Foundations for the Mathematical Modeling of the First-Year Introduction to Engineering Course Classification Scheme using Abstract Mathematics

Mr. David Reeping, Ohio Northern University

David Reeping is a junior majoring in Engineering Education with a minor in Mathematics and an undergraduate research assistant. He is a Choose Ohio First scholar inducted during the 2012-2013 school year and the recipient of the Remsburg Creativity Award for 2013 and The DeBow Freed Award for outstanding leadership as an undergraduate student (sophomore male) in 2014. Also, he is a member of the freshman honorary society (Alpha Lambda Delta / Phi Eta Sigma) and the mathematics honorary society (Kappa Mu Epsilon). His research interests involve first year engineering course analysis, authentic projects and assessments, and K-12 engineering.

Dr. Kenneth J Reid, Virginia Tech

Kenneth Reid is the Assistant Department Head for Undergraduate Programs and an Associate Professor in Engineering Education at Virginia Tech. He is active in engineering within K-12, serving on the TSA Boards of Directors and over 10 years on the IEEE-USA STEM Literacy Committee. He was awarded an IEEE-USA Professional Achievement Award in 2013 for designing the nation’s first BS degree in Engineering Education. He was named NETI Faculty Fellow for 2013-2014, and the Herbert F. Alter Chair of Engineering (Ohio Northern University) in 2010. His research interests include success in first-year engineering, engineering in K-12, introducing entrepreneurship into engineering, and international service and engineering. He has written two texts in Digital Electronics, including the text used by Project Lead the Way.
Foundations for the Mathematical Modeling of the First-Year Introduction to Engineering Course Classification Scheme using Abstract Mathematics

Introduction

First-year engineering programs tend to vary in structure at different universities, so rigorous comparisons between programs are naturally difficult to make. These differences become important for programs that intend to study internal consistency among sections or award transfer credit to incoming students. While a program would typically consider “Introduction to Engineering” equivalent to a course of a similar name, the focus of one may be communication while the other could be programming. In this case, awarding credit is a disservice to the student, especially if later courses assume prior knowledge from the introductory course. Therefore, a means to compare courses is desirable as the differences in the student’s experience can then be measured.

Moreover, the ability to evaluate one’s course in the context of curriculum development can also be daunting. The task is made more streamlined using the First-Year Introduction to Engineering Course Classification Scheme, but this tool only quantifies the content (the objectives) of the course. Assessment and any associated performance metrics are not captured directly using this methodology, so while the tool is useful for quantifying course objectives, its use as an assessment tool is limited – especially in the context of a curriculum review. Thus, by construction, the opportunity to remind the users to consider a one to one correspondence between the performance objectives and assessment is lost. Ensuring the balance of assessment and objectives is especially crucial in ensuring that student-learning objectives are being met in terms of program and ABET requirements, which is of concern to first-year faculty.

Interpreting results of the classification scheme is also quite an undertaking, despite the fact only content is being considered. In the proposed model, this process of interpretation can be expedited and may be more powerful, if presented in the right way.

Finally, the concept of defining course foci is of particular interest to the investigators. Course foci are operationally defined as areas in which content is heavily centered. Since the classification scheme is designed to consider the content of a course, defining foci represents a more accurate means to describe a course. While defining course foci was not originally proposed during the development of the scheme, a result of classifying a course would include the ability to describe the course in terms of its foci. This could be especially useful for awarding transfer credit and in the general course-by-course comparisons by allowing a course to be described in terms of a non-self-reported quality.

The advantages of formally defining such differences and relationships between the outcomes, assessment, and performance objectives prompted a study to develop a mathematical model in order to facilitate these investigations. This model would be an extension of the classification scheme and will likely factor into the process after an intermediate step that will account for other instructional variables - which can be feasibly described using mathematical techniques. Thus, the model will accept a course that has been classified in terms of its content, related assessments, and associated performance objectives in order to provide a quantitative summary.
of the course itself. In the case of multiple inputs, the model will generate a measure of similarity between the courses as well.

This paper presents a description of the classification scheme including the foundational methodology behind establishing the proposed model. An overview of mathematical modeling is presented in addition to the benefits of modeling a tool such as the classification scheme. Avenues for abstract analysis that serve as the basis for the development of the mathematics to meaningfully compare courses in the first year will then be discussed. Next steps will be noted as well.

Background

The Classification Scheme

The content within first year engineering courses is often a combination of the instructor’s preferences, learning outcomes dictated by the program, and content driven by accreditation requirements. These courses tend to occupy their own sphere of content and loosely relate to later classes – even to other “Introduction to Engineering” courses at different universities. Through an NSF sponsored study, the *First-Year Introduction to Engineering Course Classification Scheme* was developed in order to allow instructors to describe the content of their courses using a common tool.

The classification scheme is a taxonomy of all topics and outcomes that could be found in general Introduction to Engineering courses. The scheme lists and distinguishes relationships between all material covered in interdisciplinary courses offered as a general introduction to the field of engineering.

The framework for the taxonomy was developed through a mixed methods study that consisted of an analysis of syllabi for Introduction to Engineering courses and a conference workshop where participants consisted of instructors, directors, and professors of these classes gathered for a focused brainstorming session. From these two preliminary methods, two draft classification schemes were created; however, a more complete taxonomy was developed through a Delphi study.

The three-round Delphi study involved participants who were identified as interested in first-year programs. Each participant was asked to identify objectives currently found in his or her courses as well as objectives that should be included in similar courses. The second and third rounds involved presenting the current scheme and asking each participant for suggested changes. Each suggestion from the 31 participants was considered.

In addition, participants were asked to use the scheme to classify their courses to ensure that the taxonomy was useful as a tool. Further verification of the taxonomy has come from two conference workshops and its use in internal studies among sections of a course.2
The final top level of the classification scheme is pictured in Figure 1 which shows the eight main outcomes (or categories) where each of the more specific outcomes are cataloged. The complete classification scheme in a table format can be found in Appendix A.

![Diagram of First Year Engineering Courses]

**Figure 1: Top Level of the Classification Scheme\(^1\)**

**Application 1: Application of the Scheme among Multiple Course Sections**

Two Midwest universities have extensively utilized the classification scheme to reflect upon current practices and determine gaps in content.\(^2\) A self-study exercise was performed by one Midwestern university for six courses which were part of two tracks: a common introductory sequence and a sequence for honors students.\(^3\) Professors and teaching assistants of these courses classified their respective section(s) of “Introduction to Engineering” and generally had agreement in most areas within each of the eight main outcomes; however, discrepancies in topics were discovered within sections covered by each outcome.

In the self-study, the results were organized by main outcome where a three-color coding system was used to show the level of agreement between instructors.\(^3\) An outcome marked as green denoted that the outcome was covered in each section of one or more courses. An outcome marked in yellow meant that the outcome was not marked consistently. Finally, red indicated that the outcome was not covered in any of the courses (which may very well be appropriate).
After reviewing the results of the classification, recommend changes to the first year curriculum were provided.

During a debriefing conference call between the investigators and instructors, the binary value of a “covered” mark (strictly ‘yes’ or ‘no’) was of interest. Differences among instructors in the multiple sections and the possibly subjective nature of deciding whether or not a topic was covered “enough” to earn a mark certainly leads to variability among completed classification schemes for different sections of a course. Through this discussion, the value of establishing a mathematical model became apparent; this proposed model is planned to account for and measure such phenomena. Toward this objective, introducing an expansion of the binary system to include “depth of coverage” or emphasis of a topic is an integral component of the model.

Application 2: Testing the Scheme

Another study of applying the classification scheme occurred during a National Science Foundation sponsored workshop at the First Year Engineering Experience Conference in 2013. During the workshop, samples from 28 different classified courses were collected and analyzed. The study used two different methodologies, namely by course and by outcome analysis. By course analysis involves the examination of the whole course from a top-level perspective using radar charts and by outcome analysis is concerned with analyzing courses by looking at the marked outcomes themselves. Through this analysis, the prevailing traits of the sample could be culled from the 28 classified courses and succinctly summarized. This was accomplished primarily through examination of the marked outcomes, dividing the data into quartiles, and grouping. Currently, the comparisons using by course analysis are the most underdeveloped. Ideally, by course and by outcome analysis can inform one another in order to provide a measure of similarity. Together, these two analysis methods can provide some comparison between two arbitrary courses, which would lead to the development of a satisfactory mathematical model and better formulation of by course analysis.

Mathematical Modeling as a Next Step

A mathematical model is an abstract representation of a real device, object, or system using mathematical terms where the end goal is “added value.” The act of modeling would be no more than an exercise for its own sake if nothing was learned from or improved through developing the complex model. While creating an abstraction of a complicated situation or process should prove useful, it is worth stating that the models do not necessarily need to solve large problems; in fact, many simpler mathematical models are developed daily for simple activities as simple as planning a trip. Developing a model to establish a quantitative summary of a course or to give a measure of similarity among sections of a course or among multiple courses is necessary to go beyond a comparison based simply on a ‘cloud’ of checked objectives within different areas of the scheme.

The method of developing a mathematical model is ideally not a rigid step-by-step process. Instead, the activity of modeling is principled and the approach to developing a successful model is dependent on the investigator’s ability to answer guiding questions as outlined by Dym. In fact, Dym provides a top down view of the process of modeling as inspired by Carson and
Cobelli in Figure 2.\textsuperscript{7,8} The layout incorporates the guiding questions (why, find, given, assume, how, predict, valid, and verified) in relation to the components or steps in the process in which the question is most pertinently related to: the object/system/tool, the model’s variables and parameter, any predictions, the testing of the model, and the final validity.

Mathematical modeling has the potential to be a powerful tool for making predictions; therefore, a carefully constructed model of the classification scheme can be useful to not only make predictions, but also function as a tool for further refinement.

Framework of Study

The methodology employed in this study is led by abstract mathematics. In order to principle this activity and carefully align with the best practices in mathematical modeling, the process in which this model is being developed is outlined in Figure 3.

The proposed framework is appropriately tied to the principles of mathematical model outlined by Dym and the authors, Carson and Cobelli. When “identifying a system to model,” we are asking the questions: “what are we looking for” and “what do we want to know.” Further, “picking the logic and sample space” is addressing the three following questions: “what do we know,” “what can be assumed,” and “how should we look at this model.” Next, “developing the mathematics and finding common patterns” best relates to the next question, “what will our model predict?” Then, “testing the mathematics using realistic constraints” addresses the
questions of “are the predictions valid” and “are the predictions satisfactory.” Finally, “refine” is trivially related to “how can this be improved” and “define a different sample space” is a step further if the particular area of mathematics is not producing anything of value.

The intended result of this iterative process is a programmable mathematical model that has two or more inputs, classified courses, that returns a measure of similarity as an output. This measure of similarity would ideally not be a scalar; instead, it would be a detailed summary of distinct differences between the inputs found using both by course and by outcome analysis. If only one course is taken as an input, then a quantitative summary would be the output. This process is illustrated in Figure 4.
Use of the Proposed Methodology Thus Far

Identify System to Model

Clearly, the first step in the process given in Figure 4 is already well defined. The system chosen to be modeled is the Classification Scheme for First Year Engineering Courses in order to develop a valid method to summarize a course quantitatively and compare one course to another.

Pick logic and create a sample space to examine discrete properties

Ideas from well-established fields of mathematics can be applied to the classification scheme in order to achieve this type of model: namely group theory, set theory, and graph theory. The application of groups aids in the understanding of how courses relate to one another. Set theory provides the basis by which objects are counted through the concept of cardinality, and it can give implications related to sizes of specific clusters of courses. Finally, graph theory helps in classifying the structures that the outcomes in the taxonomy form when plotted with respect to order.

A portion of the work done in the abstract space is for the sake of being thorough; however, that does not mean the methodology is not useful for uncovering subtleties in the system being modeled. For example, the branch of mathematics concerned with structures called sets is called set theory, founded primarily by Georg Cantor as described his seminal paper “On a Property of the Collection of All Real Algebraic Numbers.”

Despite the initial hesitation by the mathematical community, set theory has become the basis for much of the mathematics used today, both abstractly and arithmetically. A set is defined to be a collection of objects, often numbers in the context of mathematics. The freedom of choice with respect to the membership rules of the set provides a convenient method of organizing a large amount of objects abstractly. Due to the construction of set theory, it is well suited to solve a wide variety of problems – the same is true for graph theory.

Another example of an abstract theory providing meaning solutions to physical problems is group theory. Group theory has been invaluable in its application to problems in chemistry, physics, and cryptography to describe symmetry. One type of group, point groups, involves a set of symmetry operators for a given molecule, which are especially useful in explaining the symmetries of molecules. Further, group theory has advanced physics by providing insight to the solutions of the Schrödinger equation where no analytic solution is possible. In a more recreational context, the process of solving a Rubik’s cube can also be described in terms of groups. While group theory is certainly an abstract concept, the field can aid in revealing the symmetries of physical phenomena such as shapes and motions in geometry.

Although this point may seem to be an aside, it must be stressed that development of a general theory concerning application to this taxonomy is invaluable to the generalization to other engineering courses – or any course in general for that matter. The “theorem and proof” approach can seem loosely applicable to the end user of the scheme and mathematical model, but this back-end development is the glue that holds the technique together while still being tolerant of changes to the system.
To ensure that the model is flexible, this framework is done with respect to unspecified parameters, so the interpretations that one can make are general at this stage in development. The model uses a mix of set theory and graph theory as a basis and group theory when appropriate. From a practical standpoint, the properties of the system derived during development will be incorporated in the programmable model that can ease the process of classifying a course and interpreting the results.

**Summary of the Model Development**

The classification scheme is useful as a tool to quantify course objectives, but developing a model to further describe an introductory course would prove useful in comparison among courses or sections and toward a means of assessment. Currently, there are two analyses are proposed that will provide the bulk of the model, *by course* and *by outcome analysis.*

In terms of *by course analysis,* a fully classified course produces multivariate data due to the eight main outcomes found within the classification scheme all being measured at once. One such method to display multivariate data and potentially compare courses or sections is called radar plots (or star plots), as described by Chambers et al.\(^1\) While radar plots are traditionally used to plot data from different variables which are not directly comparable, we claim that there exists some coefficient to relate one variable to another in the context of this application.

One common criticism of radar plots is the lack of meaning in shape since, if the assumption is made that the variables lack any relationship, the order in which the variables are assigned is arbitrary.\(^1\) Klippel, Hardisty, and Weaver discussed interpretations of a radar plot in their study of spatial analysis such as perceived similarity due to the existence of salient shapes or prominent features.\(^1\) As this is a unique application of the radar plot, a coefficient relating variables can ensure there is meaning to their order and the shape generated by the connecting the endpoints of the observations, which would imply that area may also have meaning.\(^1\) Such a coefficient would be in spirit of correlational methods discussed by Ward and supported by Borg and Staufenbiel.\(^1,2\)

In this case, we are considering the radar plot to be the best candidate to begin constructing a mathematical model for the classification scheme for *by course analysis.* Moreover, since the geometric representation of a course using the radar plots is currently the primary candidate for analysis, group theory can be easily applied.

*By outcome analysis* was previously done on a sample of 28 courses by examining the frequency in which the outcomes were marked and splitting the data in quartiles as appropriate. For one course, doing such an analysis is not feasible, as only one observation will be recorded. Thus, the model will incorporate set theory and graph theory to be better describe the relationships between the outcomes. *By outcome analysis* is also expected to produce a plot, but it will be more complex as the individual outcomes will be plotted instead of the total coverage in each main outcome.

The model, or intermediate steps in the development of the model, is intended to include best feedback practices, assessment, and performance objectives to complement the classification
scheme’s focus exclusively on content. Since these aspects of the curriculum are complex, the investigators are taking care in defining an appropriate metric to include them. As mentioned, it is hoped that the model can provide feedback to the user of the classification scheme to consider the one-to-one correspondence between assessment and his or her performance objectives – a convenient self-check often-missed in higher education.

The calculation of foci is also proposed and in development. Since foci can be pictured as ‘spikes’ in coverage, the classification scheme is an ideal basis for their derivation. Edits to the scheme, primarily the elimination of the binary system, will aid the investigator into a more accurate measure into the true depth of coverage identified by the user.

Once the model is developed, validation of the mathematics portion of the model is expected to be straightforward since contradictions are easily spotted. On the other hand, a diverse sample would be needed to ensure that the model is handling the inputted information correctly. If adjustments need to made, the generality provided by the “theorem and proof” approach enable the flexibility to make the needed changes. It is difficult to say how the model will be validated, but this concern can be addressed during the initial testing phase.

Implications

Once the model is complete, instructors will be able to expedite the classification of their courses and be provided with a meaningful analytical summary of their course. As this is an early and broad discussion of the mathematical model for the classification scheme, the principle of the structure is the most important to note. Moreover, the determination of course foci will enable courses to be assigned a non-trivial, non-user reported description of a first year engineering course. This trait will be of interest to those wishing to award accurate transfer credit to students and possibly to funding agencies that desire to identify specific characteristics of courses within proposals. The comparison of engineering courses will also be better defined, as desired by the driving force of this effort.

Future Directions

Primarily, the scheme will need to be edited to reduce the variability when a course is classified multiple times. To account for this in the model, it was determined that adding a “level of coverage” to the existing binary system can be a solution. Assigning an appropriate system to quantify assessments and performances objectives such that there is a nontrivial connection between them in the context of the model is also in development. It is apparent that more data is needed from diverse sources to not only test the scheme, but also to perform any meaning preliminary testing of the mathematical model.

Conclusions

Since the content of first year engineering courses is typically a combination of the instructor’s preferences, program learning outcomes, and accreditation requirements, a more formalized method of comparing courses is a necessity. The First-Year Introduction to Engineering Course Classification Scheme was created in order to facilitate these comparisons by offering a
comparison at the objective level, but in order to expand and improve upon quantifiable comparisons between sections or courses, further development is necessary. In addition, being able to determine best assessment practices and performance objectives in the context of curricula development and revision via some model is desirable.

Further applications of the scheme and rigorous mathematics, both abstract and concrete, will enable the model for the classification scheme to not only inform future hypotheses, but also serve as a tool for the comparison of engineering courses. Moreover, a convenient and programmable method of succinctly summarizing a course’s content, performance objectives, and modes of assessment - both top down and bottom up - can serve as a welcome tool for the first year engineering community.

References


## Appendix A: Complete Table of Outcomes from the Classification Scheme

### Communication (COMM)
- **I. Professional**
  - A. Client Interactions
- **II. Written**
  - A. Reports
    - 1. Lab
    - 2. Documentation
  - 3. Engineering
  - B. Email Writing
  - C. Résumé
- **III. Oral and Visual**
  - A. Presentations
  - (COMM IV.A.0 / ESTT II.D.3)
- **IV. Visual**
  - A. Posters

### Engineering Profession (ENPR)
- **I. Relevance of the Profession**
- **II. Images of Engineering in Today’s Society**
  - A. Roles and Responsibility
- **III. Professional Societies**
  - A. Student Organizations
  - (PROF VI.0.0)
- **IV. Types of Engineering**
- **V. Engineering History**
- **VI. Definition and Vocabulary**
  - A. Nature of Engineering
  - B. Nature of Technology
- **VII. Disciplines of Engineering**
  - A. Intro to Professions
- **VIII. Commitment to Discipline**
  - (ACAD VII.0.0)

### Math Skills (MATH)
- **I. Trig Review**
- II. Calculus
- III. Significant Figures
- IV. Units and Dimensions
- V. Dimensional Analysis
- VI. Linear Regression
- VII. Matrices
- VIII. Abstraction
- IX. Calculations
  - A. Statistics
    - 1. Empirical Functions
  - B. Graphing
  - C. Estimation

### Professional Skills (PROF)
- **I. Critical Thinking**
  - A. Problem Solving
  - (DESN III.0.0)
- **II. Ethics**
  - A. Codes and Standards
- **III. Teamwork**
  - A. Team Management
    - 1. Work Distribution
    - 2. Strength / Weakness ID
  - B. Team Dynamics
- **IV. Research**
  - A. Library Resources
  - B. Quantitative
  - C. Qualitative
- **V. Patent Search**
- VI. Leadership
- VII. Entrepreneurship

### Design (DES N)
- **I. Engineering Design**
  - A. Fundamentals of Design
  - (DESN I.F.3)
    - 1. Mathematical Modeling
    - 2. Physical Modeling
    - 3. Formal Design Process
    - 4. Brainstorming
    - 5. Concept Selection
    - 6. Testing Hypothesis
    - 7. Design Review
    - 8. Refine
  - B. Reverse Engineering
  - C. Research
  - (PROF IV.0.0)
    - 1. User testing
  - D. Creativity and Curiosity
  - E. Empirical Design
  - F. Authentic Design
    - 1. Engineering Feats and Failures
    - 2. Design Projects
    - (PROF III.0.0)
    - 3. Realistic Design
    -(DESN I.A.0)
- **Global Interest (GLIN)**
- **I. Grand Challenges**
  - (DESN I.F.0)
- **II. Concern for Society**
  - A. Assistive Technologies
  - B. Social Entrepreneurship
  - C. Design Safety
  - D. Sustainability
- **III. Biomechanics**
- **IV. Bioinformatics**
- **V. Virtual Reality**
- **VI. Geotechnical Engineering**

### ENGR Tech/Tools (ESTT)
- **I. Engineering Skills**
  - A. Electromagnetic Systems
  - B. Circuits
  - C. Statics
  - D. Mechanics
  - E. 3-D Visualization
  - F. Material Balance
  - G. Thermodynamics
  - H. Sketching
- **II. Software**
  - A. Programming*
  - B. Programming and Design*
  - C. Computer Aided Design*
  - D. Microsoft Office*
- **III. Hardware**
  - A. Shop Experience*
  - B. Topic Specific Tools
    - 1. Bread boarding
    - 2. Arduino Based Project
    - 3. Basic Surveying
    - 4. Laboratory
    - 5. Nanosensors

*Outcomes indexed under this outcome were omitted to conserve space. Please review the complete version of the scheme for the complete list.

---

Note that an outcome in **bold** designates that it and one or more outcomes can be marked off if certain requirements are met. These outcomes were defined to be **tied outcomes** in the paper. The classification scheme defines the non-trivial relationship between the outcomes so the user understands what would constitute an appropriate marking.