AC 2012-3925: STEPPING STONES IN LEARNING ELECTROMAGNETICS

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Abstract

The theory of *Electromagnetics* is very elegant from the perspective of physicists, but most students consider it abstruse and messy. This phenomenon can be explained with an analogy: Imagine that *Electromagnetics* is a huge tree, and Maxwell's equations are equivalent to its stem. Physicists look at the tree from inside, and all the branches are derived from the stem with perfect order. On the other hand, students look at it from the outside, what they see is a messy collection of leaves. This shift of perspective from outside to inside is rather difficult, and there are many obstacles to overcome, such as new concepts and new approaches, as well as advanced mathematics: vector calculus. We found that it is helpful to start with something familiar to the students, and then progress gradually from the perimeter to the center. In addition, with the emphasis on the similarities between different areas, the knowledge learned in the past can serve as stepping stones for the new knowledge.

I. Introduction

Electromagnetics is widely considered as a very difficult course, and students often get lost at the beginning. There are several challenges: mathematics, imagination, as well as new concepts and approaches. First, vector calculus is the language of *Electromagnetics*, and many students have poor background in this subject.[1] Second, most students can visualize the motion of particles, but they have a hard time imagining the spatial distribution of the EM field.[2] Third, a number of new concepts and theorems are introduced in a short period of time, as well as new approaches to solve problems.[3] One way to overcome these challenges is to engage new technology.[4] There are very powerful full-wave 3D simulation software packages, such as ANSYS HFSS™ and CST Microwave Studio™; on the other hand, there are also low cost 2D simulators, such as QuickField™. However, we also noticed that there are side effects, and the CAD software can also be a distraction to some students.[5]

There are two different approaches in achieving proficiency in an area: The first is the external approach by getting familiar with various experimental results, and the second is the internal approach by deriving theorems from the fundamental principles. The first approach was engaged in research and discovery, while the latter one is emphasized in teaching, as it is much more efficient. The advancement in physics is accompanied by the progress of unification.[6] For example, Newton unified celestial and terrestrial objects, Faraday related electricity and magnetism, Maxwell identified light as electromagnetic wave, and Einstein discovered the relationship between energy and matter. Without this unification scheme, science would become a collection of scattered disciplines and scientists would be reduced into artisans.

The similarity between different fields can be divided into three levels. The first is at the phenomenological level, such as the wave characteristics of sound and electromagnetic waves. The second is at the mathematical level, such as the second order differential equation used to describe the spring-mass system and the LC oscillator. The third is at the philosophical level, such as the phase transition phenomena in different systems. Our emphasis in this paper is at the mathematical level, but the implication can be reflected at the phenomenological level.

II. Kinematics vs. Electrostatics

In electrostatics there are three closely related variables: charge density, electric field and potential. In many applications one needs to deal with its one-dimensional solution, such as in a *pn* junction in semiconductor devices. Under such a circumstance, the divergence and gradient operators can be converted to the simple derivative. In this way, the mathematical formulas are rather close to the kinematic equations relating acceleration, velocity and displacement.

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}, \quad \vec{E} = -\nabla V \quad \Rightarrow \quad \frac{dE}{dx} = \frac{\rho}{\varepsilon}, \quad E = -\frac{dV}{dx} \quad \Leftrightarrow \quad \frac{dv}{dt} = a, \quad v = \frac{dx}{dt}$$

In a simplified model of *pn* junction, the charge density can be assumed to be constant in the space charge region. Its counterpart in kinematics is the motion with a constant acceleration, such as free fall. With the help of this familiar example, students can analyze the *pn* junction problem with a similar approach.

III. Electrostatics vs. Magnetostatics

Compared with electrostatics, magnetostatics is a little more challenging, as it has a vector potential. However, there are also similarities between the equations for the potentials and the fields. After learning electrostatics, students can take advantage of such similarities to learn magnetostatics. In the following diagram three equations governing charge/current, field, and potential in these two areas are contrasted, where R' stands for the distance from a point in space to a point in the source, and V' stands for the volume of the source.

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon}\int \frac{\rho}{R'} dV' \qquad \Leftrightarrow \qquad \vec{A}(\vec{R}) = \frac{\mu}{4\pi}\int \frac{\vec{J}}{R'} dV'$$

$$\vec{E}(\vec{R}) = \frac{1}{4\pi\varepsilon}\int \hat{R}' \frac{\rho}{R'^2} dV' \qquad \Leftrightarrow \qquad \vec{B}(\vec{R}) = \frac{\mu}{4\pi}\int \frac{\vec{J} \times \hat{R}'}{R'^2} dV'$$

$$\nabla^2 V(\vec{R}) = -\frac{\rho}{\varepsilon} \qquad \Leftrightarrow \qquad \nabla^2 \vec{A}(\vec{R}) = -\mu \vec{J}$$

The dipole model is very useful in electromagnetics, and there is also a similarity between the electric dipole and the magnetic dipole, as well as their potential and field distributions. In the following diagram the expressions of the potential and field are contrasted, where *p* and *m* stand for the electric and magnetic dipole moments, the similarity is striking.

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon}\frac{\vec{p}\cdot\hat{R}}{R^2}$$
$$\vec{E}(\vec{R}) = \frac{1}{4\pi\varepsilon}\frac{p}{R^3}(\hat{R}2\cos\theta + \hat{\theta}\sin\theta)$$

$\Leftrightarrow$

$$\vec{A}(\vec{R}) = \frac{\mu}{4\pi}\frac{\vec{m}\times\hat{R}}{R^2}$$
$$\vec{B}(\vec{R}) = \frac{\mu}{4\pi}\frac{m}{R^3}(\hat{R}2\cos\theta + \hat{\theta}\sin\theta)$$

Compared to the vector potential, the scalar electric potential distribution in electrostatics can be imagined more easily, and it can also be conveniently shown using the results of simulation from a CAD software package. With this scalar field as guidance, one can figure out the distribution of the vector potential of the magnetic field.

IV. Plane Wave vs. Transmission Line

There are striking similarities between a plane wave propagating in a uniform isotropic medium and the EM wave traveling in a transmission line, such as a coaxial cable. First, they share the same format of differential equations; second, the impedance of the medium can be defined in a similar way; third, the formulas of the transmission and reflection coefficients are identical. Take the example of a lossless medium, these similarities can be shown in the following diagram. The symbols of Γ and τ in these equations are the reflection and transmission coefficients, respectively; L' and C' in the group of equations for a transmission line stand for the specific inductance and capacitance, respectively.

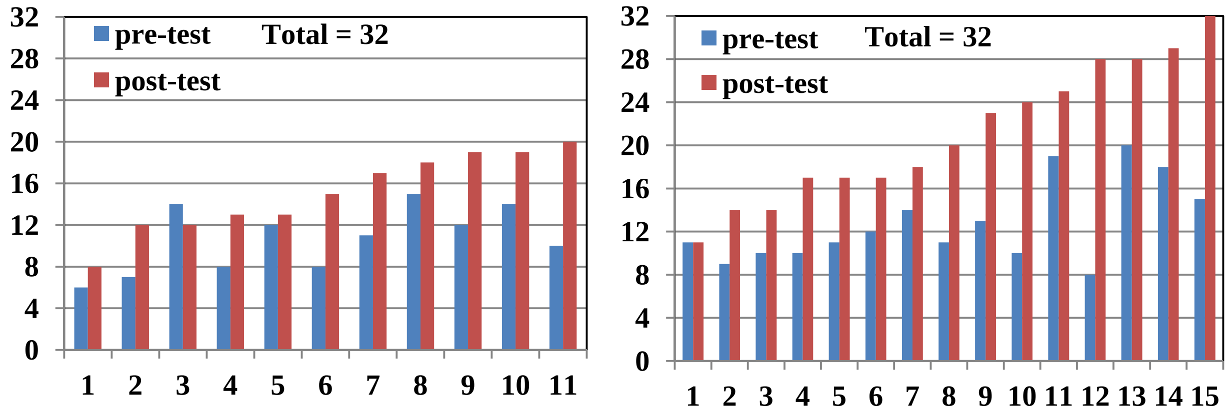
$$\frac{d^2E}{dz^2} + \omega^2\mu\varepsilon E = 0$$
$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$\Leftrightarrow$

$$\frac{d^2V}{dz^2} + \omega^2 L'C'V = 0$$
$$Z_0 = \sqrt{\frac{L'}{C'}}$$
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$
$$\tau = \frac{2Z_2}{Z_2 + Z_1}$$

V. Assessment

In order to assess the result of this course objectively, the *Conceptual Survey of Electricity and Magnetism* (CSEM) was adopted. This survey has 32 questions, and it covers electrostatics, magnetostatics, and the interaction between time-varying electric and magnetic fields. Due to relatively low enrollment, this course is offered every other year in our department, and the two most recent results are shown in Fig. 1. In fall semester of 2009, 11 students took the survey, and the average scores of the pre-test and post-test are 10.6 and 15.1, with 21.0% improvement. In fall semester of 2011, 15 students took the survey, and the average scores of the pre-test and post-test are 12.7 and 21.1, with 43.5% improvement. The data was also analyzed with two-tailed *t*-test, and the *p*-values are 1.55×10^{-4} and 2.65×10^{-5}, respectively. This result shows that there is a significant increase of the comprehension level on these concepts.



(a) Fall 2009 (b) Fall 2011

Fig. 1. CSEM assessment results in fall 2009 (a) and fall 2011 (b).

By taking advantage of the analogy between concepts and theorems in different areas, considerable amount of time and effort can be saved in lectures. In Fall 2011 four more sections on plane wave, polarization, and transmission line were covered. Furthermore, at the beginning of each lecture a short video clip was shown to the students so that they can frame themselves in the right background; in addition, this can also arouse their interest in possible applications. On the other hand, the survey result in Fall 2009 also exposed a number concepts that most students did not understand very well, such as the shielding of electric charge inside a metal enclosure. In Fall 2011 these concepts were emphasized in lectures, and students also investigated some of them by doing simulations.

VI. Conclusion

One of the challenges for students in learning Electromagnetics is that many new concepts and theorems are introduced in a very short period of time. This problem can be mitigated by emphasizing the similarities between the new concepts and the ones previously learned, and building new knowledge on the basis of what is already known. Fortunately, the theory of Electromagnetics is very elegant, and there is symmetry between electric and magnetic fields, except that there is no magnetic counterpart of electric charge. In addition, we can also find similarity between electrostatics and kinematics, as well as between the propagation of EM plane wave in a uniform isotropic medium and the traveling of an EM wave in a transmission line. The assessment result shows that this is an effective way of learning Electromagnetics, and this integrated knowledge structure can also resist the relentless erosion of time.

Reference

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