AC 2012-3507: DESIGN AND CONTROL OF A TWIN TANK WATER PROCESS

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Abstract

The paper is concerned with the design of a twin tank water process and experimental evaluation of feedback and cascade control structures to achieve a desired water level in the second tank by adjusting the water supply to the first tank (feedback only structure) and the water level setpoint of the first tank (feedback/cascade structure). Detailed, first principles-based, dynamic models of this non-linear and interactive process have been developed and compared to experimental data. Furthermore, this experimental study entails and discusses the design of the twin tank process and associated instrumentation, real time data acquisition and control in LabView, process modeling, controller design, and evaluation of the performance of different control structures in a closed loop manner. This work was performed in partial fulfillment of the requirements of the Senior Capstone Project course in controls and instrumentation of the Engineering Technology department at the University of Houston-Downtown. Student experiences are summarized and the need for effective project management methods is emphasized.

I. Process Description

The process considered is a coupled tank apparatus that consists of two acrylic containers with identical dimensions (14.2 cm long, 12.8 cm wide and 30 cm high). The tanks are named tank 1 and tank 2, and they are connected through a 0.5 in diameter PVC pipe with a valve used to regulate the flow between the two tanks. The two tanks are mounted above a reservoir which stores water (Fig. 1). Tank 2 is drained out (back into the reservoir). Water is pumped from a reservoir into the first tank by using a variable speed pump, which is driven by an electric motor.

Water levels in the tanks are measured by using two ultrasonic level sensors that perform as an electrical signal 4-20 mA, which is proportional to the height of water. The level controller in tank 2 will adjust the set point of level controller in tank 1 to maintain a desired water level in the tank 2. Tank 1 level controller manipulates the 12Vdc water pump by pulse width modulation (PWM) from an analog circuit that interfaces with NI USB6009 and Labview.

The actual twin tank process along with its computer control system is shown in Fig. 2.

Fig. 1: Twin Tank Process Diagram

Fig. 2: Actual Twin Tank Process

II. Process Instrumentation

A number of instruments are included for level measurement and control. A list of them and related design specifications are shown in Table 1 while Fig. 3 shows the hardware layout and connections. The ultrasonic level sensors can measure maximum level at 49.2 inches (1.25 m). Level sensors have a dead band. It cannot measure if the water level is within 2.5 in from the bottom of the level sensor. In our case, we calibrate the level sensor as shown in Fig. 4.

Table 1: List of Process Instruments and Design Specifications

Physical Quantity	Quantity	Symbol	Units
Tank 1, 2 dimensions	2	W x L x H	12.5 x 12.5 x 30 inches
Reservoir water tank	1	W x L x H	14 x 23.5 x 13.8 inches
Valve diameter	2	valve B, valve C	0.5 inches
Pump	1	Pump	12 Vdc - 2 Amp
Ultrasonic Sensor	2		4 - 20 mA
Power supply	1		dual voltage -12 and +12 Vdc
Power supply for level sensors	1		24 Vdc
PC desktop (or laptop)	1		
Sensor calibration USB-Fob	1		4 - 20 mA
NI USB-6009	1	USB-6009	A/D and D/A interface
PWM circuit	1	PWM	volts

Fig. 3: Hardware Layout and System Connections

Fig. 4: Ultrasonic level sensors calibration

III. LabView Programming

Data acquisition, process variable trending, Human Machine Interface (HMI) and control strategy programming were done using Labview. Fig. 5 shows the Labview block diagram of the twin tank process (top) and the HMI (bottom).

Fig. 5: Labview block diagram (top) and HMI (bottom)

IV. First Principles Based Dynamic Modeling

A detailed dynamic, first principles based, model of the process was developed and used to develop transfer functions to design the two level controllers. Model development details are presented next.

IV.1 Single tank Model

To facilitate model development, a single tank as in Fig. 6 is considered.

Fig. 6: A Single Tanks System

The liquid level model is using a dynamic material balance assuming constant density and tank area. Thus:

$$\dot{L}(t) = \frac{F(t)_i}{A} - \frac{F_o(t)}{A} \quad \frac{\text{cm}}{\text{sec}} \quad (1)$$

where,

A = the cross-sectional area of the tank,
L = the height of water level

Then, the inlet flow rate to Tank 1 depends on the voltage applied to the pump and is given by

$$F_i(t) = K_p V_p \quad \frac{\text{cm}^3}{\text{sec}} \quad (2)$$

where,

$$K_p = \text{the pump constant in } \left(\frac{\text{cm}^3}{\text{volt-sec}} \right)$$

$$V_p = \text{the voltage applied to the pump}$$

The outlet flow rate, F_o , depends on the hydrostatic pressure. Then,

$$F_o(t) = a\sqrt{2g.L} \quad (3)$$

where,

a = the cross sectional area of the valve

$g = 980 \text{ cm/sec}^2$, gravitational constant

By substituting equations (2) and (3) into (1), the non-linear, dynamic water level model is:

$$\dot{L}(t) = \frac{K_p V_p(t)}{A} - \frac{a\sqrt{2gL}(t)}{A} \quad \frac{\text{cm}}{\text{sec}} \quad (4)$$

Equation (4) is the mathematical model that describes the nonlinear system behavior. It is a first order differential equation relating inlet flow rate, F_i , and tank water level, L . The same analysis will be used to develop the non-linear, dynamic model of the twin tank process shown in Fig. 1.

IV.2 Twin Tank Non-linear Model

Consider the coupled tank as described in Fig. 1. The dynamic model is derived by taking material balances around each tank. L_1 is the level in tank 1 while L_2 is the level in tank 2. The independent variable is the voltage to the inlet pump. The water levels are dependent variables.

A model for each tank is established, next.

For the first tank:

$$\dot{L}_1(t) = \frac{1}{A} [F_i(t) - F_b(t)] \quad (5)$$

where,

$F_i(t)$ = the pump flow rate

$F_b(t)$ = the flow rate from tank 1 to tank 2 through valve B

A = cross sectional area of tank 1 and tank 2

$L_1(t)$ = water level in tank 1

For the second tank:

$$\dot{L}_2(t) = \frac{1}{A} [F_b(t) - F_c(t)] \quad (6)$$

where,

$F_c(t)$ = the flow rate from tank 2 through valve C

$L_2(t)$ = water level in tank 2

The flow out of the second tank is determined by the water level in the tank. In a way similar to equation (3),

$$F_c(t) = a_c \cdot \sqrt{2gL_2} \quad (7)$$

The flow out of the first tank is determined by the difference in levels of the two tanks

$$F_b(t) = a_b \cdot \sqrt{2 \cdot g(L_1(t) - L_2(t))} \quad (8)$$

Thus, the twin-tank system behavior is given by:

$$\dot{L}_1(t) = \frac{1}{A_1} (F_i(t) - F_b(t)) \quad \text{and} \quad \dot{L}_2(t) = \frac{1}{A_2} (F_b(t) - F_c(t)) \quad (9)$$

By substituting for the flow rates,

$$\dot{L}_1(t) = -\frac{a_b}{A_1} \sqrt{2 \cdot g(L_1(t) - L_2(t))} + \frac{K_p}{A_1} V_p(t) \quad (10)$$

or

$$\dot{L}_2(t) = \frac{a_b}{A_2} \sqrt{2 \cdot g(L_1(t) - L_2(t))} - \frac{a_c}{A_2} \sqrt{2gL_2}(t) \quad (11)$$

Equations (10) and (11) constitute the non-linear, dynamic process model. To design linear controllers, this model must be linearized around a nominal steady state. This is done in the following.

IV.3 Twin Tank Linear Model

From equation (10), the steady-state pump voltage V_{pss} that produces the desired steady-state constant level L_{1ss} in the tank 1, is obtained by setting $\dot{L}_1(t) = 0$. This yields

$$V_{pss} = a_b \frac{\sqrt{2g(L_{1ss} - L_{2ss})}}{K_p} \quad (12)$$

In a similar manner, the steady-state level, L_{1ss} in tank 1 that produces the desired steady-state constant level L_{2ss} in tank 2, is calculated by setting $\dot{L}_2(t) = 0$. Equation (11) then yields:

$$\frac{a_b}{A_2} \cdot \sqrt{2g(L_{1ss} - L_{2ss})} = \frac{a_c}{A_2} \cdot \sqrt{2g(L_{2ss})}$$

$$\begin{aligned}
\rightarrow a_b^2 \cdot [\mathfrak{L} \mathfrak{g}(L_{1ss} - L_{2ss})] &= a_c^2 \cdot (\mathfrak{L} \mathfrak{g} L_{ss}) \\
\rightarrow a_b^2 (L_{1ss} - L_{2ss}) &= a_c^2 L_{2ss} \\
\rightarrow L_{1ss} - L_{2ss} &= \left(\frac{a_c}{a_b} \right)^2 \cdot L_{2ss} \\
\rightarrow L_{1ss} &= \left[\left(\frac{a_c}{a_b} \right)^2 + 1 \right] \cdot L_{2ss}
\end{aligned} \tag{13}$$

Next, a set of deviation variables is defined as:

$$l_1(t) = L_1(t) - L_{1ss} \quad \rightarrow \quad L_1(t) = l_1(t) + L_{1ss} \tag{14}$$

$$l_2(t) = L_2(t) - L_{2ss} \quad \rightarrow \quad L_2(t) = l_2(t) + L_{2ss} \tag{15}$$

Equation (10) is rewritten as

$$\dot{L}_1(t) = -\frac{a_b}{A_1} \sqrt{2 \cdot g(L_1 - L_2)} + \frac{K_p}{A_1} V_p \tag{16}$$

Then, using Taylor series expansion,

$$\begin{aligned}
\sqrt{L_1 - L_2} &= (L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot [(L_1 - L_{1ss}) - (L_2 - L_{2ss})] \\
&= (L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot [l_1 - l_2] \\
\rightarrow \dot{L}_1(t) &= -\frac{\sqrt{2g} \cdot a_b}{A_1} \left[(L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot (l_1 - l_2) \right] + \frac{K_p}{A_1} \cdot V_p
\end{aligned}$$

At steady state, we set $\dot{L}_1(t) = 0$

$$\begin{aligned}
\rightarrow 0 &= -\frac{\sqrt{2g} \cdot a_b}{A_1} \left[(L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot (0 - 0) \right] + \frac{K_p}{A_1} \cdot V_{pss} \\
\rightarrow \dot{L}_1(t) - 0 &= -\frac{\sqrt{2g} \cdot a_b}{A_1} \cdot \frac{1}{2} + (L_{1ss} - L_{2ss})^{-0.5} \cdot (l_1 - l_2) + \frac{K_p}{A_1} \cdot (V_p - V_{pss}) \\
\rightarrow \dot{l}_1(t) &= -\frac{\sqrt{2g} \cdot a_b}{A_1} \cdot \frac{1}{2(L_{1ss} - L_{2ss})^{0.5}} \cdot (l_1 - l_2) + \frac{K_p}{A_1} \cdot v_p
\end{aligned} \tag{17}$$

Then the time domain equation for level in tank 1, in deviation form, is:

$$\dot{l}_1(t) = -a \cdot l_1(t) + b \cdot l_2(t) + c \cdot v_p(t) \tag{18}$$

where:

$$a = \frac{a_b}{A_1} \cdot \frac{\sqrt{2g}}{2(L_{1ss} - L_{2ss})^{0.5}}$$

$$b = \frac{a_b}{A_1} \cdot \frac{\sqrt{2g}}{2(L_{1ss} - L_{2ss})^{0.5}} \quad \text{Note: } a = b$$

$$c = \frac{K_p}{A_1}$$

By taking the Laplace transform, equation (18) yields:

$$s.l_1(s) = -a.l_1(s) + b.l_2(s) + c.v_p(s)$$

$$\rightarrow (s+a).l_1(s) = b.l_2(s) + c.v_p(s)$$

$$\rightarrow l_1(s) = \left(\frac{b}{s+a} \right) . l_2(s) + \left(\frac{c}{s+a} \right) . v_p(s) \quad (19)$$

Next, in a similar manner, equation (11) is linearized. From equation (11),

$$\dot{L}_2(t) = \frac{a_b}{A_2} \sqrt{2g(L_1 - L_2)} - \frac{a_c}{A_2} \sqrt{2gL_2}$$

Since,

$$\begin{aligned} \sqrt{L_1 - L_2} &= (L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot [(L_1 - L_{1ss}) - (L_2 - L_{2ss})] \\ &= (L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot [l_1 - l_2] \end{aligned}$$

It is obtained that,

$$\dot{L}_2(t) = \frac{\sqrt{2g}.a_b}{A_2} \left[(L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot (l_1 - l_2) \right] - \frac{\sqrt{2g}.a_c}{A_2} \left[L_{2ss}^{0.5} + \frac{1}{2} \cdot L_{2ss}^{-0.5} (L_2 - L_1) \right]$$

At steady state, $\dot{L}_2(t) = 0$. Thus,

$$\begin{aligned} 0 &= \frac{a_b}{A_2} \left[(L_{1ss} - L_{2ss})^{0.5} + \frac{1}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot (0 - 0) \right] - \frac{\sqrt{2g}.a_c}{A_2} \left[L_{2ss}^{0.5} + \frac{1}{2} \cdot L_{2ss}^{-0.5} (0) \right] \\ \rightarrow \dot{L}_2(t) - 0 &= \frac{a_b}{A_2} \cdot \frac{\sqrt{2g}}{2} \cdot (L_{1ss} - L_{2ss})^{-0.5} \cdot (l_1 - l_2) - \frac{a_c \cdot \sqrt{2g}}{A_2} \cdot \frac{1}{2} \cdot L_{2ss}^{-0.5} \cdot l_2 \\ \rightarrow \dot{l}_2(t) &= \frac{a_b}{A_2} \cdot \frac{\sqrt{2g}}{2(L_{1ss} - L_{2ss})^{0.5}} \cdot (l_1 - l_2) - \frac{a_c \cdot \sqrt{2g}}{2A_2 \cdot L_{2ss}^{0.5}} \cdot l_2 \end{aligned} \quad (20)$$

Then the time domain, linear model for the level in tank 2 is:

$$\dot{l}_2(t) = d.l_1(t) + c.l_2(t) \quad (21)$$

where:

$$e = \frac{a_b}{A_2} \cdot \frac{\sqrt{2g}}{2\sqrt{L_{1SS}} - L_{2SS}} + \frac{a_c}{2A_2} \cdot \frac{\sqrt{2g}}{\sqrt{L_{SS}}}$$

$$d = \frac{a_b}{A_2} \cdot \frac{\sqrt{2g}}{2\sqrt{L_{1SS}} - L_{2SS}}$$

By taking the Laplace transform:

$$s.l_2(s) = d.l_1(s) - e.l_2(s)$$

$$\rightarrow (s+e).l_2(s) = d.l_1(s)$$

$$\rightarrow l_2(s) = \frac{d}{(s+e)}.l_1(s) \quad (22)$$

From (19) and (22), we have:

$$l_2(s) = \frac{d}{(s+e)} \cdot \left[\frac{b}{(s+e)}.l_2(s) + \frac{c}{(s+a)}.v_p(s) \right]$$

$$\rightarrow l_2(s) = \frac{d.b}{(s+e)(s+a)}.l_2(s) + \frac{d.c}{(s+e)(s+a)}.v_p(s)$$

$$\rightarrow \left[1 - \frac{d.b}{(s+e)(s+a)} \right].l_2(s) = \frac{d.c}{(s+e)(s+a)}.v_p(s)$$

$$\rightarrow \frac{(s+e)(s+a) - d.b}{(s+e)(s+a)}.l_2(s) = \frac{d.c}{(s+e)(s+a)}.v_p(s)$$

$$\rightarrow l_2(s) = \frac{d.c}{(s+e)(s+a) - d.b}.v_p(s)$$

$$\rightarrow l_2(s) = \frac{d.c}{s^2 + (a+e).s + e.a - d.b}.v_p(s)$$

Since $a = b$, then

$$l_2(s) = \frac{d.c}{s^2 + (a+e).s + (e.d).a}.v_p(s) \quad (23)$$

Equation (23) describes the effect of pump power on tank 2 level. It is a second order system.

Rewriting equation (23),

$$l_2(s) = \frac{\frac{d.c}{(e-d).a}}{\left[\frac{1}{(e-d).a} \right].s^2 + \left[\frac{(a+e)}{(e-d).a} \right].s + 1} \cdot v_p(s) \quad (24)$$

To bring equation (24) into the general form of a second order system,

$$l_2(s) = \frac{K_p}{\tau^2 s^2 + 2\tau\xi.s + 1} \cdot v_p(s) \quad (25)$$

The following variables are defined:

$$K_p = \frac{d.c}{(e-d).a} \quad (\text{steady state gain})$$

$$\tau^2 = \frac{1}{(e-d).a} \quad \rightarrow \quad \tau = \sqrt{\frac{1}{(e-d).a}} \quad (\text{time constant})$$

$$2.\tau.\xi = \frac{a+e}{(e-d).a}$$

$$\text{Or } \xi = \frac{a+e}{(e-d).a} \cdot \frac{1}{2\tau} = \frac{a+e}{(e-d).a} \cdot \sqrt{(e-d).a}$$

$$\rightarrow \xi = \frac{a+e}{\sqrt{a(e-d)}} \quad (\text{Damping coefficient})$$

IV.4 Numerical results

It is desired that the tank 2 water level set point be 10 cm. Thus, at steady state conditions,

$$L_{2ss} = 10 \text{ cm},$$

The pipe on the tank outlet has a diameter of 2 cm. Then,

$$a_b = \frac{\pi.d^2}{4} = \frac{\pi.\left(\frac{2}{2}\right)^2}{4} = \frac{\pi}{4} = \frac{3.14159}{4}$$

$$\rightarrow a_b = 0.7854 \text{ cm}^2$$

If valve C is 75% open, then $a_c = 0.58905 \text{ cm}^2$

Then, equation (13) gives,

$$L_{1ss} = [0.75^2 + 1].10$$

$$\rightarrow L_{1ss} = 15.625 \text{ cm}$$

At steady state, V_{pss} is 5V. Therefore the pump constant K_p is 0.01637cm/volt-sec.

Since,

$$a = \frac{a_b}{A_1} \cdot \frac{\sqrt{2g}}{2(L_{1ss} - L_{ss})^{0.5}}$$

and

$$A_1 = (12.5 * 2.54) \cdot (12.5 * 2.54) = 1008.06 \text{ cm}^2$$

Then,

$$a = \left(\frac{0.7854 \text{ cm}^2}{1008.06 \text{ cm}^2} \right) \cdot \left(\frac{\sqrt{2 * 980 \frac{\text{cm}}{\text{sec}^2}}}{2 \cdot [15.625 - 10]^{0.5} \text{ cm}} \right)$$

$$\rightarrow a = 0.0073 \text{ s}^{-1}$$

Similarly, the values of b and d are 0.0073/sec. The value of c is 0.01637cm/volt-sec while the value of e is 0.01133/sec.

After substituting the values of a, b, c, d and e into equation (24) and comparing equations (24) and (25), it is calculated that the system gain is 4.06 cm/volt/sec, the time constant is 184 seconds, and the damping coefficient is 1.7.

V. Control Structure and Controller Design

The control objective is to maintain the level in tank 2 at a desired setpoint. A cascaded control configuration is used. The slave controller adjusts the pump voltage to maintain a level in tank 1. The setpoint for the level in tank 1 is determined by the primary controller which attempts to maintain the tank 2 level at the desired setpoint.

In summary, the block diagram of the cascade control structure is shown below in Fig. 7.

$$l_1(s) = \frac{c}{s+a} \cdot v_{pss} \quad \rightarrow \quad G_s(s) = \frac{c}{s+a}$$

$$l_2(s) = \frac{d}{s+e} \cdot l_1(s) \quad \rightarrow \quad G_M(s) = \frac{d}{s+e}$$

Fig. 7: Cascade level control configuration for the twin tank

Controller Tuning

Proportional/Integral (PI) controllers were used for level control (controllers G_C and G_{CM} of Fig. 7). The open loop Ziegler-Nichols tuning method¹ was used with satisfactory performance (see Fig. 8).

Fig. 8: Twin Tank Closed Loop Level Control

VI. Lessons Learned

This senior project work has been completed in partial fulfillment of the requirements for the bachelor of science degree in control and instrumentation at the University of Houston – Downtown. The project duration was one academic semester (Fall 2011). There were 2 students working on this team project.

To accomplish this work in a semester's timeframe, it required clear objectives, deliverables, a realistic timetable with milestones, and methods to access progress towards the desired project objectives.

Preparing, as a team, a project proposal and obtaining instructor's approval was a very important step towards accomplishing our project objectives. In addition to final project report and presentation, there was a mid-term, formal, project assessment. However, what helped immensely were the weekly team meetings with the instructor. In those meetings, accomplishments, issues and proposed methods of resolution were discussed. Also, work plans for the following week were reviewed and approved.

In addition, a mini series of lectures on teamwork was helpful. It helped understand the importance of working together towards a common goal (successful and timely completion of the project), timely communication and trust among team members.

VII. Conclusion

The paper was concerned with the design of a twin tank water process and experimental evaluation of feedback and cascade control structures to achieve a desired water level in the second tank by adjusting the water supply to the first tank (feedback only structure) and the water level set point of the first tank (feedback/cascade structure). Furthermore, this experimental study entails and discusses the design of the twin tank process and associated instrumentation, real time data acquisition and control in LabView, process modeling, controller design, and evaluation of the performance of control structures in a closed loop manner.

This work was performed in partial fulfillment of the requirements of the Senior Capstone Project course in controls and instrumentation of the Engineering Technology department at the University of Houston-Downtown. Student experiences were presented.

References

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