AC 2012-3246: INTEGRATING WRITING WITH CONTEMPORARY MATHEMATICS TO DEVELOP CRITICAL THINKING SKILLS

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Abstract

Globalization, compounded by the rapid societal evolution being propelled by advancing technology, is increasing the necessity for astute critical thinking skills, yet many students arrive at the university with these skills underdeveloped. Such higher-level thinking involves analyzing, evaluating, and creating (the topmost three levels of thinking in Bloom's Taxonomy of the Cognitive Domain revised by Anderson in 2001). Several researchers in the late 1990s into the 2000s have shown that processing new information using these thinking skills increases students' information retention. In addition, thinking critically helps prepare students to become successful global citizens because they can make the decisions and solve the problems of modern life more astutely, having both the knowledge retained and the thinking skills developed to do so.

One teaching strategy shown by researchers since the 1960s and 1970s to be an effective learning and thinking tool is writing. Writing enables the writer to capture otherwise random thoughts by placing them on a writing surface where they become concrete and thus more readily examined and manipulated. Consequently, writing should be an effective tool for enabling math students to retain the mathematical principles being developed in the classroom as well as for aiding them to improve their critical thinking abilities needed for applying their mathematical understandings to problems of the modern world.

By incorporating writing that emphasizes critical thinking into the math classroom, this study seeks an answer to two questions: *(1) how can mathematics professors, who usually are non-writing specialists, incorporate writing into the math class comfortably and effectively* and *(2) how can writing strategies used within math courses enhance both students' knowledge retention and their critical thinking abilities?* A variety of carefully tailored writing tools (such as three-minute recollection papers and targeted journal assignments, among others) and related materials are used in several Contemporary Mathematics university classes both to reinforce the math principles and to develop students' critical thinking. Pre-tests and post-tests of students' critical thinking ability are used to determine effectiveness of the tailored writing strategies.

Introduction

Globalization, compounded by the rapid societal evolution being propelled by advancing technology, is increasing the necessity for astute critical thinking skills. Daily, information conveyed by globally interacting technology bombards us, requiring understanding, analyzing, and reacting in diverse ways. Managing these requirements advantageously and well requires well-developed strategies for thinking critically, yet many students arrive at the university with these skills underdeveloped—and these are not just American students.

Since 2004 I have worked primarily with non-native speakers of English enrolled in design programs at the foreign branch campus of a US university. Some of these students have grown up in bi- or tri-lingual homes, and most have studied English speaking and writing since primary school. To enter the university, they must have minimum English proficiency scores of 80 or above on the internet-based TOEFL exam or 6.0 or above on the IELTS exam[1]. Although not the official language of the country, English is widely used and is rapidly becoming considered a necessity for successful living in the modern world by the country's leaders and the younger generations of citizens. However, the means by which students and their parents have learned English have varied considerably until the recent standardization of learning requirements in the country's primary and secondary educational systems. In addition to the English requirement, all of these design students must pass a Contemporary Mathematics (MATH 131) course, taught in English, as a liberal arts requirement for graduation. The course topics include geometric and arithmetic sequences, Fibonacci numbers, fractals, and graph theory, among others.

My experiences and conversations through the years have repeatedly shown that most of these students come to college ill prepared for college math specifically and for critical thinking in general. Until recently, most were learning math, among other subjects, by rote memorization in the primary and secondary schools. One day a Contemporary Math student expressed this difference poignantly in class: "Back then, we studied *hard mathematics;* this is *soft mathematics!"* She was referring to the college course as *soft* because it encouraged *reasoning* rather than *memorizing,* which she equated with "learning." Unfortunately, this mistaken equation is widespread among these students, and its mistakenness was proven most emphatically in August 2008 when only two of approximately 36 incoming freshmen passed the mathematics placement test that qualified them to take MATH 131. Understandably, the course could not be offered that semester.

These students were victims of a crucial misunderstanding about memorization: although it is possibly an appropriate *initial* strategy for learning, it is ineffective without also *understanding* and *owning* the material memorized. As one of the math placement test administrators explained, "Students who challenge their placement in the remedial course say they 'know' the material, just forgot, or were not ready for the test. However, the lowness of their scores does not support their claims."[2]

Since Janet Emig's seminal article, "Writing as a Mode of Learning,"[3] was published in 1977, writing has been recognized as an effective tool for learning. Consequently, incorporating writing assignments that encourage critical thinking in math should prove effective for enabling students to retain the mathematical principles being developed in the classroom. In addition, it should aid the improvement of critical thinking abilities needed for applying their mathematical understandings to problems of the modern world. One problem with using writing in the math classroom, however, is that many math professors hesitate to use writing assignments because *math, not writing,* is their field of expertise. Thus, one of the questions this study seeks to answer is: *how can mathematics professors, who usually are non-writing specialists, incorporate writing into the math class comfortably and effectively?* In addition, it investigates the related question of importance to learning outcomes: *how can writing strategies used within math courses enhance both students' knowledge retention and their critical thinking abilities?*

What is critical thinking?

According to Brittney Christensen, an English educator who writes professionally about the psychology of learning, among other topics, we usually categorize critical thinking skills as:

- *affective strategies:* means for understanding and using emotions wisely that can evolve into a thinking maturity characterized by open-mindedness to consider others' ideas and feelings;
- *macro cognitive strategies:* receiving, analyzing, and evaluating data; and
- *micro cognitive strategies:* manipulating information taken in, such as by comparing/contrasting, collecting results, or developing original hypotheses/solutions.[4]

These categories, combined with the perspectives of different disciplines, have led to numerous definitions of *critical thinking.* Some of these definitions emphasize the required mental and physical abilities, while others focus on persuasive abilities or on the *process* of thinking critically and the various individual characteristics contributing to this process.

Dr. William G. Huitt, a widely published Educational Psychologist who has analyzed many of these definitions, proposes a more generic definition "to more closely align the concept to the evaluation level as defined by Bloom et al. (1956) and to include some of the vocabulary of other investigators."[5] He defines the concept as

> "the disciplined mental activity of evaluating arguments or propositions and making judgments that can guide the development of beliefs and taking action."[6]

Such higher-level thinking involves analyzing, evaluating, and creating (the topmost three levels of thinking in Bloom's Taxonomy of the Cognitive Domain revised by Anderson in 2001) and well serves the purposes of this research because design students must learn to evaluate clients' needs and wants as well as to make judgments that will influence their creative products.

Huitt goes on to relate critical thinking to the affective and macro/micro cognitive strategies but extends it to include *behavioral* strategies as well. (See Huitt's diagram, "Model of Critical Thinking & Its Modification," and corresponding discussion in Appendix A.) This extension enables him to suggest teaching strategies for developing critical thinking behaviors because each component of the model must proceed successfully to complete critical thinking, and these different components may require different teaching methods.

> For example, if one is most interested in impacting declarative knowledge (facts, concepts, principles, etc. that are stored in semantic and episodic memory), the most appropriate teaching method is probably some form of didactic, explicit, or direct instruction. On the other hand, if the focus is on procedural knowledge it is likely that modeling and/or personal experience would be more appropriate teaching methods. Likewise, if one were trying to impact the memory of images or visualizations, then modeling, active visualizations, or working with pictures might be more appropriate. Attitudes are probably impacted most directly by socialization and the teaching method of cooperative learning. Learning the process of critical thinking might be best facilitated by a combination of didactic instruction and experience in specific content areas. Impacting conation might best be done through goal-setting exercises and action learning. Finally, overt behavior and learning to use feedback might best be accomplished using positive and negative reinforcement.[7]

A further distinction important to design students is Huitt's relating critical thinking to Bloom's Taxonomy and separating it from *creative thinking:*

> Research over the past 40 years has generally confirmed that the first four levels [of Bloom's Taxonomy] are indeed a true hierarchy. . . . However, research is mixed on the relationship of synthesis and evaluation [evaluating and creating, respectively, in Anderson's revised model; see Figure 1 on the next page]; it is possible that these two are reversed or they could be two separate, though equally difficult, activities (Seddon, 1978).[8]
>
> In my opinion, . . . synthesis/creating and evaluation/evaluating are at the same level. Both depend on analysis as a foundational process. However, synthesis or creating requires rearranging the parts in a new, original way whereas evaluation or evaluating requires a comparison to a standard with a judgment as to good, better or best. This is similar to the distinction between creative thinking and critical thinking [see Huitt 1998]. Both are valuable while neither is superior. In fact, when either is omitted during the problem solving process, effectiveness declines.[9]

Why is Critical Thinking Important?

Evaluating and synthesizing/creating are the precise skill sets that students at our university need to master to become capable, well-respected designers. But not only designers need these critical thinking skills. According to Harvard economists Richard Murnane and Frank Levy, who have been studying changes in the workforce for more than a decade, in today's workplaces, computers usually perform jobs requiring the lower-level skills, meaning that employees must think and perform at higher levels than in previous economic environments.[10]

Writing researchers in the 1960s and 1970s such as Janet Emig and James Britton, among others, have shown that using writing that requires higher-level, critical thinking skills to process new information increases students' information retention. Research by more recent scholars, such as

Garavalia, Hummel, Wiley, & Huitt in 1999, also shows that learning to handle a topic at these highest levels improves and increases students' memory of what they have learned because they must elaborate and process the information more thoroughly.[11] Thus, higher-level thinking facilitates knowledge retention. In addition, thinking critically helps prepare students to become successful global citizens because they can make the decisions and solve the problems of modern life more astutely, having both the knowledge retained and the thinking skills developed to do so.

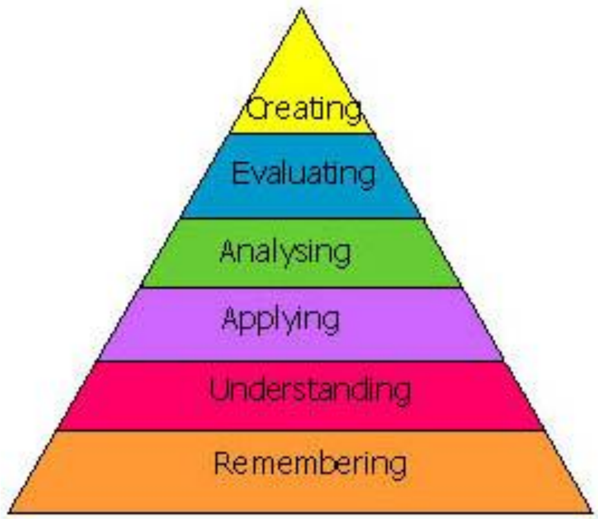

Figure 1. Bloom's Taxonomy as revised by Anderson in 2001. Source: Mary Forehand. [12]

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Research Method and Materials

Participants. Thirty-five university students enrolled in three sections of MATH 131, Contemporary Math, in Fall Semester 2011 have participated in this research thus far. The University is co-educational, with most classes containing a few males and predominantly females, ranging from freshmen to seniors. Freshmen were taking their first college course in English writing, but other students had taken two or more university English courses that required writing. Two of the 35 were honors students, who took the same course as the other students but who completed more, and more difficult, assignments. These two students were juniors. Most students were non-native English speakers from diverse countries, and all of them are, or plan to become, design majors.

Course Activities. Over several years, the teaching strategies and assignment requirements for MATH 131 have evolved to accommodate students' generally non-American cultures, their English-as-second-language (ESL) needs, and their individual learning and information-processing preferences. After a writing center instructor administers and evaluates learning style preference and brain hemispheric preference tests given during the first few days of class, the professor prepares teaching strategies that address visual, auditory, tactile, and kinesthetic preferences as needed for the specific class. Quizzes, tests, brief "recollection writings," written journal assignments, and projects are incorporated into the course. Students complete journal assignments under the guidance of the same writing center instructor, who collaborates with the math professor. While good grammar, punctuation, and clarity of wording are encouraged, students are penalized for writing problems only when their responses cannot be clearly understood.

Procedures and Materials. Although the MATH 131 professor and a writing instructor collaborated to develop the procedures followed, none of the materials used is so specialized that, once informed about specific instruments, the math professor alone could not have conducted the course. The materials and techniques included: modifying and extending the course objectives; assessing and appealing to students' *super links*[a]*;* using various short papers as gauges of students' understanding and as concept reinforcements; assigning journal questions; estimating students' critical thinking skills; administering quizzes, tests, and projects; requiring reports and presentations by the Honors Students; and emphasizing writing as an instructional tool.

Course Objectives. The math professor and the writing center instructor first collaborated to develop course objectives reflecting the higher-level thinking and learning skills demanded by critical thinking. They stated such objectives for both the topics to be covered and for critical thinking skills specifically. Both sets emphasized the higher thinking levels of Bloom's Taxonomy: analyzing, evaluating, and creating (*synthesizing* in the earlier model). (Actions related to these levels are highlighted in blue text in the following examples.) Some of the higher-level objectives for the math topics included:

- Analyze and synthesize Fibonacci Sequences by creating original artistic designs incorporating them.
- Demonstrate understanding of symmetrical properties by designing a small project using symmetrical properties found in Arabesque art.
- Use complex numbers to develop an advanced fractal, such as the Mandelbrot fractal.
- Differentiate among various graph trees, graph paths, and graph circuits by analyzing situations, choosing the appropriate structure to illustrate each situation, and correctly constructing the structure.

Objectives stating separate expectations for students' critical thinking skills also were included:

[a] A term coined by Linksman (1996) to describe an individual's quickest, easiest way to learn.

In addition, students will develop their critical thinking skills by

- Demonstrating an understanding of *taxonomy* by writing its definition in their own words *(synthesizing)*.
- Writing their responses to math journal assignments that involve the higher cognitive skills of *evaluating, analyzing,* and *synthesizing.*
- Creating ideas and projects in response to math assignments.

The complete list of course objectives was included in the syllabus given to all students on their first day of class. The professor reviewed the objectives with the students in class along with the other university policies and procedures and the course information in the syllabus.

Super Links. During the first week of the semester, the writing instructor came into the class to explain briefly and to administer two tests for determining students' preferred learning styles and hemispheric preferences[b]. These results were desired for two reasons: to make students metacognitively aware of their learning activities and their abilities to improve them and to provide the professor with information that could guide him to select teaching strategies that would appeal to the diverse students in each class. Once these tests were graded, the writing instructor returned to the class to discuss in detail the theory of *super links* developed by the diagnostic reading expert, Ricki Linksman, which Linksman explains as the fastest, easiest way for a person to master new information.[14]

In addition to relating their learning preferences to the students' personal lives as well as academic endeavors, the writing instructor introduced the course emphasis on critical thinking by explaining Bloom's Taxonomy as modified in 2001 (see Figure 1) and by linking its highest level, synthesizing/creating, to the creativity required of designers. She concluded this class by distributing and reviewing the first of three sets of journal writing assignments that students would receive during the semester. Although journal writing already was being used in this course, the math professor and writing instructor collaborated to rewrite these assignments so they more purposefully focused on the higher-level thinking skills required by critical thinking by giving students practice in analyzing, evaluating, creating, and imagining.

Short Papers. Throughout the semester the method included two types of short, in-class writing exercises called "One-Minute Papers" and "Three-Minute Recollections." One-Minute Papers typically ask students to respond to a single question by writing an answer for no more than one minute. Educator Brian Steele of Texas Tech University identifies five uses for One-Minute Papers:

[b]Although Pashler, McDaniel, Rohrer, and Bjork (2008) have uncovered numerous problems with research in this area and have established doubt for some claims made by Linksman and others regarding learning styles and brain hemispheric preferences, the author continues to discuss these theories with students to arouse their curiosity through very personal relevance of the information and to encourage their development of metacognition, as well as a reminder to the professor to use varied teaching strategies to maintain students' interest.

- emphasize course objectives,
- improve note-taking,
- encourage questions,
- clarify students' perceptions or correct their misperceptions, and
- gain feedback on course management.[15]

The One-Minute Papers used in MATH 131 typially asked a single question related to the day's math lesson and were not graded. The math professor administered them at the end of a class or at the beginning of the next class to reinforce the students' memories of something selected from the lesson, to develop their metacognition and stimulate engagement with the material, and to help him identify the students' misunderstandings. For example, he used such questions as:

- How useful or interesting was today's lesson? Why?
- Briefly summarize what you learned today.
- As a designer, how could you use information from today's lesson?
- How do you feel about your understanding of today's material?
- Explain how you would tell someone else about this topic.
- The MOST IMPORTANT thing I learned from today's lesson is _____ because _____.

In addition to these papers, the writing instructor added a similar strategy that she called "Three-Minute Recollections." As the name indicates, the response time was extended to three minutes to allow students to think more critically, and questions were more difficult than those in the One-Minute Papers. Although both sets of papers used repetition to reinforce recollection, the Three-Minute Recollections were limited to topics presented by the writing instructor concerning students' learning and brain hemispheric preferences, the concept of taxonomies, and critical thinking in relation to Bloom's Taxonomy.

In contrast to the One-Minute Papers, the Three-Minute Recollection questions tested the same concepts at different times in various ways. A single exercise contained two or three questions, sometimes focused on one concept and at other times covering more than one. Some sample questions, grouped to show concept relatedness, were:

- What is the term we discussed for "the fastest, easiest way for someone to learn anything"?
- A person's *Super Link* is determined by using a test to find a score that measures that person's _____ and _____.
- The "fastest, easiest way for someone to learn anything" consists of that person's _____ and _____.
- The results of your personal tests showed that your own preferences were _____.

- What is a *taxonomy?*
- What is the name of the learning taxonomy discussed in the last class?
- According to that taxonomy, the highest level of learning is . . .

These papers were graded to show students their own strengths and weaknesses and to encourage additional study where needed, but the grades were not recorded.

A variation on these two papers that was *not* used in this course is the "Exit Ticket." It poses a question about the day's lesson that students must answer at the end of class and must submit in order to leave. The question is designed to reinforce the lesson and to stimulate students' higher-order thinking by requiring them to summarize their thoughts in writing.[16]

Journal Assignments. Another writing component of the course was the journal assignments. As in previous semesters, three sets of journal assignments, each covering approximately one-third of the course material, were distributed at the beginning of each third of the course. Students were allowed to work on the current set for the duration of the class discussions about its topics; they also were allowed to discuss their questions and potential responses with both the math professor and the writing instructor. As mentioned previously, these assignments were designed to teach students to use writing as a means for thinking and learning; consequently, no penalties were assigned for incorrect punctuation, spelling, or grammar. Students lost points only if the content itself presented inadequate or incorrect concepts and responses or was undecipherable. The math professor and the writing instructor collaborated in both designing and evaluating the journal assignments, as they were considering both the relevance and accuracy of the mathematical content as well as the extent of critical thinking required and evidenced in students' responses. In designing the writing assignments, they focused on creating assignments that asked students to compare and contrast, to analyze, to synthesize, to evaluate, and to create by using the mathematical material from the current section and relating it to other known information. Refer to Appendix B for samples of these questions.

Critical Thinking. Early in the semester, the writing instructor administered a critical thinking pre-test to each of the three classes. This consisted of a scenario set within a social setting the students could understand from their own experiences followed by several questions that required inferring meanings and analyzing relationships and situations. Following the example of Stein, et al, (cited below) who designed their own test to identify and evaluate basic skills that they deemed essential to critical thinking, the writing instructor selected a sample test question from an American College Test (ACT) practice booklet on which to base the pre-test.

The final exam also contained one question designed to post-test students' critical thinking for comparison with the initial test (see Appendix D). This question, created by the Writing Center Instructor, involved a situation related to a math concept the students had studied and required them to reason out answers to questions about the concept.

Because there is considerable debate about the effectiveness of tests of critical thinking as well as disagreement about what to test and how to test it, these two tests were regarded as only rough indicators of students' abilities. For a comparison of types and weaknesses in 12 popular critical thinking tests, refer to the Appendix in Stein, Haynes, and Unterstein.[17]

Quizzes, Tests, Projects. As usual, the math professor administered both announced and unannounced quizzes and periodic tests to assess students' learning. In addition, students completed math-related projects, including a major final project that was exhibited in the university's annual Physics and Mathematics Day. The process for completing the major project was more structured than it had been in past semesters: to diversify project topics, students were required to choose from a list of math topics covered, and no more than two students per class were allowed to choose the same topic. Periodic progress reports given informally during class also were required to assure students were not waiting until the last minute to begin this work. Students who had chosen end-of-semester topics received enough individualized advance instruction to begin their work with the rest of the class. An approximately one-page formal report regarding the math principles involved and the student's development of the project was required, and students were allowed to work with the writing instructor on this paper to insure that the English requirements were met as well as the content requirements.

Reports and Presentations. In addition to the formal report required with the major project for all students, the two Honors Students were assigned special-topic reports that they presented to the class. The math professor chose topics beyond the scope of the course for the average student, in accordance with the Honors Students' advanced academic standing. Oral presentations to the class that usually included PowerPoint or other visual materials helped reinforce the Honors Students' understanding of the advanced concepts while introducing the remaining students to the advanced work.

Writing as Instructional Tool. Primarily from experiences in English courses, most American students come to regard writing as a formal, public communication that demonstrates not only one's mastery of the topic or field written about but also one's mastery of the English language requirements for formal writing. In this form of writing, one is concerned with the polished, final product.

In this research, the polished, final product is *not* the type of writing used or recommended because the intention is to use writing as *a tool for learning math.* To emphasize this use, the focus is not on *both* the math and the English, as would be required for a formally written document, because requiring students to master two competencies (i.e., math *and* English) reduces the total mental focus available for either competency. Instead, the writing more nearly resembles earlier stages in the overall writing process: a type of prewriting that we might call "partially polished freewriting."

Noted writing researcher Janet Emig was among the first to explain the concept behind using writing as a tool for learning. She proposed that writing simultaneously, or nearly simultaneously, addresses the three primary methods humans use for understanding and handling reality (i.e., *learning*): by doing, primarily with the hands; by seeing; and by representing symbolically (as with words or mathematical symbols), which requires mental engagement.[18] As Emig explained,

> . . . the symbolic transformation of experience through the specific symbol system of verbal language is shaped into an icon (the graphic product) by the enactive hand. If the most efficacious learning occurs when learning is re-inforced, then writing through its inherent re-inforcing cycle involving hand, eye, and brain marks a uniquely powerful multi-representational mode for learning.
>
> Writing is also integrative in perhaps the most basic possible sense: the organic,

> the functional. Writing involves the fullest possible functioning of the brain, which entails the active participation in the process of both the left and the right hemispheres. Writing is markedly bispheral . . .[19]

During the first stage of the writing process, called *prewriting*, the writer focuses on exploring his or her initial ideas about the topic nonjudgmentally by capturing them on paper as they occur to make them visible because seeing ideas helps us discover relationships and organize thoughts. Such techniques as *freewriting* or *brainstorming* are commonly used during this stage. Both involve random listing of ideas as they occur during a set time, without correcting grammar, punctuation, or syntax. These "editorial processes" inhibit thinking about the topic and, in extreme cases, can result in an inability to write called *writer's block.* Formal investigation of the topic through reading and research usually occurs after prewriting. The process then proceeds into drafting, re-drafting, and revising of the document a number of times that depends upon the writer's unique situation. For example, discovering additional information about the topic may stimulate re-drafting, as may learning about the needs of the audience for whom the document is written. Finding that one has misunderstood a crucial concept may require revising it as well.

To use writing as a tool for learning math, the math professor controls the level of re-drafting performed by the writer (i.e., the math student). This control permits the professor to grade as much or as little of the grammatical and syntactical element as he or she may be comfortable evaluating. Ideally, these elements are minimized to maximize the focus on the student's coming to understand the mathematical concepts through individualized writing. Crucial to the success of this process is the professor's notifying the student of the expectations concerning the English mechanics. Rubrics for written assignments are one way of doing this. When they are used, a good rule of thumb for evaluating the English mechanics in non-English courses is that they should count for no more than one-third of the total points for the assignment.

In the MATH 131 courses in this research, the students' generally weak math backgrounds were known from pre-test scores required when they entered the university. Consequently, the writing instructor was quite lenient in evaluating English mechanics, announcing to classes that journal writings and short papers needed only to communicate the students' ideas "well enough" to be understood without ambiguity. Even in the formal reports required for final projects, the focus remained on mathematical and design elements with only the mechanical elements deemed most important for these classes listed in the rubric distributed with the final project assignment and used to grade the projects. These elements are highlighted in Figure 2 below.

Results and Discussion

Critical Thinking Tests. Critical thinking is extremely difficult to measure because it involves complexly intertwined skills that simultaneously cross the boundaries of many disciplines. In addition, "correct" answers may be highly subjective or context- or individual-dependent, making assessment slippery at best.[20] With these caveats in mind—and well knowing that many students have previously relied on rote memorization that may have inhibited developing critical thinking skills—the math professor and writing instructor hoped that the pre- and post-tests could suggest rough degrees of their students' abilities to think critically as well as suggest *types* of critical thinking skills present both before and after the course.

RUBRIC FOR MATH 131 FINAL PROJECT

Mathematics (50%):

Project effectively develops one of the math areas studied *(10 points)*............................... ________

Project includes a short typewritten report (1-2 pages) that explains the math used in the project and how/why the student developed the project as he/she did *(10 points)* .... ________

Project and/or paper develop the math ideas using at least two sources other than the math textbook *(10 points)*.. ________

Sources chosen are appropriate, authoritative, and are cited correctly *(10 points)*.............. ________

Organization flows logically and mathematical reasoning is sound *(10 points)*.................. ________

Creativity (25%):

Your idea expands the math used in your project in an *original* way *(15 points)* .... ________

You carefully select project components to emphasize the mathematics *(10 points)*.......... ________

Artistic Design (evaluated by an artist—25%):

Your project is professional in appearance with no spots, smudges, etc. *(5 points)*............ ________

Your project is attractive and eye-catching *(5 points)* .... ________

Project contents are arranged and spaced appropriately *(5 points)*..................................... ________

You use care and precision in the project's components, producing a work of high quality *(5 points)*... ________

Your project explains/shows its topic clearly, understandably to a general audience (i.e., delivers its message well) *(5 points)*... ________

Total Score .... ________

Figure 2. Rubric for Final Project and Accompanying Formal Report

The critical thinking pre-test that we used was a sample ACT Critical Thinking test question that was chosen for the likelihood that its scenario would be familiar and understandable by our students. We made minor word modifications to the original text to make it more comprehensible by non-native speakers of English. Both the sample question and the minor modifications we made were approved by ACT and used with their permission (see Appendix C). The pre-test consisted of eight multiple-choice questions related to a printed discussion about three ethical viewpoints regarding spending one's earnings by giving to charity. It required students to compare and contrast both actual and implied statements to determine the best of four possible answers for each question. Since the writing instructor monitored the test-taking, students could ask questions about words and phrases they did not understand. Consequently, this scenario-reading format could possibly help reveal language problems potentially contributing to students' reasoning problems. Except for one student who exited the test for unknown reasons within the first five minutes after it was issued, no students in any of the three classes asked for help with the language used, suggesting that English was not a significant impediment to the students' reasoning processes. Table 1 gives the pre-test results for each of the three classes.

Points scored / Class	0	12.5	25	37.5	50	62.5	75	87.5	100	Average Class Score
10:50 (11 students)	1	0	2	2	2	2	1	1	0	46.59
13:40 (11 students)	0	1	1	1	4	1	2	1	0	52.27
16:30 (12 students)	0	1	1	1	7	1	1	0	0	46.88
Total Students	1	2	4	4	13	4	4	2	0	34 students
% All Students	3	5.9	11.8	11.8	38	11.8	11.8	5.9	0	100.0%

Table 1: Critical Thinking Pre-Test Results

Of interest in these results are the extreme scores: no student scored 100, and only one student answered all questions incorrectly. That no student excelled in the test was expected, given the students' second language usage and various cultural backgrounds. In retrospect, the score of the student who missed all questions was predictable once the professors had worked with him long enough to discern his overall attitudinal problems with school. His abilities consistently seemed far greater than his performances indicated, and his attitudes toward time management and class attendance were unusually casual and undisciplined.

The average score for each class hovered around 50 percent, or roughly half of the questions answered correctly, which could be considered respectable and possibly "on track" scores. In each of the classes, the first four questions and Question 6 were least often missed (see Table 2 below). These questions involved actual statements made and more concrete elements of the arguments. Apparently, the students have some experience in evaluating truth and falsehoods as well because many answered Question 6 correctly. The last four questions were definitely harder, requiring one to consider elements and ideas other than those specifically given in the scenario. Significantly, most students missed Questions 5, 7, and 8, all of which required them to reason beyond the given elements of the arguments, signaling some problems with more advanced levels of critical thinking.

The post-test consisted of a single question on the final exam related to the math topics covered during the semester that required students to analyze, evaluate, and synthesize. The scenario described in general terms Gustav Fechner's discovery of the *Golden Ratio Hypothesis*, and three questions related to it required students to synthesize their knowledge about the scientific inquiry process with the topic (1) to analyze and identify a true statement, (2) to analyze and extend an argument to select a relevant statement, and (3) to compare/contrast and evaluate potential results of the research. Table 2 below shows the class-by-class results of this test.

Question Number/Purpose	Number of Students Who Missed A Given Question (Total Response Sets: 34)				
	10:50 Class	13:40 Class	16:30 Class	All Classes	
1. Extend a stated argument to its meaning	2	2	2	6	17.65%
2. Analyze the purpose behind an argument	4	5	4	13	38.24%
3. Analyze an argument by identifying one of its unstated assumptions	8	5	4	17	50.00%
4. Analyze an argument by identifying what it does NOT do or say	4	3	9	16	47.06%
5. Extend an argument to include a hypothetical position and evaluate its ability to weaken the argument	10	5	7	22	64.71%
6. Evaluate the truth or falseness of an argument to identify a reasonable objection to it	5	6	6	17	50.00%
7. Analyze and evaluate an argument to explain its relevance to the debate	8	7	9	24	70.59%
8. Compare/contrast three arguments to group their positions into "agree" and "disagree"	5	8	10	23	67.65%

Table 2: Numbers of Students Who Missed Each Pre-Test Question

Points scored / Class	Not taken	0	33.335	66.665	100	Average Class Score
10:50 (11 students)	N/A	2	4	0	5	57.58
13:40 (11 students)	N/A	0	0	4	7	87.88
16:30 (12 students)	1	0	3	1	7	78.79
Total Students	1	2	7	5	19	34 students
% All Students	2.94	5.88	20.59	14.71	55.88	100.0%

Table 3: Critical Thinking Post-Test Results

When we compare the pre-test results in Table 1 with the post-test results in Table 3, immediately we can see that students generally scored better on the post-test, with 55.88% of all students tested earning 100 compared with not one student scoring 100 on the pre-test. As much as we might like to claim so, unfortunately, this difference does not necessarily indicate the "success" of incorporating critical thinking writing assignments into the course. Rather, differences in students' health and moods on given days, as well as differences in the tests themselves possibly play significant roles in this overall outcome. On the other hand, because of its relatedness to the course material, the post-test may indicate that students' better understanding of the fundamental principles of the scenario that was presented in the test as a result of their having studied the material in MATH 131 enabled them to exercise better critical thinking skills.

To attempt comparing differences in students' critical thinking performances on similar types of tasks, we tallied numbers of students who missed each particular type of post-test question, as was done for the pre-test in Table 2 above. Those results appear in Table 4 below and illustrate a general strength for identifying an unstated assumption to identify a true statement. Mentally extending an argument beyond explicitly stated information was the most difficult task for these students.

Question Number/Purpose	Number of Students Who Missed A Given Question (Total Response Sets: 33)				
	10:50 Class	13:40 Class	16:30 Class	All Classes	
1. Analyze and identify a true statement by identifying one of its unstated assumptions	6	2	4	12	36.36%
2. Analyze and extend an argument to select a relevant statement	2	0	0	2	6.06%
3. Compare/contrast and evaluate potential results of the research	6	2	3	11	33.33%

Table 4: Numbers of Students Who Missed Each Post-Test Question

Next we compared the critical thinking tasks that students were asked to perform in each test. Although the critical thinking tasks necessarily differed because of the different scenarios involved, the required critical thinking skills were similar in the question sets shown in Table 5 below:

Pre-Test Question Number/Purpose (34 responses)	Post-Test Question Number/Purpose (33 responses)	% Correctly Answering Pre-Test	% Correctly Answering Post-Test
3. Analyze an argument by identifying one of its unstated assumptions	1. Analyze and identify a true statement by identifying one of its unstated assumptions	50.00	63.64
7. Analyze and evaluate an argument to explain its relevance to the debate	2. Analyze and extend an argument to select a relevant statement	29.41	93.94
8. Compare/contrast three arguments to group their positions into "agree" and "disagree"	3. Compare/contrast and evaluate potential results of the research	32.35	66.67

Table 5: Comparison of Pre- and Post-Test Critical Thinking Skills Requirements

As Table 5 shows, in each instance the percentage of students correctly answering the question type in the post-test exceeded the percentage of these students who had correctly answered the pre-test question requiring a similar critical thinking skill set. In addition, that the smallest percentage increase occurs in the earlier questions, which required lower-order cognitive skills, suggests that some gains may have been made in students' abilities to use cognitive skills ranking higher in order on Bloom's Taxonomy.

Short Papers. During the semester, One-Minute Papers were used primarily to provide the professor with feedback on students' misunderstandings and effectiveness of teaching strategies. In contrast, three Three-Minute Recollections were administered relative to the critical thinking component of the course. Twenty-three students completed all three of the Three-Minute Recollections, which intended to reinforce the concepts of individual learning preferences and of various thinking levels, with critical thinking emerging in the three highest levels of Bloom's Taxonomy. Each recollection was administered at the end of a class session without prior announcement. The questions in the first paper focused on the taxonomy materials covered in class and in handouts. The second paper reviewed the Super Link information and tested students' memory of their own Super Links. It was given after students should have completed a journal assignment requiring them to work with their individual Super Link results. The final paper required knowledge of both the taxonomy and Super Link information.

Of the 23 students completing all three papers, 14 students (61%) either improved their scores by the final paper or held constant their scores across all three papers. Table 6 below shows the breakdown of results by class section. Out of 12 students whose grades indicated progress by the last paper, five students, or 41.7%, earned progressively higher scores, suggesting that being required, unexpectedly, to complete the first Three-Minute Recollection may have been a wake-up call that influenced these students to pay increased attention to their handouts and in-class discussions about these supplemental topics. Among the seven students whose final score was the highest (58.3%), it is possible that they were pleased enough with their first score that they believed the material was easy and they did not need to study it. Having failed to study sufficiently, they performed worse on the second recollection, which possibly convinced them to review all of the supplemental materials before the third paper. Alternatively, because only one of the scores among the first and second papers in this group was zero, it is possible that chance or differences in students' individual interests related to the materials played roles in this group.

	10:50 Class (11 Students)		1:40 Class (11 Students)		4:30 Class (12 Students)		Student Totals	% Progress
No. Students Completing All Three Recollections	6	54.5%	8	72.7%	9	75.0%	23	
Students Earning Progressively Higher Grades	0		3		2		5	41.7
Final Grade is Student's Highest Grade	3		2		2		7	58.3
Constant Grade of 2/3	0		0		1		1	
Constant Grade of 3/3	0		1		0		1	
Progress Over 3 Papers	3/6	50.0%	5/8	62.5%	4/9	44.4%	12/23	52.0%

Table 6: Comparison of Students' Three-Minute Recollection Results By Class Section (Shaded cells indicate students who made progress by the final paper)

Journals. All students received ten journal assignments requiring use of analyzing, evaluating, synthesizing, and creating, among other cognitive skills. In addition to these ten assignments, honors students received an additional ten, more advanced, assignments (20 assignments total).

Each set of assignments involved just the topics being covered in that portion of the course, and students had about one month to complete each set. Table 7 below illustrates the cognitive skills required by the assignments: most assignments required several cognitive skills, so any given assignment number may appear in more than one column. The columns shaded in pale yellow reflect the numbers of chances students had to practice the skills identified with critical thinking. As these numbers show, students who successfully completed all journal assignments had ample practice in critical thinking skills; consequently, these writings were expected to be the primary or most influential tool used in the course for improving students' critical thinking, if any improvement were to occur.

Unfortunately, only five of the 34 students in the classes (one had withdrawn at mid-semester), or 14.7%, turned in all of the assignments. The honors students were not among these five. If the hypothesis that increased practice would better develop critical thinking skills were correct, these five students could be expected to outperform the other students on the post-test because they had the advantage of more practice in exercising critical thinking skills. This expectation was supported by the findings of Tsui, whose work positively associated critical thinking development with "substantive writing," among other teaching strategies.[21]

In reality, this is not what happened. Of the 34 participants remaining who took the critical thinking post-test, 11 of those who *did not* complete all of the journal assignments earned 100s on the critical thinking post-test (32.35%). On the other hand, just two of those 34 students both completed all journal assignments and earned 100s (5.88%). Clearly, the additional journal practice did not translate into higher post-test scores, which brings into question whether the students' critical thinking skills increased during the course.

	Analyzing	**Applying**	**Comparing**	**Creating/Synthesizing**	**Evaluating**
	1.1	1.1	1.1	1.4	1.1
	1.3	1.3-A	1.2	1.2-A	1.2
	1.2-A	3.3	1.3-A	1.3-A	1.3
	1.4-A	1.4-A	1.4-A	1.4-A	1.2-A
	2.1	2.1	2.1	2.2	2.2
	2.2	3.3	2.3-A	2.1-A	3.1-A
	2.3		3.1	2.3-A	
	2.1-A		3.2	3.1	
	2.2-A			3.2	
	2.3-A			3.3	
	3.1			3.1-A	
	3.2				
	3.3				
	3.2-A				
	3.3-A				
Total Assignments	15	6	8	11	6

Table 7: Breakdown of Journal Assignments by Targeted Cognitive Skills
(Shaded cells indicate the higher-order skills needed for critical thinking)

As previously mentioned, considerable debate exists regarding the effectiveness of critical thinking tests, what to test, and how to test it. According to the S.P.J.A. award-winning writer, researcher, and educator, Kathleen Northridge, some of the difficulties in evaluating critical thinking lie in the complexly interacting skills it involves and in its interdisciplinary nature.[22] In addition, the literature review conducted by Bataineh and Zghoul uncovered "conflicting findings in terms of the relationship between critical thinking and a host of other factors. . . . [Despite these conflicts, there] is now quite a consensus that students' critical thinking capacities can be improved through instruction and practice."[23]

Projects. The students were required to select from a list of predefined topics for their final projects, and no more than two students could choose the same topic. These topics included: Fibonacci Sequence; Golden Number; arithmetic sequence/geometric sequence; money (simple interest, compound interest, annual percentage yield); symmetry (reflections, rotations); fractals; graph theory (Hamilton graphs, Euler graphs); and the Konigsberg Bridge Problem (circuits, Eulerian cycle, Unicursal Circuit, traceable graph, multigraph). Students were required to develop their projects independently and to write a one- to two-page summary of how they developed the idea for the project and how they incorporated the relevant mathematics. This activity required considerable planning and hands-on work, and it was the most creative of all activities required in the course; consequently, it proved to be a good exercise in critical thinking that actively engaged the students while reinforcing the mathematics. Because these projects and reports were displayed in the annual Physics and Mathematics Day, when visitors from both inside and outside the university are invited to view the projects and to discuss them with their creators, correct spelling, punctuation, and grammar, as well as clarity in explaining the concepts were required in these reports. Students were encouraged to seek help with these papers in the Writing Center.

Conclusions

Throughout the course, two primary strategies demonstrated how mathematics professors, who usually are non-writing specialists, could incorporate writing into the math class comfortably and effectively. Although one of these strategies involved collaborating with a writing specialist that may not be available in every situation, emphasizing writing *as a learning tool* that relies more heavily on *freewriting* than on polished documents, can free both the professor and the student from the pressure of achieving "perfect English," thus distributing more energy to *thinking* than to grammatical considerations.

Similarly, using grading rubrics showing no more than 25-30 percent of the points attributable to English mechanics, reduces the anxiety of evaluating English for the mathematics professor and reduces the distraction of manipulating English precisely in a non-English course for the student. This happens because most of the grade weight rests on mastering mathematical concepts and manipulating them rather than on enforcing rules from a non-math discipline with which both student and professor may be uncomfortable and thereby distracted from the course objectives.

As discussed, the research on critical thinking has shown many difficulties with assessing these higher-order thinking skills, and the disappointing comparison results between students in this course who practiced all the prescribed journal writing versus those who did not is a case in point. Explicit reasons for this result cannot be ascribed with any degree of certainty. Perhaps one semester is simply too short a time in which to develop students' consistently present and accessible critical thinking skills. Certainly, future research is needed to investigate more deeply the use of writing in developing critical thinking skills.

Despite this problem, incorporating writing into the mathematics course significantly and intentionally increases the quantity and degree of critical thinking required of students. Additionally, this study describes a technique that can be adapted for use in engineering mathematics as well as in the mathematics used by designers, for example, as shown in this study, by targeting course objectives to the desired course outcomes and by focusing on the three upper-level cognitive strategies in Bloom's Taxonomy (*analyzing, evaluating*, and *creating*). Further, incorporating short papers, journal assignments, written reports, and creative projects actively and repetitively engage students with mathematical concepts, thus enhancing students' knowledge retention. Attempting to think about such concepts at the higher cognitive levels of critical thinking by using writing to help students capture, manipulate, and integrate new concepts with their own thoughts provides an increased degree of mental exercise as we consciously and frequently introduce and guide students into the deeper types of thinking that mark a well-rounded education and that distinguish the stronger, more effectively functional citizen in the modern world.

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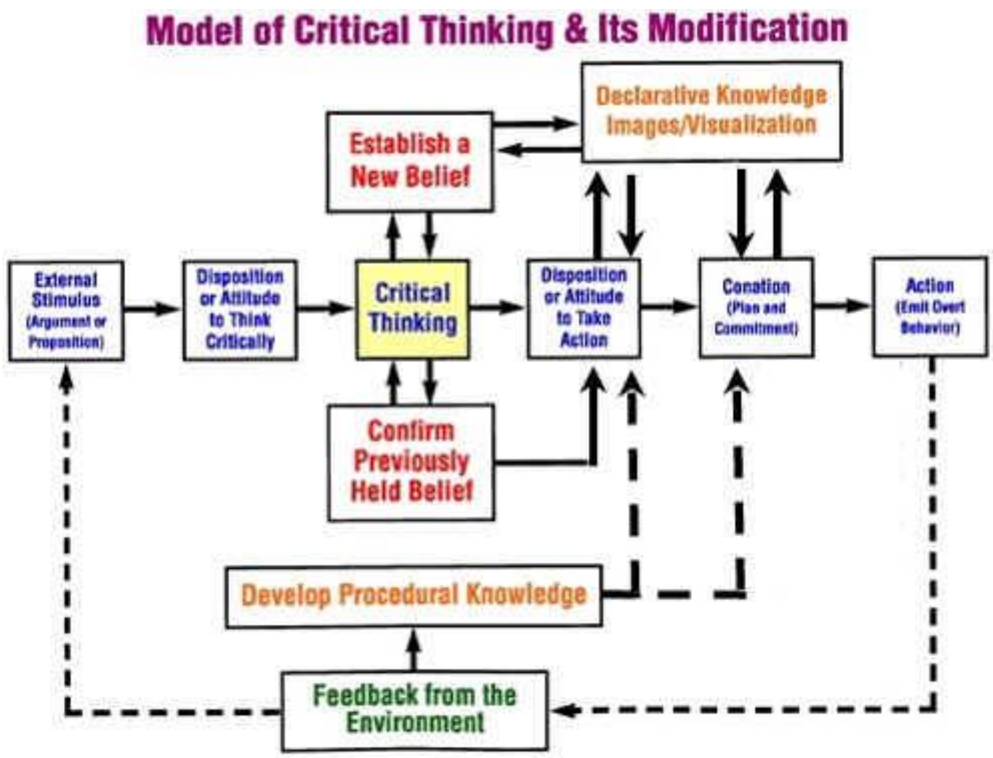
22. Northridge.

23. Bataineh and Zghoul.

APPENDIX A

Huitt's Model of Critical Thinking and Its Modification



This diagram is based upon critical thinking research conducted between 1991 and 1995 by Mertes, Scriven and Paul, Ennis, and Lipman that added the elements of beliefs, behaviors, and/or contexts to definitions of critical thinking. It supplements the cognitive processes of critical thinking with affective, conative, and behavioral aspects that also affect critical thinking. Beginning with the leftmost box in the diagram labeled in blue text *External Stimulus*, the individual receives information to evaluate.

To evaluate the stimulus, the individual must possess "an affective disposition to use critical thinking" that initiates the progression of thought (denoted by the yellow-shaded box labeled *Critical Thinking).*

At this point, one of two decisions is made (illustrated by the boxes labeled in red immediately above and below the *Critical Thinking* box): (1) *Confirm Previously Held Belief* or (2) *Establish a New Belief.*

If the previously held belief is confirmed, the individual behaves characteristically until additional critical thinking changes that belief, thus requiring a behavioral adjustment, at some future time. If a new belief is adopted, "[t]his will be established as a component of declarative memory in its semantic form although there may be episodic information associated with it. There may also be images or visualizations formed or remembered as part of the critical thinking process," noted by the rectangle labeled "Declarative Knowledge: Images/Visualization."

At this stage, the individual decides to act (i.e., to accept the new belief) and plan belief changes necessary to adopting the new belief as a behavioral guide. Goal setting and self-regulating are required to activate the action planned (i.e., the overt behavior emitted in the "Action" box on the right).

Once the individual acts, environmental feedback results in increased knowledge that is then available to guide the individual's actions toward the goal resulting from the newly accepted belief. Previous behaviors may be corrected as new situations require additional critical thinking to align the new belief with the individual's actions.

To effectively develop one's critical thinking, each component shown must be addressed, possibly using different teaching methods for each:

> For example, if one is most interested in impacting declarative knowledge (facts, concepts, principles, etc. that are stored in semantic and episodic memory), the most appropriate teaching method is probably some form of didactic, explicit, or direct instruction. On the other hand, if the focus is on procedural knowledge it is likely that modeling and/or personal experience would be more appropriate teaching methods. Likewise, if one were trying to impact the memory of images or visualizations, then modeling, active visualizations, or working with pictures might be more appropriate. Attitudes are probably impacted most directly by socialization and the teaching method of cooperative learning. Learning the process of critical thinking might be best facilitated by a combination of didactic instruction and experience in specific content areas. Impacting conation might best be done through goal-setting exercises and action learning. Finally, overt behavior and learning to use feedback might best be accomplished using positive and negative reinforcement.

Source: Huitt, W. (1998). Critical thinking: An overview. *Educational Psychology Interactive*. Valdosta, GA: Valdosta State University. Retrieved 1 Jan. 2012 from http://www.edpsycinteractive.org/topics/cogsys/critthnk.html. [Revision of paper presented at the Critical Thinking Conference sponsored by Gordon College, Barnesville, GA, *March, 1993*.]

APPENDIX B

Sample Journal Assignment Questions

1.1. Super Links: Analysis and Metacognition

Observe the teaching of any one of your professors **except Dr. XX** during two class periods and note what he/she does that fits in with your preferred learning style and hemispheric preference. Use the same professor for both class observations. In your journal discussion about this experience,

- state your preferred learning style and hemispheric preference,
- identify what the professor did to appeal to your "Super Link" (or failed to do if that is the case), and
- discuss alternative things the professor could have done to appeal to your "Super Link."

To get credit for answering this question, you must include answers to all three bulleted points in your writing.

1.2. Fibonacci Numbers: Comparing and Evaluating

Explain for an uninformed reader how each of these works: the ***recursive definition*** and ***Binet's formula.*** Determine which of these is better for finding Fibonacci numbers and defend your choice.

1.3. Money, Money, Money! (Analyzing and Evaluating)

For your birthday, your wealthy uncle gave you 15,000QR that you want to invest wisely for one year. You discover that Bank A will pay you 0.5% per month simple interest. Bank B, on the other hand, will pay 5% APR compounded monthly. Should you put your money into Bank A or Bank B and why?

1.4. Geometric and Arithmetic Sequences: Synthesizing/Creating

Design, draw, and color in your journal two original patterns. One pattern must use a geometric sequence, and the other must use an arithmetic sequence. Label each pattern and explain how you created it using the relevant sequence.

1.2-A (HONORS). Fibonacci Numbers: Thinking more deeply

Prove the Binet formula. (HINT: Use online research.)

1.2-B (HONORS). Fibonacci Numbers: Thinking more deeply

Scenario: You are painting apartment buildings. They vary in height: some are one floor tall, and others are two, three, four, or five floors high. On any one building, you paint each floor either yellow or red, but you never paint two consecutive floors (one above the other) red.

Questions:

- How many ways can you paint each type of apartment building?
- What do you notice about the painted buildings?

1.3-A (HONORS). Money: Thinking more deeply

Create a chart to compare simple interest, compound interest, and APY. You intend to invest 5,000QR for 10 years at 4% simple interest or at 4% compound interest. Calculate the APY for both the simple and compound interest for each year and include it in the chart.

1.4-A (HONORS). Geometric and Arithmetic Sequences: One Step Further

This assignment builds on the arithmetic sequence and geometric sequence patterns that you created in Assignment 4, so you will need to refer to them.

After creating your first two patterns, draw one more pattern using the same instructions as in Assignment 4, except this time use a Fibonacci sequence to generate the pattern. When this pattern is completed, compare it with your first two. What do you notice about the "speed" at which your pattern develops? Write about what you notice and why you think this happens.

2.1. Symmetry: Analysis

For this assignment, choose **ONE** of the following bulleted items and sketch or paste a picture of it with this assignment in your journal to help you visualize and think about the item:

- A picture of a place of worship
- A painting you like
- The pattern on a rug
- A drawing of a window display
- A magazine advertisement
- The capital letter Q

Think about the symmetry of your chosen object and write about it in your journal. As you write, include answers to the following questions:

(a) Is your item symmetrical? Why or why not?
(b) Reflect your item along an axis. What happens? Why?
(c) Rotate your original, non-reflected item. Does the result differ from the reflected item? Why or why not?

2.3. **Fibonnaci and Fractals: Analysis**

Explain the connections between the Fibonnaci Theory and the Fractal Theory.

2.1-A (HONORS). Symmetry: Analysis and Synthesis (Creating)

Design a pattern to which you can apply a glide reflection that is symmetric.

3.1. The Mandelbrot Set: Analysis

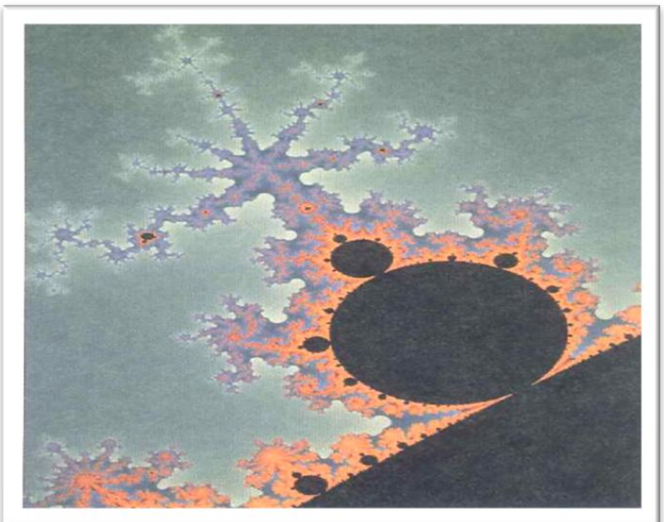
Understanding Mandelbrot sequences is very important for designers because, according to your textbook, "the Mandelbrot set provides a bounty of aesthetic returns for a relatively small mathematical investment . . ." (457).

Your chapter on the Mandelbrot set explains three especially important concepts related to it:

- **Escaping Mandelbrot Sequences,** in which the terms get larger quickly, rapidly moving away from the origin. These terms are *not* in the Mandelbrot set, so we give them nonblack colors: "hot colors" (red, yellow, orange) for more slowly escaping sequences, and "cool colors" (blue, purple) for more rapidly escaping sequences.

- **Periodic Mandelbrot Sequences,** which are numbers in the sequence that, at some point, begin to alternate between 0 and -1 in a cycle. In these sequences, the seed is a point of the Mandelbrot set, so it is colored black.
- **Attracted Mandelbrot Sequences,** in which the terms of the sequence get progressively closer to a fixed or constant number that is the *attractor* of the sequence; alternatively, we can say that the sequence is *attracted* to that number. The seed is a point of this Mandelbrot set, so it is colored black.

Using the information given above, examine Figures 12-23(c) and 12-24(a) from your textbook, which are shown below, and analyze the colors of each to identify its probable escaping, periodic, and attracted sequences. **Include in your discussion what you think the color green means in Figure 12-24(a).**



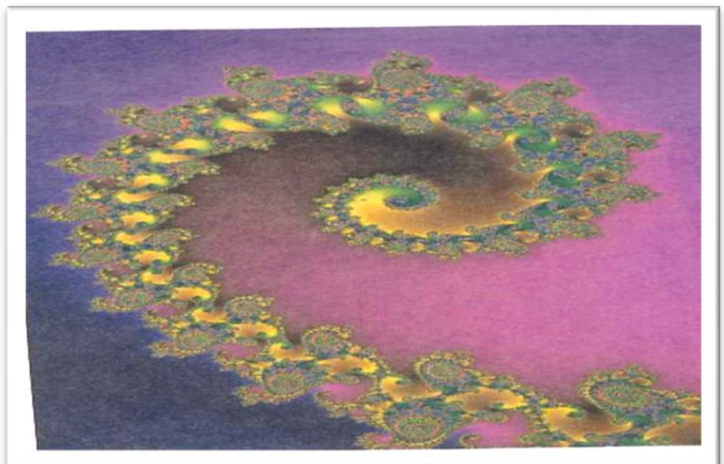


Figure 12-23(c) (Page 452)	Figure 12-24(a) (Page 453)

3.3. Graph Theory: Application, Analysis, and Synthesis

(1) Draw an original graph; be sure it has AT LEAST
- One Euler Circuit and
- One Hamilton Circuit.

(2) Label the vertices and give the labeled path for each circuit. (Be sure to identify each circuit as either *Euler* or *Hamilton*.)

(3) Assign monetary values of your choice to all of the edges in the graph. Use Krushkal's Algorithm to find <u>the minimum cost </u>to ship a DHL box to all vertices. (Note that the lease expensive route may not necessarily be the Hamilton circuit that you just created.)

3.1-A (HONORS). Geometric Fractals: Synthesis and Creativity

Invent a new geometric fractal using geometric figures such as rectangles, trapezoids, hexagons, etc., BUT NOT CIRCLES. Measure ANY THREE from the following list of measurements contained in your new fractal:

- The number of sides or edges
- The number of geometric figures
- The length of one new side or edge
- The length of one new geometric figure
- The length of all new sides or edges
- The length of all new geometric figures
- The area of one new shape
- The total area of all new shapes
- The area of one new geometric figure
- The total area of all new geometric figures

Using the three measurements chosen above for your new fractal, determine if the sequences are attracting or escaping.

APPENDIX C

Critical Thinking Pre-Test Scenario and Question Set

(Used with permission)

Passage:
Three friends, *Keepit, Givit,* and *Wait,* are discussing whether to make regular voluntary donations to charitable organizations.

Keepit: I should not contribute to charities. What good would it do? My small contribution would never be noticed in a charity's million-dollar budget. But that same amount of money would be very noticeable if kept in my own family budget; that's where it makes the biggest difference and where it does the most significant good. In any case, our first moral obligation is always to the well-being of our own families. My family would rightly resent my favoring strangers over them. Given my level of income, any money of mine that is not needed for their present well-being should be saved for their future.

Givit: People have a right to have their most basic needs satisfied. Rights involve obligations. So anyone who has more than enough money to satisfy his or her own basic needs has a constant moral obligation to help meet the most basic needs of others. Consequently, we are each morally obligated to contribute to charities, and to refuse is blameworthy.

Wait: I haven't decided what to do yet. I agree that it is good to contribute; still, it is not morally required for us. Our money comes from wages we earn by our own labor, using our own abilities. And so long as people don't use the money to harm others, they are entitled to put earned wages to whatever use they choose. People who contribute hard-earned money to charities deserve praise. But no one should be blamed for not contributing such money.

Givit: Some people are not as lucky as you: their abilities are fewer, or their legitimate needs are greater. For example, some people are born with serious physical or mental disabilities; others require expensive medical treatments. Why should they suffer for such accidents of fate? When our economic system provides you with luxuries while failing to meet their most basic needs, you are getting more than your fair share. I'm not saying that money should be taken from you by force, but I am saying that you have a constant moral obligation to help right such wrongs.

Wait: Your principles go too far. Suppose we do have a constant moral obligation of the kind you describe. Then even if people act morally, they will find themselves with a continuing obligation to *keep* giving until they can just barely satisfy their own most basic needs. Be honest. We are all planning to buy season football tickets, which are not basic needs. Do you think we are obligated not to buy the tickets and give the money to charity instead?

Keepit: The economic system may treat some people unfairly, but that does not mean that I am required to help them at the expense of my own family. In our country, the wealthiest 5 percent own 35 percent of the country's wealth, so obviously they have more money than they can use for their own families. And if they would contribute just a tenth of that wealth, charities would have all the money they need. Thus, there is no need for ordinary people like us to contribute, so there is no obligation.

1. Keepit's stated principles demand that:
A. people who have no families have no moral obligations.

B. Keepit is not morally required to contribute earned wages to charities.

C. every action is worthy of either praise or blame.

D. it is fair to pay people on the basis of their abilities as well as their labor.

2. Wait states that so long as people don't use the money to harm others, they are morally entitled to put earned wages to whatever use they choose. In making this statement, Wait is:
A. trying to establish that Wait, Keepit, and Givit are not morally required to contribute to charities.

B. trying to establish that it is good to contribute to charities.

C. trying to establish that charities should receive money from sources other than earned wages.

D. arguing against Wait's own claim that those who contribute hard-earned money to charities deserve praise.

3. Givit's argument assumes, although it does not actually state, that (circle a *letter—not a Roman numeral)*:
I. Keepit, Givit, and Wait each have more than enough money to satisfy their most basic needs.
II. contributing to charities is a way to help some people satisfy their most basic needs.
III. if people refuse to contribute to charities voluntarily, governments should force them to contribute.
A. I only

B. II only

C. III only

D. I and II only

4. Keepit states that the truly wealthy have more money than they can use for their own families. Which of the following is **NOT** true of Keepit's statement?
A. It is part of Keepit's attempt to establish that there is no need for ordinary people to contribute to charity.

B. It is consistent with Keepit's claim that charities would have all the money they need if the truly wealthy would contribute one-tenth of their wealth.

C. It supports Givit's claim that we each have a moral obligation to contribute to charities.

D. It is part of Keepit's attempt to prove Givit wrong.

5. Which of the following, if true, would most substantially weaken *Keepit's* argument for not contributing?

A. Keepit does not really want to contribute.

B. If Keepit were to contribute, the contribution would go entirely to a needy family who otherwise would not have received assistance.

C. Rights involve responsibilities.

D. The wealthiest 5% own much more than 35% of the country's wealth.

6. Keepit's argument for the conclusion that there is no need for ordinary people to contribute to charity is subject to a reasonable objection on the grounds that:

A. the wealthy must spend some of their money on their own families.

B. it may not be possible to persuade the wealthiest 5% to contribute one-tenth of their wealth to charity.

C. Keepit assumes that the wealthy have not earned their wealth.

D. the conclusion is not relevant to Keepit's main point.

7. Which of the following best explains why Wait's reference to the season football tickets is relevant to a logical evaluation of Givit's argument?

A. It implies that Givit is a hypocrite.

B. It illustrates a possible consequence of Givit's position concerning the extent of the obligation to help those in need.

C. It demonstrates an inconsistency in Givit's position concerning one's obligations to one's family.

D. It indicates that Givit overestimates the willingness of others to join with him in giving substantial charitable aid.

8. Keepit and Wait clearly agree, while Givit clearly denies, that:

A. Keepit should not be blamed for refusing to contribute.

B. contributing to charities is an ineffective way to help those in need.

C. the first moral obligation is to one's own family.

D. the present economic system is fair.

APPENDIX D

Critical Thinking Post-Test Question

10. *Read the following scenario to answer the three questions that follow. Circle the letter of your answer choice for each question.*

Psychologists and psychobiologists have long believed that the golden ratio plays a major role in humans' perceptions of beauty. To test this idea, in the late 19th Century Gustav Fechner displayed many rectangles of different proportions to hundreds of people and asked them to choose the most pleasing ones; he discovered that a large majority of people preferred the rectangles with proportions closest to the golden ratio. Today we know Fechner's discovery as the *Golden Ratio Hypothesis.*

I. Based on what you have read above, which one of the following statements is true?
   A. Fechner established the Golden Ratio Hypothesis as a fact.
   B. While Fechner's research is important, it is not conclusive.
   C. The Greeks' use of the golden ratio influenced the research of psychologists and psychobiologists.

II. Based on the description of Fechner's research study given above, we can say that . . .
   A. his study was limited because he investigated people's reactions to rectangles without including other shapes and objects.
   B. we consider people with perfectly symmetrical facial features to be more beautiful.
   C. his discovery should be renamed the *Golden Ratio Rule.*

III. The term *Golden Ratio Hypothesis* suggests that
   A. Fechner conducted his research scientifically by using a large number of people.
   B. Fechner's work proved his claim that the golden ratio influenced people's perceptions of beauty.
   C. The idea that people think that shapes and objects using the golden proportion are more beautiful was supported, but not proven, by Fechner.