Synchronous Machine Winding Layout & Flux Animation Tool

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Abstract

This paper describes the development and application of a tool created in MathCAD® to illustrate the internal workings of a synchronous machine on video. Upon receiving a set of parameters and preferences, the tool creates an interactive animation of the currents, magnetic flux, and physical rotation of the machine. The tool even recommends the best settings to obtain a movie that loops to simulate continuous rotation in a fast or slow motion. This enables the student to see what a finite element program might reveal about a synchronous machine but requires only the same MathCAD® readily available on a university-wide license.

The program has an attractive level of sophistication. For example, its inputs are the following: physical dimensions, number of poles and slots, air gap width, frequency, positive and negative sequence inputs, appropriate motor or generator sign convention, power and torque output, resistances, reactances, simulation time and time step, and calculation density. The tool then builds and displays an appropriate physical cross-section diagram of the stator created from the given dimensions, showing a correct number of slots with their windings properly pitched. It calculates a complete lumped parameter d-q model, displaying its particulars on request. The tool then creates the rotor from the given dimensions, with windings appropriately placed, pitched, and skewed. The rotor has a calculation burden appropriate for the animation.

The program calculates flux linkage, illustrating it as an animated distribution of arrows similar to the manner in which finite element programs show this. The density of the points for flux linkage analysis and the information that their arrow distribution shows is another user-controlled option, based on assigned permeabilities and calculated topology. Calculations are based on a Biot-Savart model of the magnetic, consistent with Maxwell's equations. The program updates these with every time step as the machine rotates. A plotting algorithm, specially developed to illustrate the results, shows an animated illustration of the machine's currents, magnetic fluxes, and rotation.

The tool calculates a companion phasor diagram to help explain the machine's state and the interaction of its voltages, currents, and lumped parameter model of internal behaviors. This collaborating phasor diagram can be displayed next to the animated cross section illustration of the machine. Animation controls, as with all inputs and controls, are available on a graphical user interface.
The tool is intended not to replace finite element analysis, but to provide an illustration of appropriate results gained at no extra cost to the college student who is already using MathCAD®. It is appropriate for use in classroom and instructional settings and likely pertains to the college undergraduate or graduate levels. In presenting this paper, we will show the animation, we will provide assessment data from use with students, and we will provide a link for download of the tool.

We have employed this program in college classroom demonstrations, but not for student use on projects yet. Student reaction, taken anecdotally so far and not rigorously sampled, enthusiastically supports the program’s value for illustrating machine behavior and for gaining familiarity with the output of a finite element program. Faculty reaction to this program has likewise been quite enthusiastic, reinforcing with comments its value as a means to illustrate elementary synchronous machine functions without the price tag of a finite element software package.

The Overall Objective

The objective of this Animation Tool is to assist the studying engineer in understanding the electrical and magnetic interactions of a 3 phase synchronous machine. This educational motive is achieved by permitting a simple way to enter Synchronous Machine (S/M) parameters and immediately see interactive results all in the familiar software environment of MathCAD®. A visual depiction of an axial cutaway view of a S/M is automatically produced and is accompanied by a corresponding phasor diagram. To further enhance S/M understanding, the tool visually depicts changes to electrical and magnetic parameters as the user gradually advances through full electrical cycles or utilizes the incorporated assisted animation tool. It was the desire to learn that prompted the development of this tool, and it is the desire to inform that keeps it going, hopefully in an enjoyable way.

Classroom objectives obtained from the tool may include an understanding of winding layout in conjunction with various pole, slot, and pitch combinations. Numerous arrangements could be presented quickly to permit efficient use of classroom time to illustrate the basics of S/M physical layout. Once animated, objectives may focus on how a three phase system produces a balanced, rotating magnetic field. Students can see how the stator-produced rotating field interacts with the rotor magnetic field to yield a range of power output levels. The phasor diagram provides performance feedback as the user changes machine parameters, thus permitting another set of related objectives. A variety of classroom objectives can be drawn to ultimately result in enhanced student understanding of the 3 phase S/M. Here, we have only touched on the most basic objectives, but I believe many more will be extracted as the reader explores the details.

Overview

This paper begins as a user’s guide to familiarize the operator with what inputs are necessary and what results can be expected. The intent is to get up and running as quickly as possible. Once
familiar, those who are curious about how the tool works or wishing to make modifications can proceed to the sections that explain the tool’s background operations.

First, user inputs of S/M parameters are covered followed by an explanation of the derived graphical depiction and phasor diagram. This includes a description of interactive user controls and animated video production. The behind the scenes explanation follows to describe how user-specified parameters are transformed into a rotating 3 phase S/M, one component at a time.

I. S/M Parameter Entry: User Settings & Auto-Validation

The MathCAD®-based S/M illustration tool opens to present the user with customizable settings from which the machine and phasor diagrams are derived. These include S/M Physical Layout, Electrical Parameters, and Flux Analysis Settings. A description of each follows.

Beginning with the S/M Physical Layout (Figure 1), compatible values for Slots and Pitch are presented as the user specifies Poles, then Slots, then Pitch. This guides practical parameter specification and provides immediate feedback when settings do not correlate. Additionally, Air Gap distance is adjustable but only affects the way the rotor is drawn; no other values are determined from the Air Gap distance for this illustrative tool.

![S/M Physical Layout](image)

**Figure 1**

Electrical Parameter specification immediately follows to allow selection of start time and step time per electrical phase angle advancement, phase sequence and frequency, sign convention, motor output, and internal resistances and reactances. See Figure 2.
If a 2-pole S/M is specified, a Start Time of zero ensures it is aligned with the phasor diagram, i.e. the physical rotor and rotor magnetic north will be aligned with the direct (+d) axis depicted by the phasor diagram. This is an intentional characteristic of the program in order to assist in understanding the connection between phasors and physical machine. Time Step defines how far to advance, in seconds, each calculated snapshot of the S/M in motion.

Positive Sequence (a-b-c) electrical phases are used by default; Negative Sequence (a-c-b) can be selected. Also, Motor Convention is used by default such that positive current corresponds to producing power and torque at the rotor. Generator Convention is optional and simply reverses the direction of positive current such that it is entering the S/M from the rotor. It should be mentioned that typical S/M analysis will reverse the positive flux linkage direction in combination with reversed positive current direction (Generator Convention) in order to keep the right-hand-rule relevant. However, because the primary textbook of reference did not reverse the flux linkage direction in Generator Convention, it is not done in this tool. Therefore the “left-hand-rule” applies!

Finally, internal machine parameters are specified to largely contribute to phasor diagram development. These parameters will affect rotor angle with respect to electrical phase position in the S/M diagram. For example, the less power a motor produces, the more aligned the rotor will be with the driving flux linkages; and conversely, a motor producing substantial power will show a significant angle between stator-produced and rotor-produced flux linkages. Plotted current and flux linkage magnitudes are affected but also are adjusted in order to achieve the goal of illustration. All machine characteristics are in per unit with terminal parameters represented by complex vectors.

Flux Analysis Settings (Figure 3) permit user specification of where to calculate flux linkage direction and magnitude. For typical demonstrative/educational Pole, Slot, and Pitch settings, these values can remain unchanged and still provide a favorable flux linkage display. However, for unusual machine parameters, such as a S/M with 48 slots or more, a user may desire to adjust
these settings for a more favorable display. Flux Analysis presents the greatest calculation burden of this tool, so settings may also be utilized to take advantage of the fastest processor or customized for the slowest.

Flux Analysis Settings

<table>
<thead>
<tr>
<th>Setting</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ψ_radials_per_slot</td>
<td>Number of lines per slot along which flux will be calculated</td>
</tr>
<tr>
<td>ψ_inner_start</td>
<td>Radial distance from the S/M center to the innermost point at which a flux line is placed</td>
</tr>
<tr>
<td>ψ_step</td>
<td>Radial distance between flux line analysis points</td>
</tr>
<tr>
<td>ψ SCALE FLUX</td>
<td>Adjust the flux arrow size. Typical values are 2 to 5.</td>
</tr>
<tr>
<td>Use Air Gap Permeability</td>
<td>By default the air gap is ignored during flux calculation; check the box to incorporate it</td>
</tr>
</tbody>
</table>

Figure 3

Flux Linkage is analyzed along several points on a radial grid. The radial lines of analysis are concentric with the S/M plot and the user can specify how many radials of analysis to calculate per slot. The inner-most distance from the S/M center to begin flux linkage analysis is specified by ψ_inner_start where the center is 0 and the outer edge of the back iron is 100. ψ_step is the distance between analysis points, and ψ SCALE FLUX allows adjustment of the flux linkage arrow sizes to permit the best illustration. The last option permits user selection to consider the low permeability of the air gap when determining flux linkage magnitude. Selecting this results in noticeable attenuation of flux linkage in the air gap but does not affect flux linkage between the ends of the rotor poles and stator.

II. S/M Illustration and Phasor Diagram

Immediately after all user settings are entered, the program will produce the S/M and Phasor Diagrams as in Figure 4. This worksheet area permits simplified user interaction with an array of command buttons for time-stepping and phasor diagram interaction. For now, we will skip the small Animation portion of the worksheet which appears just before these diagrams when expanded.

Synchronous Machine Diagram

For demonstration purposes, a 2-Pole, 12-Slot, 5/6 Pitch S/M was built and reset to time zero. For all S/M representations, the stator is automatically oriented such that the positive A-phase current axis lies at the 3-o’clock position.

Winding colors red, blue, and green correspond to A, B, and C phase currents, respectively. The same color layout can be found in the small current plot immediately above the S/M plot. The vertical black bar in that plot indicates the instantaneous current magnitudes for the time displayed. In this case, time zero is shortly after phase b reached its negative peak value. As time is stepped forward, the sinusoidal current waves will shift left as the black bar remains fixed.
The winding radii in the plot indicate current magnitude, so it can be seen that phase B (blue) is nearly at maximum magnitude, while phase A (red) is not quite at its peak value, and phase C (green) is at a lower value. Dots and X’s are used to illustrate current direction either out of or into the screen, respectively.

Although smaller, the rotor has fixed windings also with indicated current direction. Right-hand-rule application will reveal flux direction and corresponding rotor pole orientations. In the example shown, rotor-produced flux is oriented at the 8 o’clock position.

Additionally, the plot header shows instantaneous current magnitudes, depicted time, rotor mechanical angle ($\theta_{rm}$), and torque ($\tau$). All values are per unit except rotor angle, and the reported torque will indicate whether the S/M is “Motoring” or “Generating.”

Purple arrows on the plot indicate stator-produced flux direction and magnitude. Together, these arrows show how the stator windings produce a rotating magnetic field having the specified number of Poles. It can be seen that the orientation of this stator-produced magnetic field compared to the orientation of the rotor-produced field, which maintains peak intensities fixed at the rotor pole ends, illustrate the difference in motor or generator operation. More closely aligned fields correspond to low torque in or out. If user specifications result in the fields spreading apart, more torque is involved in or out as applicable. Correspondingly, total magnetomotive force (MMF) is illustrated by the offset, orange-circular ring shape on the plot. An MMF point value of zero would lie directly on the inner-most point of the stator iron.

Positive peak MMF points touch the outer edge of the stator while negative peak values approach the center of the entire plot. So in Figure 4, maximum MMF magnitudes are produced near the 1 and 7 o’clock stator positions at the time depicted.

Phasor Diagram

The Phasor diagram presents A-phase terminal voltage, current, and flux linkage as well as internal voltage and impedance. The diagram depicts the d and q-axis, and d-q components of the A-phase current, impedance, and flux linkage. The +d-axis is aligned with the physical rotor and rotor-produced magnetic field while the +q-axis leads this by 90°. Certain vectors are drawn offset to illustrate additive properties of vector components. For example, offset vectors show flux linkage relationships such as:

$$\psi_d + \psi_q = \psi_{af} + \psi_{aarm} = \psi_a$$

Winding vs. Phasor Diagram

The intent is to view the S/M and Phasor diagrams side-by-side for enhanced interpretation. Together, both diagrams present terminal parameters and complex power used by motoring or produced by generating. Convention is clearly displayed. If a 2-Pole motor is drawn and reset to time 0, the Phasor Diagram +d-axis and Rotor will align exactly. Also, the Peak Flux phasor will align with the positive peak of the Total MMF ring on the S/M diagram, and A-phase current will be in the electrical phase position as indicated by the Phasor diagram. Only the S/M diagram changes with time, while the Phasor Diagram remains constant.
S/M and Phasor Diagram User Controls

A set of controls appears above the S/M diagram as shown in Figure 5. The Manual Time Stepper buttons allow the user to reset the calculation time to what the user has entered as START_TIME (0 ms typical) or step backward or forward through time at the user-specified time step interval (tstep). The Phasor Diagram Controls permit panning up, down, left, or right and zooming in or out. A Reset button returns the diagram to default settings.

Assisted Animation

Since a major purpose of this illustrative tool is to show the electrical inner-workings of a 3 phase S/M in motion, the program takes full advantage of MathCAD® Animation capabilities. The “Animation” section (Figure 6) appears just above the S/M Diagram and produces recommended settings for animations designed to repeat smoothly for an indefinite time. Suggested Frame values will produce one frame short of a single complete electrical cycle, which results in an un-interrupted continuous rotation when played on repeat. Two frame rates (Frames/Sec) are suggested for discernable rotor speeds of 1/8 or 1/4 of a mechanical revolution per second.

In the MathCAD® window, select Tools > Animation > Record… The “Record Animation” window will appear where Frame settings can be entered just as the animation tool presents. This permits the user to simply transcribe the values to produce the most viewable animation.

Once Animation settings are specified and with the Animation dialog still active, select the worksheet area to record to video, and then select Animate. With the suggested settings, the worksheet will produce images for 2π electrical degrees of operation (minus one frame to permit fluid repeat play). This group of images can then be saved to an avi-type video file. Play the resulting avi video in Windows Media Player on repeat.

Another useful feature of Windows Media Player is single frame stepping. To do this, pause the video and select anywhere along the Seek bar to make it the active control. Now you can use the left or right arrows to step through the video one frame at a time.
Sample avi videos and the interactive MathCAD file can be downloaded from:
https://drive.google.com/folderview?id=0B1Tj3WSO7b7MUnJ4VG5sOWhvRlE&usp=sharing

**Video 101**: 4 Pole, 24 Slot Motor, Motor Convention, Positive Sequence

**Video 105**: Steps through various complex power values to show how MMF orientation is different when motoring vs. generating. Motor convention is used.

**Video 106**: Like Video 105, this steps through various complex power values but uses generator convention.

**Video 199**: 6 Pole, 18 Slot Motor, Motor Convention, Positive Sequence
III. Behind the Plot: How It Works

Now, for those interested in the math behind the machine, this section takes you from user-specified parameters to S/M diagram output. This section will also assist anyone wanting to make slight modifications to the program achieve desired results.

Code Components

Returning to the point where the user has specified all desired settings, the tool has everything it needs to produce an operating S/M and phasor diagram. The next sections of the MathCAD® file, which can be found in the worksheet collapsed (but expandable) areas, contain the building blocks of the machine. Basic math and drawing functions simplify and expedite subsequent calculations. Next, electrical and magnetic values are determined, and finally, a multitude of arrays are derived, which when plotted bring the S/M to realization.

Electrical

Calculation implements the simplest electrical model of a S/M as shown in Figure 7. The figure shows motor convention where positive I_a delivers power from the stator to the rotor; generator convention simply reverses the positive direction of the current.

![Figure 7](image)

Electrical Calculations

In order to go from user-specified parameters to end product, steady state phasors are calculated to describe the machine’s initial electrical state. Time-based functions are then easily derived from the phasors.

Figure 8 illustrates the phasor calculations and includes implementation of Motor or Generator Convention and derivation of q and d-axis angles, currents, and flux linkages. Because the worksheet layout calls for the user to input complex power and terminal voltage, E_x is first computed to determine q and d-axis angles. This could easily be modified to permit alternate parameter entries.
A summary of phasors calculated from entered parameters is shown in Figure 9.

Time Functions

Time-based functions permit instantaneous electrical parameter calculation at any specified time. These functions return a complex vector based on the corresponding phasor and time arguments. See Figure 10.
Starting with stator winding currents, $i_a$, $i_b$, and $i_c$, the common magnitude $|I_a|$ is applied for this balanced 3 phase system. Phase shift offsets are based on the angle of $I_{a0}$ and the user-selected Phase Sequence.

$\Psi_{d\text{MAG}}$ & $\Psi_{q\text{MAG}}$: Flux linkage magnitudes along the d and q axis are constant over time since these axes are attached to the rotor (for this analysis and typically). And from the point of view of the rotor the stator currents on a S/M are always in the same position. The animation illustrates this.

$\theta_r(t)$: The initial rotor angle is tied to the d-axis, then electrical rotor angle is determined with electrical rotor speed and time.

$\theta_{rm}(t)$: Rotor mechanical angle depends on Poles and Phase Sequence and changes with time, of course.

$\Psi_{a\text{fP}(t)}$ & $\Psi_{a\text{armP}(t)}$: Flux linkage along phase A stator axis due to field and armature currents apply rotor mechanical speed.

$\Psi_{af}(t)$, $\Psi_{dp}(t)$, & $\Psi_{qP}(t)$: Flux linkage along a, d, & q axis begin at respective phasor angle and rotate at rotor mechanical speed.

Notice the use of Euler’s formula to easily rotate a complex vector by an angle to produce a new complex vector. When multiplied by a complex vector, the left hand side of (2) provides a concise way to rotate the point by angle $\alpha$. This operation is used extensively to produce the S/M plot.

$$e^{\pm j\alpha} = \cos(\alpha) \pm j \sin(\alpha)$$ (2)
The Bare Synchronous Machine

Now with the steady state phasor description of the S/M and capability to produce complex vectors to describe necessary electrical and magnetic states of the machine at any specified time, all that remains is to build a graphical depiction of one instantaneous snapshot. The program can then recalculate and redraw for each new time step.

Build the Stator – One Slot

The drawing of the Stator can be used to describe the fundamentals of depicting this S/M on a MathCAD® 2D plot. First the Slot vector is formed, which contains eight complex vectors to describe point locations on a real vs. imaginary axis grid as in Figure 11. Variables are:

- \( R_i \): radius of inner stator (realize outer edge of back iron is 100 units)
- \( r_w \): winding radius
- \( \beta \): slot opening at inner radius (in radians)
- LAYERS: specifies how many layers of windings, one or two. One layer for a full pitched stator, two layers otherwise. You can see that the depicted slot is for two layers because its depth is twice its width (\( \beta \)).

![Figure 11](image)

Rotate and Copy

This single slot that is built along the real axis for simplification is then rotated by Euler’s formula (2) and copied as many times as needed (Figure 12). A pair of for loops combine rotated copies of the Slot vector to form the SlotPlot multi-column array. The first, outer, for loop builds one column per slot while the inner for loop multiplies each point by an angle corresponding to which slot is being formed. Integer multiples of Slot pitch, \( \sigma \), determine rotation angle.
Build the Rotor

The rotor is built in a very similar fashion. The diamond points on the black-line rotor in Figure 13 depict each point that is connected by a line. Line connections are controlled by grid properties. The blue rotor illustrates how easy it is to rotate the rotor with the RotorPlot function and a time argument. It is shrunk to 80% of the full size to prevent overlap in this illustration.

Wind the Rotor

Rotor windings are positioned with a separate drawing array and reveal fixed rotor winding currents. Current direction is indicated by an X or a dot on each winding. The right-hand-rule indicates flux derived from these windings. A magnetic north pole is shown in Figure 14.
Wind the Stator

With user-specified Poles, Slots, and Pitch, the stator can be wound to show the position of all phase windings. Figure 15 illustrates how current phase corresponds to color, current magnitude corresponds to the radius of the colored portion, and direction is shown by an X or a dot. A, B, and C phase currents are represented by red, blue, and green colors respectively. Both stators below contain 4 Poles and 24 Slots. The stator on the left is 5/6 Pitch, which demands the 2 winding layers, whereas the stator on the right has a Full Pitch layout with one layer. Notice that in both cases, the positive a-axis is exactly horizontal on the screen even though this requires a slight shift in slot positions. Again, credit Euler’s formula (Equation 2).
Flux Linkage Calculations

This section breaks down how the MathCAD® tool calculates flux linkage, which may be the most interesting and substantial feature of the program. Here, the flow of operations is presented while cumbersome details of how each calculation is made have been left out. Detailed comments to include descriptions of each array and most calculations can be found in the actual MathCAD® worksheet.

Flux Linkage Analysis Points (FLAPs)

Based on user-inputs of Flux Analysis Settings, locations of where to analyze flux linkage over the entire area of the S/M are determined. User settings permit easy specification of where to place these analysis points. Points can be located anywhere from the center of the S/M to the outside edge of the back iron of the stator and can be arranged in a sparse or dense manner according to the desired outcome.

Vector From Windings to \( \Psi \) Points

With analysis points set, an array is formed to contain the distance and direction from each stator winding to each Flux Linkage Analysis Point (FLAP). An associated array maintains unit vectors of this array to simplify subsequent calculations.

Determine Permeability

Permeability of each FLAP is determined based on its location; salient pole rotor position is accounted for as it rotates air gap position with time. Permeabilities of the stator iron, rotor iron, and air gap are 1.0, 0.9, and 0.1 per unit, respectively. Since the flux linkage calculated for this illustration is based solely on stator currents, the 0.9 value given to the rotor iron vs. the stator iron is to account for the smallest air gap between rotor and stator.

Biot-Savart Law

The Biot-Savart law takes Ampere’s law of determining a magnetic field produced by a current and applies it to a distribution of current beyond that of a long, straight wire. With the dispersion of wires inherent to the wound stator, we have a nonsymmetric arrangement of currents that contribute to specific magnetic fields measureable at each FLAP. The Biot-Savart law permits flux linkage to be summed from each winding one at a time.

Biot-Savart Applied

Because all currents in this model flow perpendicular to the screen, the math is significantly simplified. We really only need the distance and direction from each winding to each FLAP, and this is provided by the complex vector array previously calculated.
Result of One Winding

Flux linkage from one stator winding is distributed to each FLAP. The linkage is applied as a complex vector having a direction perpendicular from the flow of winding current in accordance with the right-hand-rule. Linkage magnitude is based on permeability of the FLAP location and distance from the source winding. Attenuation due to distance is inversely proportional to flux path permeability.

Sum the Flux

The flux linkage contributed from each stator winding to each FLAP is totaled to result in one complex flux linkage vector at each FLAP. A nest of for loops provides a way to algebraically sum the contribution of currents from each winding to each FLAP.

Make it Plot-able

A simple set of functions convert the complex vectors of summed flux linkage into a separate plot-able matrix of arrows that illustrate magnitude and direction when plotted on a real-imaginary grid. Since these arrows are similar to those of the phasor diagram, the details are reserved for explanation in the Phasor Diagram section that follows shortly below.

Synchronous Machine Plot

With all values calculated and converted into plot-able matrixes, the revealing S/M Plot illustrates how 3 phase stator currents produce a magnetic field and how that field interacts with the rotor. When stepping through short time intervals, it can be seen how the stator appears as a fixed magnetic field from the standpoint of the rotating rotor—hence the desire of engineers to employ the rotating d-q axes reference frame for analysis. The physical arrangement of windings and associated currents produce a symmetric rotating field.

Magnetomotive Force (MMF)

The currents in the stator windings produce a rotating sinusoidal MMF field of constant amplitude. This field rotates such that its peak magnitude lies exactly on the A, B, or C-phase stator current axis at the instant each respective phase current peaks. The magenta flux lines orient directly outward or inward at the positive and negative peak locations of stator MMF. A separate, rotor-produced sinusoidal MMF field, also of constant amplitude, maintains peaks aligned with each rotor pole. So both MMF fields and the rotor itself rotate at the same speed such that everything appears stationary from the rotor reference frame; i.e. it’s synchronous. If the MMF fields lie exactly on top of each other, no work is being done—no motoring, no generating, no torque, no real power. An outside force such as an engine driving the rotor (generating) or a load on the rotor (motoring) causes a fixed angle between stator and rotor-produced MMFs; this angle is typically called the torque angle or load angle. A minimal angle between MMFs equates to minimal work, while a substantial angle equates to maximum work.
The MMF depicted on the S/M plot as an orange offset ring is the combined MMF, or total MMF, of the stator and rotor together. At time zero on a 2-pole machine, it aligns with the total flux linkage, or air gap flux linkage, \( \Psi_a \), on the phasor diagram. Under motoring operations, total MMF lags the stator MMF and leads the rotor MMF. In contrast, generating operations show that total MMF leads the stator MMF but lags the rotor MMF.

To see more details about how these vectors are drawn on a 2D plot, click on the S/M diagram. This will show the x and y-axis arguments, which can then be traced backward through the program.

Phasor Diagram

The companion to the S/M plot is the phasor diagram. It provides a snapshot summary of parameters and permits panning and zooming for customized viewing. Once the phasors are calculated, a set of functions are used to produce a plot-able version of the phasors with included directional arrows.

Each phasor is a complex vector, which describes a single point on a real-imaginary coordinate plane. In order to plot a vector to the point, a column vector is built as shown in Figure 16. The first row contains zero, which states that the vector arrow is to start at the real-imaginary axes intersection. The second row is the complex vector itself. Next, the first leg of the arrow is added as the third row. This is simply a small, constant value added at a pre-determined distance, ARROWSIZE, and rotated relative to the existing complex vector angle using Euler’s formula (Equation 2). The second point of the arrow is calculated the same way and added as the fourth row. The fifth and final value draws the arrow back to the actual complex vector value being illustrated.

![Phasor Diagram](image)

**Figure 16**
Once these drawing points are determined, the plot-ready arrow can easily be moved and rotated either by adding a constant value to each value in the column array or by multiplying each point by $e^{j\alpha}$, where $\alpha$ is the rotation angle in radians (Euler’s yet again).

Phasor diagram panning and zooming operations are carried out by controlling the 2D plot limit values when the Left, Right, Up, Down or In/Out pan/zoom buttons are used. The buttons control integer multipliers and offset values that affect the plot limit values. Select the phasor diagram to see.

Pedagogical Use and Assessment

This program has been employed in the classroom as a demonstration in the introductory synchronous machines instruction of the junior and senior undergraduate curriculum. It has also introduced the appropriate topics in our first-year graduate courses. We have not yet used it in our service course for junior-level mechanical engineering undergraduates. It has not been used as part of student projects or laboratory work yet. In the classroom, it served to illustrate important points about synchronous machine behavior. Students readily understood the presentation format, an illustration method common to finite element programs. Showing the magnetic field’s paths and the magnetic flux density throughout the machine while the machine operates gave important insight, according to student comments. Then we engaged the animation. That really gained the students’ interest and opened their understanding to how the magnetic behavior of the machine leads to important notions of induction, force, torque, and speed. Their comments, and all of them positive, were far more numerous about this software than about any other aspect of the lessons on synchronous machines. The hands-on lab with ten-horsepower synchronous generators was a close second.

When shown to other instructors, their comments endorse the usefulness of this program. They relate how they can more easily introduce the interaction of the magnetic and the mechanics, as well as the electrical behavior of the machine. As all of their students already have MathCAD® and are familiar with it, learning the software environment is not a problem, as minor as such a notion is in the classroom demonstration environment. We will next semester develop ways to use this software as something that the students can engage hands-on.

Summary

Although videos provide an interesting way to view the S/M in motion, perhaps the most useful part of the program is the immediate feedback it provides. A student can manipulate machine parameters and view induced changes on the phasor diagram in an instant. The S/M plot illustrates the meaning of the phasor diagram by producing a virtual, physical piece of machinery and magnetic field corresponding to the phasors. The importance of angle and magnitude relationships among phasors is illustrated with the correlated diagrams as well.

When first studying the dynamics of a 3-phase S/M, many texts elaborate on certain areas but do not always seem to link them together clearly. Optimistically, this tool will help tie together electrical machine concepts and bring physical relevance to the phasor diagram, all in an
entertaining way. To download the program for your educational endeavors, please visit https://drive.google.com/folderview?id=0B1Tj3WSO7b7MUNJ4VG5sOWhvRIE&usp=sharing.

Bibliography