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Addressing Barriers to Learning in Linear Circuit Analysis

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Abstract

Some possible barriers to student success in introductory linear circuit analysis courses are analyzed, particularly for DC circuits and general circuit topology issues. We argue that certain concepts actually needed to solve circuit problems are often not taught as explicit principles, and that conventional instruction often fails to address typical conceptual misunderstandings. In particular, we discuss the concepts of hinged circuits, redundant circuit elements, and voltage and current-splittability of circuit problems, the replacement theorem, iterative calculations, and the several types of problems to which one can actually apply voltage and current division. We emphasize the importance of comparing and contrasting when teaching concepts, particularly for the cases of voltage and current sources, short and open circuits (as special cases of voltage and current sources, respectively), voltage and current dividers, series and parallel connections, and voltage and current measurements. We highlight the importance of contrasting the various functions of terminals in a circuit. We propose various models that can promote understanding of basic electrical concepts, such as a microscopic Drude model of conduction, a "balls in tube" analogy to explain the constancy of current through circuit elements, and a "control loop" model to explain the operation of voltage and current sources. We use the DIRECT 1.0 concept inventory of Engelhardt & Beichner to assess conceptual understanding, administering it as both a pre- and post-test in 20 sections of a linear circuits class involving over 1100 students over two years. Pre-test scores are around 50% as found by others. Post-test scores typically rise to only 59% (averaged over many instructors), showing that conventional instruction does not address qualitative misunderstandings very effectively. By introducing targeted instruction in one section to address misconceptions, the post-test score rose to 69% in Spring 2013 (higher than any other section) and with further refinement reached 79% in Fall 2013.

1. Introduction

Introductory linear circuit analysis is one of the most widely taught courses in engineering, as it is frequently taken by many majors other than just electrical engineering. As in many such courses, the failure rates can be undesirably high. In teaching this course to over 600 students a year over many years we have found average DEW rates (i.e., percentages of students receiving grades of D's or E's in the course or withdrawing) of 23%, varying widely from 3% to 53% depending on individual instructor and section. Student difficulties may arise from a variety of causes, such as insufficient mathematical skills, lack of rapid feedback on their work, insufficient use of active learning strategies, and varying levels of motivation (especially if it is outside their major area). However, I will argue in the following that some of the analysis principles actually needed to solve circuit problems are often not taught explicitly, and that there is frequently a failure to address fundamental qualitative misconceptions that students hold about electricity.

Considerable work has been done to study misconceptions among high school and university students in physics classes related to electricity.[1-9] Typical misconceptions involve treating batteries as current sources rather than voltage sources, believing that current is "consumed" as it travels through circuit elements, believing that no voltage can exist across an open circuit (as opposed to no current through it), and failures to identify series and parallel connections correctly (esp. the latter). Other common misconceptions involve not understanding the significance of short circuits, “battery superposition” (the idea that more batteries deliver more power regardless of how they are connected), a “sequential” model in which circuit elements only affect the current that is “downstream” from them, and so forth. It has been argued that some such misconceptions are related to ontological lateral misclassifications, such as “electricity” being a substance rather than a process. These types of misclassifications are more resistant to instruction than other types of misconceptions.[10,11]

In the following, we describe several instructional approaches that we believe are potentially very helpful in overcoming misconceptions and improving student performance in a circuit analysis course. These include comparing and contrasting the behavior of similar but different objects (such as voltage and current sources), providing physically-based microscopic models and analogies to help students understand different electrical quantities, and making commonly used principles and ideas used by experienced circuit solvers explicit and formalized, rather than relying on students to reason out such ideas for themselves. Using the DIRECT 1.0 concept inventory of Engelhardt & Beichner[1,12] as both a pre-test and post-test in all of the studied sections of our EEE 202 Circuits I course, we show that use of some of these approaches can result in significantly improved conceptual understanding.

2. Effects of Conventional Instruction on Conceptual Understanding

Previous studies with the DIRECT concept inventory used students who had completed high school or university-level physics courses, but did not study the effect of subsequent engineering courses in linear circuits on student understanding. To determine the extent to which typical instruction in linear circuit analysis (defined as that used by 14 different instructors at Arizona State University), we administered this inventory through the Blackboard course management system or (in Spring 2014) through our Circuit Tutor web site as both a pre-test (in the first week or two of instruction) and as a post-test (in the last week or so of instruction). The pre- and post-tests were completed by 1287 and 990 students, respectively, in 20 sections from Summer 2012 through Summer 2014. Completing both pre- and post-tests generally counted as one homework assignment to ensure participation, but the scores did not affect students’ grades (and students were aware of that fact). The average pre-test score was 49.4% (σ = 18.0%, N = 1287), which is similar to but slightly lower than the average found by Engelhardt & Beichner for university students of 53%.[1] The average post-test for students whose instructors used conventional instruction was 57.2% (σ = 20.6%, N = 856), an increase of about 8%. It therefore appears that commonly used instructional approaches, in conjunction with the textbook, fail to overcome many student misconceptions.

3. Instructional Approaches to Address Misconceptions

In the following Sections 3 and 4, we describe some methods we have developed to promote better qualitative understanding of DC circuit concepts. These methods were developed based on the experience of one of the authors (BJS) in teaching this subject. The effects of these methods on conceptual understanding when implemented by that author in two class sections are discussed in Section 5 below. Section 3A focuses on microscopic models and analogies for current flow, and Sections 3B-3E focus on comparing and contrasting items such as voltage and current sources, short and open circuits, series and parallel connections, and measurements of current and voltage.

A. Microscopic Drude Model of Conduction and Analogies

We posit that misconceptions such as current consumption, the idea that current is stored inside a battery, the idea that electric field inside a current-carrying conductor is zero (which probably originates from electrostatics training), and sequential models in which elements only affect current downstream from that element are linked to a failure to appreciate the microscopic origins and natures of current and voltage. Most (if not all) circuit textbooks give no discussion at all of such models, and it is likely that very few instructors do either (being pressed for time and perhaps being less familiar with this topic themselves in some cases). I have found that presenting a free electron (Drude) model of conduction can be very helpful in understanding macroscopic behavior (students may or may not have been taught such a model in their physics course, but it appears rare that they retain this knowledge). An important starting point is to emphasize the near-universal electrical neutrality of macroscopic objects in nature, due to the strong Coulomb force that tends to discharge objects other than insulators. A good question to ask students is whether (dirty) water flowing down a pipe carries any electrical current. Many think it does, due to the charged ions in motion within it (failing to appreciate that the water is neutral due to a *local* balance of positive and negative ions). The idea of neutrality is in fact central to understanding Kirchoff's current law (KCL), which is often misattributed to conservation of current or charge. Current and charge *could* be conserved while still charging a node (displacement current then leaves the node to balance the conduction current entering it), but this does not normally happen because the tendency toward charge neutrality (short dielectric relaxation time) prevents it.

The Drude model involves explaining how electrons move randomly at high speeds between positively-charged crystal ions, scattering from each other and from lattice vibrations and defects. The acceleration of the electrons by the field is discussed (emphasizing that there *is* a field in the conductor), with the resulting constant acceleration between collisions and energy loss to collisions that results in a constant drift velocity. Current density is derived, as is an expression for the resistivity of the material (which is related to resistance of an object made from that material). This derivation emphasizes that resistance is a property of an object, not a function of applied current or voltage as students sometimes believe (perhaps based on naïve use of the equation $R = V/I$, without realizing that both V and I vary proportionally in a resistor!). We also relate the voltage drop to the field, and explain voltage as the driving force for current. Further, it is pointed out how (different) electrons exit one end of a conductor as other electrons enter the other end to maintain the essential neutrality.

To augment this detailed model, a simpler and more easily visualized model is also presented of electricity flowing through a wire being analogous to rigid balls in a tube. The tube (wire) is *always* filled with the balls (electrons), but pushing one in at one end must immediately expel a different one at the other end. Moreover, all balls move at the same time, it is *not* the case that one ball moves, followed by the next one, and so forth. A similar analogy can be given in terms of an incompressible fluid such as water in a pipe (though one must emphasize in that case that there is no empty tank to be filled; all conduits and tanks are already full before current starts to flow). The entire discussion can occupy a modest portion of a lecture, and yields considerable dividends in giving students a real basis to understand electricity. These ideas lead very intuitively to an understanding of the need for complete circuits, the constancy of current in a closed loop, and the idea that batteries store energy, not current.

B. Explaining Voltage and Current Sources

A very common misconception is that batteries act as current sources rather than voltage sources.[1] For example, students often think that adding a second light bulb in parallel with a first one being powered by an ideal battery will cause the first bulb to dim, because the battery current must now split between two bulbs. In reality, of course, the battery current doubles and the first bulb is unaffected, since it still has the same voltage. We believe this problem can be traced to the fact that current sources are seldom if ever taught in physics courses. This situation is understandable given that voltage sources such as wall outlets, batteries, and power supplies are much more common in everyday life than current sources. Current sources are commonly used in electronic circuits, but beginning students are less familiar with those. We argue that one cannot really understand why a voltage source (e.g., battery) does not act as a current source until the different behaviors of the two sources have been expressly compared and contrasted. The above example (adding a second bulb in parallel with the first) should be discussed for *both* cases, showing how the first bulb does dim when a current source is used, but does not when a voltage source is used. Both the currents and voltages should be discussed in both cases. Such an example can open students' eyes to the essential difference between the two sources. Without such examples, students may even believe that voltage sources supply voltage but not current, and that current sources supply current but not voltage. This idea can be negated by pointing out that power cannot be supplied unless the source provides both current and voltage. The difference is which of the two quantities is held constant, and which varies.

The above ideas can be greatly reinforced by presenting a hypothetical model for a voltage source in which an invisible agent constantly measures the voltage across the source, turning a knob to increase the output current when the voltage gets too low, and to decrease the current when the voltage gets too high. With this control loop idea, students are immediately led to realize that the current is not fixed (or zero), but varies as needed. I further find it very effective to ask students (repeatedly) in lecture, "What is the current through a voltage source?" and get them to chant out loud in response "Whatever it needs to be! (to maintain its fixed voltage)." After doing this enough times on enough different occasions, students find it much harder to make the mistake of ignoring the current through a voltage source when writing a KCL equation for a node, for example. An analogous control-loop model (and chant) is used for current sources, whose voltage is "whatever it needs to be." One should also address the tendency of students to use the value of a current source (in A) as its voltage drop when trying to write a Kirchoff's voltage law (KVL) equation through a current source (or the analogous mistake for voltage sources with KCL equations). The need for consistent units should be emphasized as a method to repudiate this practice.

Some other important aspects of voltage sources should be addressed explicitly as well. Students have a tendency to think that the positive side of a 3 V source is at a potential of 3 V above ground, regardless of whether its negative side is attached to ground or not. Or, they may assume the negative terminal to be at a voltage of –3 V. Thus, we need to emphasize that *all* voltages are relative, and that voltage sources establish *differences* in voltage across their terminals, not absolute voltage values (which do not even exist). Further, students often try to misapply Ohm's law to voltage sources, assuming that doubling their current doubles their voltage. The Drude derivation can explain the physical origin of Ohm's law, though a microscopic discussion of how batteries actually work might be necessary to eliminate this misapplication. Finally, the misconception of "battery superposition" should be explicitly addressed by discussing what happens when voltage sources with the same value are connected in series or in parallel. An example can be used where two batteries are connected positive terminal to negative terminal, showing that the series combination then delivers no power at all.

C. Explaining Short Circuits and Open Circuits

Short and open circuits are rarely discussed explicitly in textbooks, but probably need to be if we expect students to arrive at a correct understanding of them. Short circuits should be explained as being voltage sources with values of zero volts (preparing students to turn them off in superposition problems), and open circuits as current sources with values of zero amps. It is important to emphasize that like voltage sources, short circuits do not generally have zero current, but can carry large currents! Similarly, students have a very common misconception that no voltage can exist across an open circuit because there is no current flow. This idea probably results from a naïve application of Ohm's law to the situation, without realizing that the R in question is infinite. This fallacy can be addressed by discussing how Ohm's law relates to both short and open circuits (with $R = 0$ or $R = \infty$, respectively). It is important to point out explicitly that shorted elements have no voltage drop (and therefore no current either, unless they are current sources or charged or magnetically-coupled inductors). Similarly, open-circuited ("dangling") elements can have no current, and therefore no voltage drop, either (unless they are voltage sources, charged capacitors, or magnetically-coupled inductors). Just as it is important to compare and contrast voltage and current sources to achieve a proper understanding of each, it is important to do the same for their limiting (zero-value) cases of short and open circuits.

A good example to explain how open circuits can have (large) voltages across them is to explain how modern series-connected strings of Christmas lights work. Specifically, when one bulb burns out and becomes an open circuit, the entire 120 V supply voltage drops across that one bulb, causing a thin oxide separating the two leads at the base of the bulb (one is wound around the other) to (destructively) break down and become permanently conducting. The bulb is then transformed into a short circuit, slightly increasing the voltage across each remaining bulb. This concrete example can help cement the concept of voltages across open circuits in students' minds. A further method (once voltage and current division have been discussed) is to discuss how both division phenomena behave when one of the two resistances becomes either zero or infinite.

D. Explaining Series and Parallel Connections

Students often have great difficulty correctly identifying elements that are in series and parallel, even when they can state the definitions of each precisely. Moreover, many textbook definitions are not accurate or complete for the case of series elements, leading to many misconceptions. For example, students may think that elements must have a node in common to be in series, when they need not! Both elements can be part of a series chain of elements, without directly sharing any node. A complete definition states that: a) *Two* elements are in series if one (and only one) end of each element is attached to a common node, and no other conducting element is attached to that node (or, equivalently, that a node connects one end each of exactly two conducting elements); and b) If elements A and B are in series, and elements B and C are also in series, then elements A and C are in series (transitivity). (The above definition can be extended by changing "elements" to "subcircuits.") Students often think that elements connected in a chain are in series, even though something else is connected to one of their shared nodes. Further, they tend to ignore that one element being shorted prevents it from being in series with anything.

An accurate definition of parallel elements (or subcircuits) is relatively easy: A set of elements are all in parallel if and only if they are all connected to the same two nodes. Students generally have more trouble identifying parallel elements than series ones, since they do not have to be physically proximate to each other, and often confuse the geometric concept of being parallel with the circuit connection concept. It is therefore important to emphasize explicitly that it is not a geometrical relationship, but that the word means that two currents can run from one node to the other through separate ("parallel," but not in the geometric sense) paths. A good example is to show that geometrically parallel elements may be connected in *series* rather than in parallel. It is also important to emphasize that the elements must be *directly* connected to the same two nodes, not for example through a third element such as a voltage source in one case. Of course, it is important to emphasize that series elements must have the same (physical) current (not just the same value of current), and that parallel elements have the same voltage drop *across* them (again, absolute voltage does not exist).

We have developed tutorial software that includes both interactive instruction on the above concepts as well as exercises in which students are asked to identify both series and parallel sets of elements in randomly generated circuit diagrams of varying levels of complexity. Such exercises are not given in most conventional textbooks. These exercises use color coding of nodes to help understand parallel connections much better, and have been found to be quite effective in improving understanding on this point.[13-15] Beyond considering series and parallel elements, it is also useful to explain how these concepts generalize to series and parallel subcircuits (one-ports), of which a single element is a special case. This generalization can help students understand why, in a combination of two resistors in parallel, the combination of which is in series with a third resistor, neither of the first two resistors is in series with the third resistor; only the subcircuit formed from their parallel combination in series with it.

E. Explaining Voltage & Current Measurements

Some textbooks explain how voltage and current measurements are performed, but many others do not. We feel it is beneficial to explain these ideas explicitly in lecture, including the idea that a circuit must be *broken* to measure current (without using a clamp-on ammeter). Understanding how these quantities are measured can lead to a better understanding of what they really are. Similar comments are made for impedance measurements below.

4. Circuit Topology Concepts

A number of concepts related to circuit topology are not typically taught explicitly, but we argue that these ideas can help students form a more systematic understanding and better skill in problem solving. We argue that these ideas need to be named and presented explicitly (rather than just in the context of working examples) to help students acquire them successfully. We introduce several less commonly used or novel terms to do so.

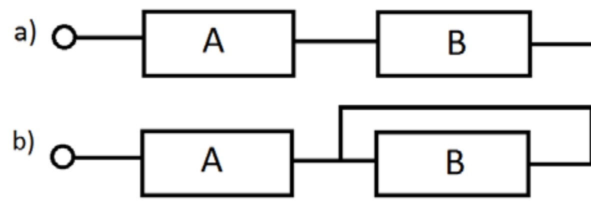

Fig. 1. Schematic of (a) dangling subcircuits (boxes) and (b) one dangling (left) and one shorted hinged subcircuits (boxes).

A. Hinged Circuits

The first example involves what have been described in circuit theory textbooks as (electrically) *hinged circuits*, which are those that *can* be drawn such that two parts are connected by a single wire.[16] Equivalently, removing some node from the circuit leaves it disconnected. By KCL, no current can flow through the connecting wire, so that the two halves of the circuit are essentially independent problems (unless they are otherwise coupled by having a dependent source in one portion whose control variable is in the other portion, or by being magnetically coupled). The subcircuit on either side of the hinging node must be either shorted or dangling, depending on whether its other terminal is also connected to the hinging node (see Fig. 1). This concept should be explicitly taught to students as a great illustration of the consequences of KCL, and an example of how KCL need not be applied only to a single node, but can be applied to *any* closed surface. Further, a hinged subcircuit can have no influence on the other subcircuit (as long it is isolated as mentioned above), so it can be removed from a circuit if the "sought quantities" (circuit variables that one wishes to find) are not in the removed subcircuit. All shorted and dangling circuit elements are hinged, but subcircuits comprised of more than one element can also have this property. We introduce the term "hinged" to students to help them understand this concept by naming it, and find that they readily adopt the terminology and use it themselves.

B. Redundant Elements

A generalization of the hinging concept is that of *redundant* circuit elements (or subcircuits). We use this term to describe any circuit element that is in parallel with an ideal voltage source, or in series with an ideal current source. A special case of this (and one that is worth explaining to students) is a voltage source either in parallel or series with a current source, as shown in Fig. 2. The element in parallel with the voltage source (whether a current source or a passive element) changes the current supplied by the voltage source, but can no effect on its voltage, and therefore on the rest of the circuit (absent magnetic or dependent source coupling as noted above). Removal of this "redundant" element therefore changes nothing, if one is not interested in the current and power supplied by the voltage source. Similar comments obviously apply to any passive element or voltage source in series with a current source, since the current is already fixed and only the voltage/power of the current source is affected by the redundant element. Very few books discuss this situation, with an exception of Davis.[17] However, books that do not discuss this situation sometimes still include problems where passive elements are redundant, which may leave students confused as to why they have no effect on their answer. Further, errors in remembering how a

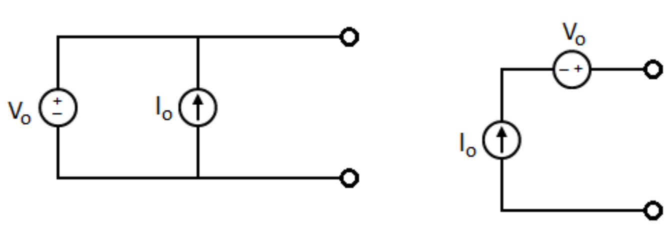

Fig. 2. Schematics of redundant sources.

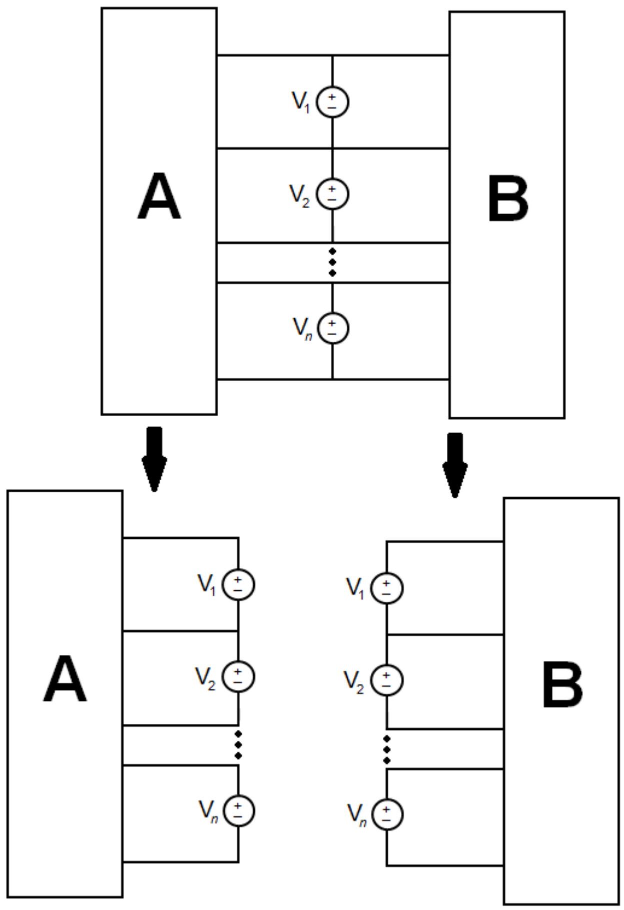

Fig. 3. General concept of voltage splittability. Top: Original splittable circuit, where the boxes are arbitrary circuits. Bottom left and bottom right: The two split halves that result.

Thévenin or Norton circuit is constructed can be avoided if students learn to recognize that putting the Thévenin impedance in parallel with a voltage source or in series with the Norton current source is redundant, and therefore not the logical form of such circuits.

Some reflection shows that hinging is actually a special case of redundancy, where the ideal voltage or current source simply has zero value (leading to shorted or dangling hinged elements, respectively).

C. Voltage and Current Splittability

A further generalization of the above ideas is that of voltage and current splittability. We define a voltage-splittable circuit as one in which replacing all of the voltage sources that form part of a loop by short circuits results in the circuit becoming hinged. Equivalently, removing the voltage sources in the loop and their associated nodes leaves the circuit disconnected. This situation is illustrated in Fig. 3. Such a circuit can be split into two pieces where the other has been replaced by an open circuit, duplicating the voltage sources in each piece as shown in Fig. 3. The voltages in each piece are unchanged as a result. A moment's reflection shows that redundant circuits (due to a voltage source) are a special case of voltage splittability where the chain contains only one voltage source.

Similarly, a circuit is current-splittable if replacing some set of current sources (which all exit a closed surface) by open circuits results in the circuit being hinged, as illustrated in Fig. 4. This type of circuit can be split into two pieces by replacing the other piece by short circuits and duplicating the current sources in each piece, as shown in Fig. 4. The currents in each piece are unchanged as a result. Redundant circuits (due to a current source) are a special case of current splittability where the set contains only one current source. In general, we can perform voltage or current splitting repeatedly on the same circuit until it is not splittable any further (perhaps yielding a number of independent circuits from the original).

The above ideas are not discussed explicitly in most circuits texts, yet problems may require students to "invent" these ideas on their own. Some students will be able to do so, but others will struggle with this requirement and would be greatly aided by an explicit exposition of the concepts (using simple examples such as a single voltage source with circuits connected on either side of it, which are redundant using our terminology).

D. Replacement Theorem & Iterative Calculations

Another useful idea that should be made explicit is the "replacement theorem," the idea that once we have solved for a particular voltage or current of an element, we can replace that element by a voltage source having its known voltage or a current source having its known current without affecting the remainder of the circuit.[17] This approach is particularly useful when requiring students to do multi-step "iterative" calculations such as those described in the next section, and also in finding initial conditions in transient solutions.

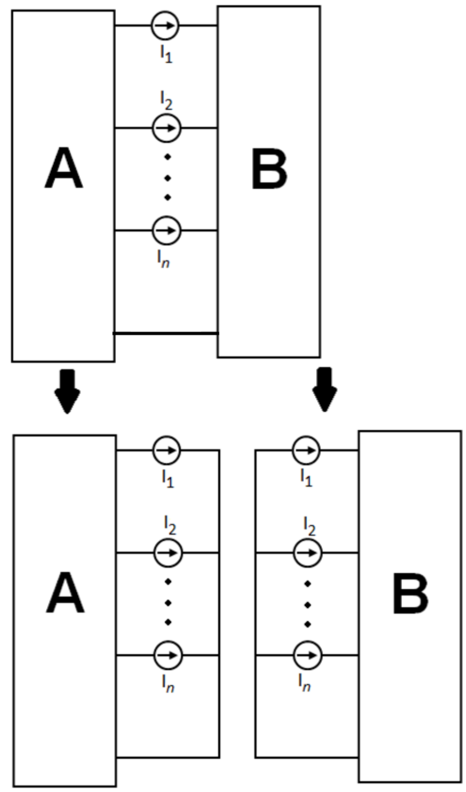

Fig. 4. General concept of current splittability. Top: Original splittable circuit, where the boxes are arbitrary circuits. Bottom left and right: The two split halves that result.

E. Applications to Voltage & Current Division

In explaining these central concepts, it is important to emphasize the intimate link between series connections and voltage division, and parallel connections and current division, respectively. In particular, it is advisable to stress the *absence* of voltage division in the parallel case and the absence of current division in the series case, as some students incorrectly think this happens. This is another situation where it is useful to compare and contrast the different behaviors in voltage and current division. Logically, the two cases are best compared using resistance/ impedance for voltage division and conductance/admittance for current division, but in practice many people use ohms universally, which can create a barrier to understanding. In this case, one has to stress that the voltage is larger across the larger resistor in voltage division, but the current is larger through the *smaller* resistor (larger conductance) in current division.

A useful device that can be used to illustrate these ideas graphically is that of a linear load line construction, similar to that often used for nonlinear devices in electronics (see Fig. 5). This construction nicely illustrates how varying a resistance in a divider causes its voltage to vary. A similar construction (though probably not often

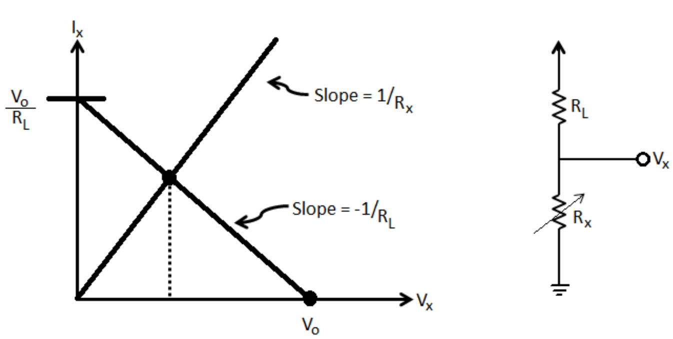

Fig. 5. Linear load line construction to help understand voltage division.

used in electronics) is possible for current division, by showing a fixed current source feeding a parallel combination of a load resistor and a variable resistor.

Most textbook treatments discuss voltage and current division only in the context of single loop and single node-pair circuits, respectively. While this is the most obvious case where this method can be used, it is far from the only one, and in fact students are often expected to apply these methods in problems that do *not* involve single loop or single node-pair circuits, without ever being told that it is possible to do so. We identify at least three classes of such problems. *Simplifiable* circuits are those that can be reduced to single node-pair or single loop circuits by combination of series or parallel elements, without losing the sought quantity of interest [Fig. 6(a)]. In this example, the four resistors at left can be combined in series and then in parallel without disturbing the desired voltage V_o, which can then be found by voltage division in the remaining single-mesh circuit.

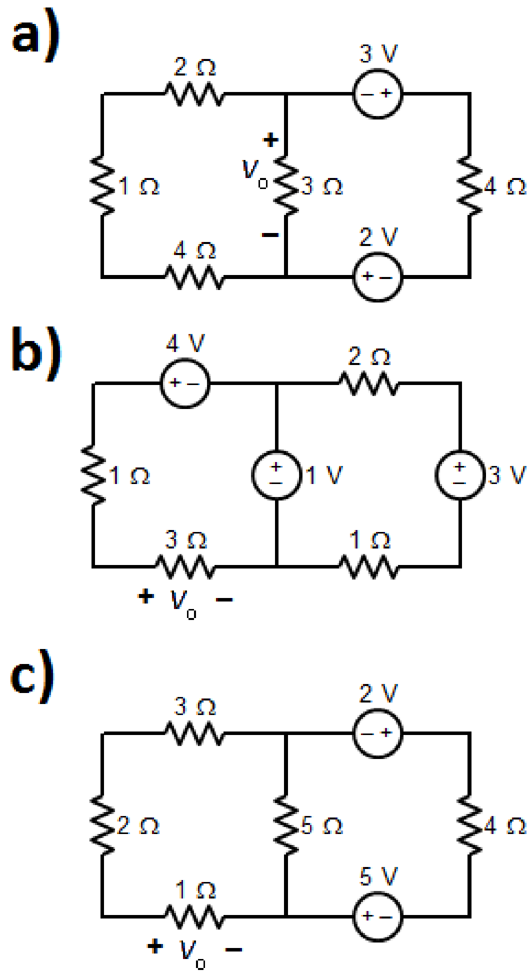

Fig. 6. Three types of circuit that can be solved by voltage division, even though they have more than two nodes. (a) Simplifiable circuit. (b) Voltage-splittable circuit. (c) Iteratively-solvable circuit. In each case we need to solve for the voltage V_o. Similar examples can be given for current division.

Voltage or current *splittable* (or redundant) circuits may permit use of voltage or current division after splitting (even though the original circuit had many nodes and/or meshes). This technique can be viewed as another type of circuit simplification (similar to combining series and parallel elements) [Fig. 6(b)]. In this example, the circuit can be split into two halves, each having a clone of the original 1 V source. The left half can then be solved for V_o using voltage division, as it is now a single loop and isolated from the other portion (which does not need to be solved).

Further, these methods can be applied *iteratively* in circuits that do not permit direct application, using the replacement theorem to go "backwards" after simplifications that remove the sought quantity of interest [Fig. 6(c)]. In this third example, the four resistors on the left can be combined first in series and then in parallel, leaving a single mesh circuit that can then be solved for the voltage across the *equivalent* resistor using voltage division. Next, one can use the replacement theorem to replace the rest of the circuit by a voltage source having the voltage just determined. Finally, the original set of resistors is restored, after which the 5 Ω resistor is redundant to the new voltage source and can be removed. The remainder of the circuit can then be solved by voltage division. Pointing out these approaches explicitly to students can avoid the common complaint that "the homework problems are nothing like the examples."

F. The Various Roles of Terminals

A point of frequent confusion, in the first author's experience, is the multiple roles that terminals play in circuits. A set of two terminals is often used to indicate where a sought voltage is to be measured (e.g., the output of a circuit). In this case, of course, the terminals ideally draw no current and have no effect on the solution of the circuit (in particular, they do not change existing series relationships). However, the same symbols are also used to denote terminals through which current can and does (usually) flow, such as those from which we "view" an impedance, or those that form the output terminals of a Thévenin or Norton equivalent circuit. In these cases, elements that would be in series without the terminals are no longer in series, because something other than an open circuit branches off their shared node. This confusion seems to be a common source of student errors. In particular, the failure to understand this issue for the case of viewing impedances may be due to a lack of understanding of how impedances are measured (e.g., by forcing a voltage and measuring a current, or by forcing a current and measuring a voltage). An explicit discussion of such measurements would therefore be helpful (along with the previously mentioned discussion of voltage & current measurements). Further, the two types of terminals need to be explicitly compared and contrasted to get students to understand the differing roles that terminals play in a circuit. They could be asked to solve the same circuit where the terminals play different roles, for example. Our existing computer-based series-parallel identification tutorial[14,15] has recently been extended to include a new section of exercises where different types of terminals are present, to drive home this concept.

5. Effects of Emphasizing Qualitative Principles

A number of the instructional strategies discussed above (but not all) were employed in the author's section of EEE 202 in Fall 2013 to determine if they improve conceptual learning. In particular, I (meaning the first author) emphasized macroscopic charge neutrality, presented the Drude model and ball-in-tube models of current flow, discussed control-loop models of independent sources* and compared and contrasted them*, emphasized that voltage sources establish voltage differences, explicitly discussed the properties of open and short circuits, used the Christmas lights example to illustrate open circuits*, discussed series & parallel connections carefully and had students complete the series-parallel exercise (though many other instructors also did the latter). I also

Table 1: Means (and Std. Deviations) for Pretest and Posttest Scores.

Section	*N*	Pretest	Posttest	
			Raw	Adjusted
1	18	14.1 (4.5)	15.8 (5.1)	16.0
2	25	13.4 (4.8)	18.1 (4.1)	18.8
3	17	17.1 (5.6)	18.6 (5.0)	17.0
4	18	16.2 (6.8)	19.2 (6.1)	18.1
5a*	79	14.9 (5.2)	19.7 (5.5)	19.5
5b**	55	14.5 (4.6)	22.4 (4.6)	22.4
6	67	12.6 (4.2)	16.1 (5.3)	17.2
7	103	13.3 (5.2)	16.5 (6.8)	17.2
8	27	16.0 (5.2)	18.2 (5.3)	17.2
9	41	18.9 (5.2)	18.8 (6.2)	16.1
10	38	14.6 (4.7)	18.2 (5.1)	18.1
11	29	13.3 (4.5)	15.9 (5.9)	16.7

*Spring 2013 section **Fall 2013 section discussed the hinged circuit concept, discussed redundant circuit elements* (but not more general splittability), emphasized the nature of voltage and current division, and differentiated the different roles of terminals in different cases. (The items marked with asterisks were done for the first time in Fall 2013; I had covered the remaining items also in Spring 2013). I included some qualitative questions on exams and discussed some in class, but generally avoided discussing problems that were too similar to the concept inventory questions (with possibly one or two exceptions). The effect of using these methods can be seen by comparing the results for different instructors in Table I (I was instructor #5, and data for my Spring and Fall 2013 sections was treated separately in this analysis). All instructors for the course followed the same basic curriculum, though they each decided on their own methods of presentation and emphasis. No other instructors specifically followed the approaches discussed here; they used the same approaches they have used historically. Whereas students knew that their grades on the pre-test and post-test did not affect their grades, this was equally true for both experimental and control sections. Thus, any changes between the scores for these sections should still be a valid metric of the effects of the intervention.

To analyze the data, a statistical analysis was performed. First, an analysis of variance (ANOVA) was conducted on the pretest scores to see if there was a difference among the instructors' classes. Indeed, there was a difference, $F(10,427) = 5.99$, MSE = 24.59, $p < 0.001$. To statistically control for these pre-existing differences among the instructors' classes, an analysis of covariance (ANCOVA) was conducted on the posttest scores using the pretest scores as a covariate. There was a statistically significant effect for instructor, $F(10, 450) = 2.2$, mean square error (MSE) = 23.38, $p = 0.017$. Post-hoc pairwise comparisons revealed that students who had instructor 5 in Spring 2013 had significantly higher adjusted posttest scores than students who had instructors 1, 6, 7, 8, 9, and 11, when adjusted for pretest score differences. (The adjusted post-test score was highest for instructor 5, but not with statistical significance for instructors 2, 3, 4, and 10.) In Fall 2013, using more of the instructional strategies recommended here, I had higher adjusted posttest scores than all other sections, $F(10, 426) = 6.48$, MSE = 23.05, $p < 0.001$. Further, a comparison of the posttest scores in my Spring and Fall 2013 sections showed that Fall outperformed Spring, $F(1, 132) = 9.15$, MSE = 26.62, $p < 0.01$. with mean scores of $M = 22.4$ and $M = 19.7$, respectively. Thus, it appears that using these methods can result in significantly better conceptual learning, and using more methods improves the results.

To date, the instructional approaches described above have been used only by one instructor. Our plan for broader implementation is to incorporate these approaches into an interactive computer-based tutorial, so that other instructors can easily assign such work without having to heavily revise their lecture approaches. The interactive tutorial could incorporate simulated or "virtual" laboratory experiments, where students could gain "hands-on" experience related to the ideas we are presenting. This tutorial will be incorporated into our existing Circuit Tutor software package,[13-15] which we plan to distribute through a textbook publisher at some point to ensure its sustainability.

6. Conclusions

Effective instruction requires that we address typical student misconceptions directly, rather than just assuming that telling them about a correct model is enough (it isn't). Comparing and contrasting the behavior of similar but different objects is a powerful method to do so, as well as introducing microscopic models for the foundations of conduction processes. Further, it is important to give students explicit and systematic concepts such as voltage and current splittability wherever possible, rather than relying on their reasoning abilities to deduce such ideas themselves. Introducing some of these instructional techniques has been found to result in significant improvements in student learning of basic electrical concepts, which almost certainly will be important in their future work with electrical circuit analysis and design. Many additional conceptual difficulties remain to be enumerated and addressed, particularly for reactive elements, transient circuits, AC circuits, op-amp circuits, and Laplace transform analysis, for example, and further such work is planned.

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