Challenges in Transforming Brittle to Flexible Structures

Dr. John M Mativo, University of Georgia

Assistant Professor UGA

Dr. Siddharth Savadatti, University of Georgia
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Abstract

Thermoelectric Generators (TEG) are typically rigid. The devices are made of several unit cells that comprise of two brittle elements each. Each element is known as a leg and is either positive or negative. The brittle nature of the legs makes the unit cells inflexible. A need exists to develop flexible TEGs for use on non-planar waste heat emitting sources. Unit cell geometry and materials affect the rigid behavior of a unit cell. The approach used in this study towards achieving TEG flexibility is to manipulate the unit cell’s geometry that allows deflection without collapsing.

Two models were developed to study the extent of the unit cells’ flexibility. A successful model would allow deflection without fail when both compressive and shear loads were applied to the unit cell structure. Further, it would also maintain a parallel relationship between the top and bottom plates of the unit cell. Two modifications were made to the original unit cell structure by applying conducting polymer at the plate/element interface; and introducing diagonal elements. The first model was an indeterminate truss that used a symmetrical element design approach. The model maintained parallel relationship of the top and bottom plates of the unit cell and its diagonal absorbed loads and allowed minimal deflection of \(<0.01\) of the diagonal member length. The second model was a determinate truss that used asymmetric diagonal element design approach. The determinate results produced better deflection results than the indeterminate model. The top and bottom plate parallel relationship was maintained, and the diagonal member absorbed both shear and compressive loads providing slightly greater deflection. A further analysis suggests a mechanism design to achieve higher deflection. The paper will discuss the first two models. In conclusion, the paper points out how engineering education could benefit from exposure and participation in such a design process even though students were not involved in this study originally.

Introduction

Thermoelectric generators convert heat to electricity. Current geometry and materials used in designs shown in figures 1 and 2 result to rigid devices. The geometry ensures no moving parts while the materials provide a high figure of merit (ZT). \(ZT=S^2\sigma/k\), where \(k\) is thermal conductivity, \(\sigma\) is the electrical conductivity, and \(S\) is the Seebeck coefficient. The figure of merit, \(ZT\), is dimensionless and is formed by multiplying \(Z\) with the average temperature, \(T\). While current configurations of a typical TEG are used adequately on planar surfaces, they do not adapt well on non-planar ones. Unfortunately, many potential waste heat recovery surfaces are non-planar and would not be served adequately by current TEG designs. Literature shows that nearly 60% of the world’s useful energy is wasted as heat [1, 2]. Flexible TG devices have great potential applications in waste heat recovery from both planar and non-planar sources such as truck engines, automobile exhaust systems, aircraft engines, and aircraft landing gears.
typical TEG design is an assembly of unit cells (figure 1) forming a Thermoelectric Device (TED) (figure 2).

![Figure 1: Unit Cell](image1)

![Figure 2: Thermoelectric Device](image2)

In addition to adapting well on heat recovery surfaces, flexible TEGs components will be able to absorb mechanical/thermal loadings and vibrations that result from in-service conditions that could include mechanical vibration, mechanical and/or thermal cycling, and thermal shock. This additional design feature will prevent premature failures (figure 3) from aforementioned loadings.

![Figure 3: Shear stress problems on brittle thermo-elements](image3)

**Flexible Thermoelectric Generators**

Several attempts have been made to develop flexible TEGs. Examples include flexible foil structures, wavy-slit technology, carbon nanotubes (CNT), and graphene nanoribbons (GNRs). Flexible foil substrate technology relies on embedding thermo-elements in epoxy [3]. This design is constrained by the level of epoxy thickness. Foil substrates are typically made of flexible epoxy film categorized as thin or thick and vary in thickness with an average 50 µm for thin and about 190 µm for thick. Thermocouple strips capable of generating voltage are
embedded in the epoxy film [3]. Glatz et al (2006) argued that because of their limited thickness, thin film deposited materials have to be laid out lateral rather than vertical inducing thermal losses through the supporting material and limiting the integration density. He further observed that placing the thermocouple onto a thin membrane reduces thermal loses but does not allow for thermally contacting the cold and hot side via the top and bottom surface of the thin device. He and his team therefore suggested and developed a thermoelectric wafer in a 190 µm thick flexible polymer mold formed by photolithographic patterning. They reported satisfaction in their experimental work as they proposed a model for optimization of vertical micro thermoelectric generators [4, 5]. Since the epoxy level is normally low, a TEG of this design tends to have low power capacity.

Shiozaki, et. al. proposed a flexible thermopile generator with slits (FTGS) to address concerns of non-planar surfaces. Device of this nature have thermocouples placed on a polyimide sheet. Each thermocouple is placed at 45° angle vertically, effectively separating p from n thermocouples. The cold junctions are formed by bending the thermopile sheet to a wavy form. The design by Shiozaki et al forms the wavy and slit flexible thermopile generator [6]. Similar to Shiozaki’s approach to developing flexible thermoelectric generators, Lon E. Bell registered patent #6,700,052 B2, in March 2, 2004, where he claimed “a flexible thermoelectric comprising: a plurality of thermoelectric elements; and first and second substrates sandwiching the plurality of thermoelectric elements and having electrical conductors that interconnect ones of the plurality of thermoelectric elements, wherein at least one of the first and second substrates is constructed of a substantially rigid material, said substrates configured to flex in at least one direction” (see figure 4). One of the challenges facing the wavy-slit design is its reconfiguration of thermal and electrical continuity from floating elements. This design adds weight because of rerouting continuity components. Another potential challenge with this design is the clamping positions of the device between its heat source and sink for proper conduction.
Carbon Nanotubes (CNTs) are mechanically strong and light weight. Their high thermal conductivity coefficients and electrical conductivity coefficients properties, however, pose a challenge in the design of the thermoelectric devices. Koplow et al observed that for highly efficient devices, efficient generators should consist of materials with high Seebeck coefficients to provide significant voltages, low electrical resistivities to minimize internal losses, and low thermal conductivity to minimize heat losses. The solution to CNTs dilemma does lie in doping where some materials aspects are manipulated to suit a desired function. Dragoman et al observed that the Seebeck coefficient ($S$) is strongly dependent on the CNT conductance $G$ being the transmission coefficient carrier through the CNT. The mobility of CNT is $\mu=(G)l_{fp}/Ne$, where $l_{fp}$ is the mean free path of the carriers and $Ne$ is the charge density. Therefore, the Seebeck coefficient and the mobility are related through conductance $G$. CNT mobility decreases with temperature, while $S$ increases rapidly at low temperatures and increase slowly in the 200 – 300K range [9].

In the Graphene Nanoribbons (GNR) technology, thin strips are increasingly being explored for use in TEGs. Their high electrical and high thermal properties place them in a close category with CNTs. However, a unique difference exists such that thermal conductivity is significantly decreased under tensile strain, but is insensitive to compressive and torsional strains. [10]. Current TEG configurations offer rectangular shaped elements made of Bismuth telluride alloys that are proven thermoelectric materials [8].

**Feasibility study**

Authors sought to first establish whether a structural configuration of figure 1 could be transformed to become flexible. In this study, authors created a MATLAB CALFEM tool to
determine such feasibility. A five element four joint truss was developed to depict a TEG unit cell (see figure 5). The unit cell comprised of two vertical elements, one top element that runs across the vertical elements, and two diagonal elements. This design satisfied the unit cell truss model equilibrium using equation 1

\[ 2j = m + 3 \]  

where \( j \) is the number of joints, and \( m \) is the numbers of structure members or elements.

![Figure 5: Truss TEG](image)

Compression load was applied to the truss’ left (y2) and right (y4) top nodes on the vertical elements, and a horizontal load is applied on the top left (y2) node as well. The objective was to determine the extent to which a structural deflection would be achieved without failure. Through the manipulation of diagonal element mass, some degree of deflection was achieved.

**Problem formulation**

First, all variables were identified as element length, \( l \); stiffness, \( s \); and cross sectional area, \( A \). The stiffness was the result of load divided by the deflection in the plane of loading. The cross sectional area multiplied by density and length results to mass.

**Objective function:**

\[ \min f(x) = (y_2 - y_4)^2 \]

**subject to:**

\[ g = A_4 + A_5 \leq \text{street space} \]

where \( y_2 \) and \( y_4 \) are displacements in the y direction of the TEGs unit cell two top nodes, and \( A_4 \) and \( A_5 \) are cross sectional areas of the diagonal elements.

With its five element-four joint truss structure, the TEGs unit cells’ eight translational displacements \((xy)\) were evaluated using the matrix form (2) and equations 3 to 5. Equations 6 to 9 were used in determining stresses within the elements.
Where \( f \) is applied force, \( k \) is element stiffness matrix and \( u \) is the corresponding displacement. Equations 3 to 5 are derived from 2 above indicating how to calculate for \( u \)

\[
u = k^{-1}f
\]  

(3)

Centrally loaded element modeled as a spring with an equivalent stiffness of

\[
k = \frac{AE}{l}
\]  

(4)

rewriting equation 3 becomes

\[
u = \frac{fl}{AE}
\]  

(5)

reaction forces are evaluated as

\[
\{r\} = [k]\{u\} - \{f\}
\]  

(6)

local internal forces are calculated as

\[
f_{ix} = k(u_{ix} - u_{jx})
\]  

(7)

\[
f_{jx} = k(u_{jx} - u_{ix})
\]  

(8)

then normal forces are calculated

\[
\sigma = \frac{f}{A} = \frac{k(u_{ix} - u_{jx})}{A} = E \left( \frac{(u_{ix} - u_{jx})}{L} \right)
\]  

(9)

where \( \sigma \) is the average stress in any two-force member and \( k \) is stiffness constant.

**Solutions based on analytical formulations**

With uneven loads applied to the two top nodes, an initial search was to determine whether an intersection existed between the two diagonal elements which would inform us that at some point, the element displacement was equal. This search was made by studying the elastic modulus for diagonal elements four (E4) and five (E5). An intersection of E4 and E5 demonstrated that a solution to this study was feasible. Figure 6 is a MATLAB output depicting an intersection of E4 and E5 indicating even displacement of the two diagonal elements showing a parallel geometry maintained between the top plate and the ground, and hence attaining an area of further investigation.
A closer review of the relationship between the two elements in figure 6 was done by studying a section of the area that had a complete interaction as depicted in both figures 7 and 8. It was determined that there lay an intersection that could be studied to further investigate the extent to which a deflection could occur in the given structures.

**Figure 6**: Elastic modulus interaction for diagonal elements 4 and 5

**Figure 7**: Line of interaction for y2 (top left node) and y4 (top right node)
Once established that the study was feasible, two models (figures 9 and 10) were created to examine specific loads and structural behavior. Figure 9 depicted an indeterminate structure while models 10a and 10b depicted determinate structures. Equal compression loads P1 and P2 were applied on left node (x2, y2) and right node (x4, y4) respectively. The shear load, P3, was applied to figures 9 and 10a only at the left node (x2, y2) position.

**Figure 9**: Indeterminate  
**Figure 10a**: Determinate  
**Figure 10b**: Determinate

Indeterminate truss design examination showed success in maintaining a parallel relationship between the top plate and the ground. The deflection was minimal and was majorly influenced by stiffness of the two diagonal elements. The determinate structure with compression loads only was able to maintain a parallel relationship between the top plate and the ground while the determinate structure with shear load was not able to maintain a parallel relationship between the
top plate and the ground. Table 1 below presents results for different loading conditions on the determinate structure.

Table 1: Results of different loading conditions for a determinate type truss designs

<table>
<thead>
<tr>
<th>Applied Loads (Newtons)</th>
<th>Figure 10a</th>
<th>Figure 10b</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1=P2=5; P3=26</td>
<td>x2=7.366e-5</td>
<td>x2=9.999e-9</td>
</tr>
<tr>
<td></td>
<td>y2=-9.999e-9</td>
<td>y2=-9.999e-9</td>
</tr>
<tr>
<td></td>
<td>x4=7.360e-5</td>
<td>x4=9.999e-9</td>
</tr>
<tr>
<td></td>
<td>y4=-0.620e-9</td>
<td>y4=-9.999e-9</td>
</tr>
<tr>
<td>P1=P2=9; P3=26</td>
<td>x2=7.367e-5</td>
<td>x2=1.799e-8</td>
</tr>
<tr>
<td></td>
<td>y2=-1.8e-8</td>
<td>y2=-1.8e-8</td>
</tr>
<tr>
<td></td>
<td>x4=7.361e-5</td>
<td>x4=1.799e-8</td>
</tr>
<tr>
<td></td>
<td>y4=-6.999e-8</td>
<td>y4=-1.8e-8</td>
</tr>
<tr>
<td>P1=P2=13; P3=26</td>
<td>x2=7.368e-5</td>
<td>x2=2.599e-8</td>
</tr>
<tr>
<td></td>
<td>y2=-2.6e-8</td>
<td>y2=-2.6e-8</td>
</tr>
<tr>
<td></td>
<td>x4=7.362e-5</td>
<td>x4=2.599e-8</td>
</tr>
<tr>
<td></td>
<td>y4=-7.799e-8</td>
<td>y4=-2.6e-8</td>
</tr>
<tr>
<td>P1=P2=13; P3=26</td>
<td>x2=7.371e-5</td>
<td>x2=5.199e-8</td>
</tr>
<tr>
<td></td>
<td>y2=-5.2e-8</td>
<td>y2=-5.2e-8</td>
</tr>
<tr>
<td></td>
<td>x4=7.364e-5</td>
<td>x4=5.199e-8</td>
</tr>
<tr>
<td></td>
<td>y4=-0.104e-8</td>
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</tbody>
</table>

Educational Impact

This analysis and design project did not include any undergraduate student contributions. The research was specifically designed for evaluating transformation of rigid and brittle structure into flexible ones. This process’ ability to demonstrate structural analysis and design for brittle materials using computational methods is critical for both undergraduate and graduate engineering education at this critical point in the U.S. history where our lead in science and engineering have been seriously threatened. Therefore, engaging students in a learning atmosphere where computational analysis and simulation is conducted will equip them with the tools they need to predict outcomes which they can verify with experimentation as well as better prepare them for future challenges. Various ways can be employed in incorporating computational methods in undergraduate curriculum early and often:

- Both simple and complex models like these conducted in MATLAB can be done in introductory engineering courses giving students a head start and a taste of engineering research early in their education.
- Combination of computational methods coupled with experimentation can allow better understanding of the place of numerical methods in engineering design and analysis.
Discussion and Conclusion

As noted earlier, the indeterminate model yielded a desirable deflection of maintaining a parallel relationship between the top plate and the ground, however it had a minimal deflection of <0.01 diagonal member length. This result could be expected because of symmetrical geometry and use of same material.

The first determinate model (figure 10a – with shear load applied) results show that a parallel relationship between the top plate and the ground is not achievable using the configuration shown. The second determinate model (figure 10b – only compressive loads applied) indicates equivalent deflections for the nodes in x and y directions. These results are also conceivable.

The discussion continues to be, what geometry can enable a structure made of “chalk” like material significantly transform from a rigid form to a flexible form without minimizing heat transfer from the top plate to the ground? It is hoped that the paper would generate ideas and possible recommendations towards this effort.

Further, student involvement in an analysis and design process will provide insights to real life challenges that engineering at large faces. It would also offer the opportunity to explore models in the effort of searching for a solution. Once a desired model is established, an experiment designed to test the model would cement learning by comparing the numerical model to experimental results.

References


