not be affected by a negative angle and the distance will be doubled to make it more important than the angle. For this competition the lowest score wins.</p>	<p>No definition</p> <p>Combination</p>

Team 5 was coded sophisticated for their definition of best floater and naïve for their definition of most accurate. For both competitions they were coded routine for sampling and combination of a statistical or mathematical analysis into a procedure. For both competitions they were coded sophisticated for their communication of the procedure to decide a winner. This team did not

provide a definition for most accurate but did provide a clear definition for best floater. This team used their prior knowledge of how gliders float to develop their definition.

This team explained their variable selection and sampling in their introduction. Variable selection for this team involved using the variable time in flight and distance from the start for best floater. They used distance to target and angle from target for their variables for most accurate. Sampling for this team involved picking variables and then using all of the data from those variables.

This team used an equation with averages and weighting for their procedure for best floater and most accurate. The use of absolute value for most accurate was a thoughtful decision but their equation involved adding unlike units with feet and degrees without any mention of a points system.

This team clearly states how it is possible to decide on a winner after using their equations. For the best floater competition the highest score will win and for the most accurate competition the lowest score will win. Also, their communication throughout the letter is detailed with a rationale for their decisions.

## **VI. Potential usage**

The task model that was developed can be used for instructional and assessment purposes. As an instructional tool it allows teachers to be more aware of the thought processes that students will go through and potential areas of difficulty that they might have. As an assessment tool it allows teachers to provide timely feedback to students as they are working on the MEA. It also allows for a way to code student solutions to the MEA as Naïve, Routine, or Sophisticated.

Each team was coded using the task model. In order to provide a snapshot of the range of strategies used by student teams, we have provided the results for all teams in Table 6. The left section of the table gives the results from the best floater results and the right section provides the most accurate results. The key for the abbreviations is provided at the bottom of the table.

Table 6. Team CTA results.

Team	Best Floater							Most Accurate						
	Variables	Math/Stat Used	Subtasks					Variables	Math/Stat Used	Subtasks				
			D	S	C	C				D	S	C	C	
1	T, O	mean, max, range, points system	R	R	N	R		D <sub>T</sub> , A	mean, min	R	R	N	R	
2	T	mean, outlier, min, max	N	S	R	S		D <sub>T</sub>	mean	N	S	R	S	
3	T	mean, max, min, inequality	N	R	R	R		D <sub>T</sub> , A	mean, +2Var, weighting	N	R	N	R	
4	D <sub>S</sub> , T	max, ×2Var	N	N	R	N		D <sub>T</sub>	mean, min	N	R	R	R	
5	T, D <sub>S</sub>	mean, min, max, weighting, ×2Var	S	R	R	S		D <sub>T</sub> , A	abs, mean, weighting, max, min, +2Var	N	R	R	S	
6	T	mean, inequality	N	R	R	N		D <sub>T</sub>	min, mean	R	R	R	N	
7	D <sub>S</sub> , T, D <sub>T</sub> , A	mean	N	N	N	N		D <sub>T</sub> , A	mean	N	R	N	N	
8	D <sub>S</sub> , T	mean, max, +2Var	N	R	N	N		D <sub>T</sub> , A	abs, mean, min, +2Var	N	R	N	N	
9	D <sub>S</sub> , T	mean, max, +2Var	N	R	N	R		D <sub>T</sub> , A	mean, min, abs, +2Var	N	N	N	N	
10	T, D <sub>T</sub>	mean, max	N	N	N	N		D <sub>T</sub> , A	mean, min	N	N	N	N	
11	T, A	mean, min, ×2Var	N	R	N	R		D <sub>S</sub> , D <sub>T</sub>	mean, min, ×2Var	N	N	N	R	
12	D <sub>S</sub> , T	mean, min	N	R	N	N		D <sub>T</sub> , A	mean, min, +2Var	N	R	N	R	
13	T	mean	N	R	R	N		D <sub>T</sub>	mean, min	N	N	R	N	
14	D <sub>S</sub> , T	max, rate, mean, ÷2Var	R	R	R	R		D <sub>T</sub>	min, mean	N	R	R	R	
15	D <sub>S</sub> , T	max, mean, +2Var	N	R	N	R		D <sub>T</sub>	mean, min	N	R	R	R	
16	T, O	max, weighting, +2Var	N	R	N	R		D <sub>T</sub> , A	max, abs, -2Var	N	R	N	R	
17	T	mean	N	R	R	N		D <sub>T</sub> , A	mean, min, abs, +2Var	N	R	N	N	
18	D <sub>S</sub> , T	mean, max	N	R	R	R		D <sub>T</sub> , A	mean, min	N	N	R	R	
19	T	mean	N	N	N	N		D <sub>T</sub>	mean	N	N	N	N	
20	T, D <sub>T</sub>	max, mean	N	N	N	N		D <sub>S</sub> , T, D <sub>T</sub> , A	mean, min	N	N	N	N	
21	D <sub>S</sub> , T, O	mean, max	N	R	R	N		D <sub>T</sub>	mean, precision	N	R	R	N	
22	T, D <sub>S</sub>	mean, max, ÷2Var	N	R	N	R		D <sub>T</sub> , A	mean, min	N	R	R	S	
23	T	mean, max	N	R	R	R		D <sub>T</sub> , A	mean, abs, +2Var	N	R	N	N	
24	T	max	R	R	N	N		D <sub>T</sub> , A	not communicated	N	R	N	N	
25	T, O	weighting, max, +2Var	N	R	R	N		D <sub>T</sub> , A	mean, abs, min, ×2Var	N	R	N	N	
26	T	symbolic equation	N	R	N	N		D <sub>T</sub> , A	Pythagorean theorem	N	R	N	N	
27	T, O	not communicated	N	R	N	N		D <sub>T</sub> , T	not communicated	R	N	N	N	
28	T	mean, max	R	R	R	N		D <sub>T</sub>	min	R	R	N	N	
29	T	mean, max	N	R	N	N		D <sub>T</sub>	mean, min	N	R	N	N	
30	T, D <sub>S</sub>	mean, min, max, ÷2Var	R	R	N	N		D <sub>T</sub> , T	mean, ÷2Var, min	R	R	N	N	
31	T	mean, max	R	R	R	R		D <sub>T</sub>	min	R	R	R	R	
32	T	mean	N	N	N	N		D <sub>T</sub>	mean	N	N	N	N	
33	T	mean, max	N	N	R	N		D <sub>T</sub>	mean, max	N	N	R	N	
34	T, D <sub>T</sub>	max, inequality	R	R	R	N		D <sub>T</sub> , A	mean, min, inequality, ×2Var	N	R	R	R	

**Key:**  
*Variables:* D<sub>S</sub> = Distance from Start, T = Time in Air, D<sub>T</sub> = Distance to Target, A = Angle from Target  
*Math/Stats Used:* +2Var = addition (or other operation) of two variables; abs = absolute value  
*Subtasks (Definition Sampling Combination Communication):* N = Naïve; R = Routine; S = Sophisticated

The task model can be used by researchers to have a better understanding of the processes that teams go through when working on a form of the engineers design process during MEAs. The task model can also be used to identify student misconceptions and how students define different constructs in the problem. Appropriately defining constructs in engineering can be an essential part of the design process. It is important to understand how to engage students in productive thinking to develop quality operational definitions.

## **VII. Discussion**

Cognitive Task Analysis was used to create a task model that details the subtasks necessary to complete the Paper Airplane MEA successfully. Task models can identify the knowledge, thought processes, and goals that underlie a task. Expert solutions to the MEA were used as a starting point for developing the task model. Then researchers analyzed teams' solutions to the MEA, audio recordings of teams working, and researcher field notes by looking for main themes or subtasks. Main subtasks were identified and discussed to reach agreement. High school students' work from the Paper Airplane MEA was coded on each of the subtasks based on three categories of naïve, routine, or sophisticated to further refine the task model.

The multi-tiered research design used, based on the Models and Modeling Perspective (MMP), is a useful framework for developing instructional and assessment tools for MEAs. While students are developing conceptual models through iterative work on MEAs, teachers and researchers as well can iteratively develop conceptual tools and revise their ways of thinking.

Using Cognitive task analysis along with MEAs can increase their ability to be used as a formative assessment tool. The task model provides useful information for teachers on what students' solutions and discussions will involve. Teachers can observe students as they ask questions and interact with other students. This will enable teachers to be able to detect and repair students' misconceptions revealed during the tasks and give students timely feedback about alternative ways of thinking to improve their models. As student learning improves and students are more engaged this may spark more students interest in STEM fields.

It remains to be seen if task models for other MEAs can be useful or if themes across task models would be similar. This framework is as good as we have currently and can be used as a starting point for other MEAs. Further research could improve on the task model for this MEA and the process of development of task models. Further research could be done on which subtask for this MEA is the most important for the quality of an overall solution to the MEA using the Quality Assurance Guide.

## **VIII. Conclusion**

The domain of engineering builds on students' curiosity about the world, how we interact with the environment, and students' interest in designing, building and taking apart objects. Interdisciplinary processes involve applying math and science learning in engineering.<sup>2</sup> Helping students to be well prepared engineers when they enter the workforce is essential. MEAs are increasingly showing promise in motivating and engaging students in learning STEM concepts. MEAs can also be a useful format for students to learning about the engineering design process.

The development of the task model and its subsequent potential use for the analysis of student work on the MEA provides information relevant for researchers and teachers. Benefits of developing a task model for a MEA for teachers include having a tool for assessing student work, as well as being able to provide timely feedback to students when they are working on the MEA. The benefits for researchers include having a better understanding of students' problem

solving procedures and being able to identify student misconceptions and mathematical constructs.

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