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Learning to Integrate Mathematical and Design Thinking in Engineering

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Introduction

In the engineering profession, engineers encounter a wide range of problems, ranging from well-defined to ill-defined, which require different combinations of mathematical and design approaches and skills. Today's future engineers enter college with pre-college experiences which may lead them to have misconceptions about the nature of engineering problems [1]. Oftentimes, they perceive that engineering problems have linear problem solving processes, are well-defined, highly constrained and are quick to solve. To interrogate this misconception, the researchers set the following research goal: To empirically examine the "interplay" between mathematical thinking and design thinking, as experienced by students engaged in open-ended design tasks, and identify situations where mathematical thinking may impede design thinking, and vice versa.

The study that ensued recruited first-year engineering students to spend three hours independently designing a playground for a fictitious neighborhood. Students are asked to "think aloud" as they work in isolation solving this open-ended and ambiguous task. Verbal protocol analysis is the primary research approach and allows the researchers to uncover invisible thought processes. The thought processes are then analyzed using a coding scheme informed by: (1) Cardella's modified version of Schoenfeld's framework for mathematical thinking [2, 3], (2) a framework for design thinking which is informed by previous playground design task studies and (3) emergent themes from the dataset.

This paper will focus on the design and mathematical ways of thinking that first-year engineering students exhibit when solving the playground design task. We anticipate that the findings from this study will inform the way that engineering courses and engineering course activities are designed. If the focus of engineering problem solving is to develop advanced mathematical and design skills in future engineering professionals, students should be taught how to engage mathematical and design "ways of thinking" when solving problems. This work may also have implications for precollege engineering learning environments, which can help students to recognize the mathematical and design knowledge at play in their everyday problem solving experiences.

Design exploration

Design exploration occurs when the designer considers the given task and seeks multiple ways to solve the problem. In design exploration, the designer may spend time generating many different solutions without committing to one. The constraints of the problem provide a boundary within which the designer works. In this space, there may be rapid iterations between design possibilities. However, iterations between design possibilities may be limited by the time, available resources or the experience of the designer [6, 7]. Time might operate as a constraint that could restrict design exploration because the designer may feel that they cannot invest in deeply considering many potential solutions. So instead, they may explore fewer solutions or simply select the best solution and continue with the design process. Additionally, resources may limit design exploration in that a lack of available resources may cause the designer to limit the ways they approach solving the design problem. Also, if there is not adequate familiarity with the diverse ways that available resources can be used toward developing solutions for the tasks, idea generation may also be limited.

Designer experience may impact design exploration. Designers with varying levels of expertise use different design processes [8, 9]. In a previous study, comparisons have been made to observe the diverse processes used by novice and expert engineering designers [10]. Within the context of engineering, designers typically follow a process called the engineering design process. In its simplest form the engineering design process is an iterative process during which the problems is understood, information is gathered, ideas are generation, screened and selected and the solution is modeled, evaluated and communicated [10].

Pre-College design learning experiences and beyond

First-year engineering students are exposed to different engineering design learning experiences prior to entry in to college engineering programs [1]. At the K-12 education level, there is no uniform method by which students learn engineering, especially considering the formal and informal contexts within which engineering can be learned [11]. Therefore, each student's different engineering design learning experience might also have a unique engineering design process or approach associated with it. Some students learn from more hobby-based experiences or learning may occur when they are given a problem to generate a solution. They might test their solution and improve until a proper solution is found [12, 13]. While other students may be exposed to engineering tasks in science or mathematics courses, where they are taught to approach the problem using a scientific method or by using a more linear "plug and chug" process [14]. In some instances, students may have been exposed to the importance of teamwork and collaboration when solving engineering tasks [15]. These experiences may be had during participation in a STEM based afterschool club, such as First Robotics [16]. In these experiences, students learn about how people with different skills work together to more fully address more complex design problems.

There is a diverse set of engineering design experiences, skills and knowledge that pre-college students acquire. As students matriculate, one of the main skills that engineering college graduates obtain from their education is the ability to efficiently and effectively solve a problem [17]. When students with pre-college engineering exposure enter the college engineering classroom, they may experience the tension of trying to reconcile previous experiences with those taught in college [1]. Regardless of their previous experience, there has been an increasing focus on encouraging design exploration and reducing fixation [18] while also teaching students to better integrate mathematical thinking into their engineering design process[6, 9, 19]. What is not fully understood is how students address this tension and the integration of mathematical and design thinking as they are learning to be engineers.

Research Questions

Given the gap in understanding of how first-year engineering students exhibit mathematical and design thinking as they are transitioning from the pre-college to the college engineering environment, the following research questions were developed to guide the investigation.

1. How do students respond to open-ended, ambiguous design tasks?
2. How do mathematical thinking activities impact design thinking activities?
3. How do students' thinking processes differ based on mathematics, design and engineering backgrounds?

This paper specifically presents the findings and discussion in order to investigate the first research question by providing evidence for the diverse ways that students respond to open-ended, ambiguous design tasks. Three objectives guided analysis of the data and will provide the over-arching structure of the findings section:

(a) How do students use their first hour of problem solving in this design task?
(b) How do students score their design and mathematical thinking abilities?
(c) What are their pre-college mathematical and design experiences?

Research Methods

Context

At a large, research focused institution in the Midwestern region of the United States; students begin their engineering careers as first-year engineers, before applying to a specific engineering discipline, within the college. Most first-year engineering students complete two introductory engineering courses, which expose them to compound engineering problems. These problems develop students' problem solving skills as the problems increase in complexity, ill-definedness and context dependence, over the duration of the first course. Students initially work on short, close-ended problems as they develop basic skills in using Excel and statistics, then they work on a model eliciting activity, which helps them learn to integrate mathematical thinking with problem solving processes [20, 21], and finally they work on an eight-week design project where students are tasked with not only solving a problem but identifying (and making an argument for) a problem to address [22].

Study Design

Students are recruited to spend three hours independently designing a playground for a fictitious neighborhood and are compensated for their time. Students are asked to "think aloud" as they work to solving this open-ended and purposely ambiguous task. Verbal protocol analysis and video analysis technique provide a research approach which allows the research team to uncover invisible thought processes. The thought processes are then analyzed using a coding scheme informed by: (1) The second author's modified version of Schoenfeld's framework for mathematical thinking, (2) a design thinking framework informed by previous playground design task studies and (3) emergent themes from the dataset. All of the participants are given a pseudonym in [10] order to preserve their anonymity. Still images of the students as they completed the task are presented in this paper with their informed consent. This study is approved by the Internal Review Board at the participating university.

Participants

This study is part of larger study that is currently underway. To date, 14 first-year engineering students have participated. Findings from their participations in the study will be shared. Figure 1 illustrates the basic demographics of the First-year Engineering students, who participated during their first semester of their engineering programs. Female engineering students have higher representation in this study than their average representation in undergraduate engineering programs. To date, we have had a much greater number of female students express interest in and qualify for participation in the study as compared to males. Also, as can be seen in the table, the majority of the students who participate identify as ethnically White. As recruitment for the

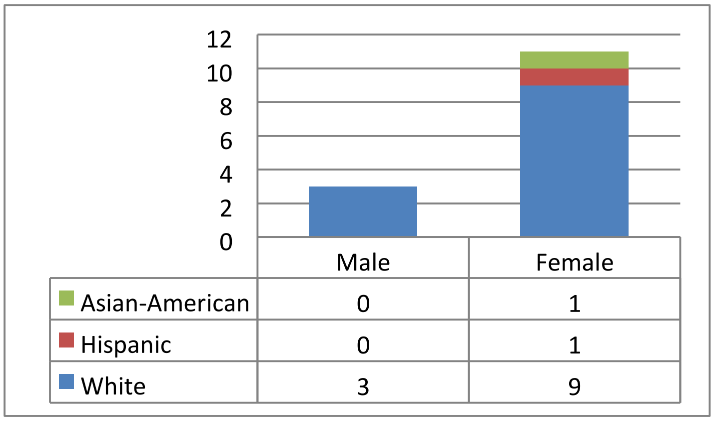

Figure 1:First-year Engineering Participants (n=14)

study continues, the research teams plans to achieve more equal sex representation and ethnic diversity in the sample.

Diversity of experiences and knowledge of mathematical thinking is also considered as students are selected to participate in this study. In order to achieve this, prospective research study participants were asked to provide: (a) a list of all the mathematics courses which they completed in college, (b) their current mathematics course enrollment and (c) their approximate GPA from high school mathematics courses. Selected students were categorized as either having a “low” or “high” mathematics backgrounds. In general, students enter engineering programs having taken Calculus 1 before matriculating to college. Therefore, students who were enrolled in Calculus 1 during their first semester of college were categorized as having a “low” mathematics background. Students whom were enrolled in Calculus II or higher, were categorized as having a “high” mathematics background.

Analysis

Verbal protocol analysis[23]of portions of the think aloud transcripts and interviews along with video analysis techniques will allow the research team to identify the ways in which participants engage in mathematical and design thinking in the engineering design process[24, 25]. In this study, the participants work independently on the three-hour playground design task [26] and are required and encouraged to think aloud as they completed the design task. Artifacts for this study include: audio and video data, drawings, sketches, researcher field notes, internet browsing history, and background information on the students’ mathematical and design experiences, which was collected prior to the start of the design session. Each of the participants design task artifacts will be scored using the Quality Scoring Instrument [26].

A key difference between this administration of the playground design task and earlier studies using the playground task is that participants in this study were able to access information from the internet during our study (this approach is consistent with [27, 28]). As was in the case in previous studies using the playground task, our participants could also request specific information from the administrator (i.e. the cost of wood and screws). A follow-up interview protocol is used to gain insight in to the students’ prior experiences and conceptions about design and mathematics.

In order to identify mathematical and design thinking behaviors in the available data, the research team integrated a coding scheme that included fixation codes and design step and mathematical thinking codes from similar studies [26, 29] along with emergent themes from the pilot data. During the early stages of the study, the coded transcripts of pilot data were analyzed in order to investigate the design of the study and its alignment with the stated research objectives. With respect to fixation, new codes were developed to explore two aspects of fixation: 1) fixation on example designs and underlying principles [4, 5] and 2) fixation as an attachment to solutions, ideas and concepts developed early in the design process. In this study, fixation is viewed as double edged, in that a student may become fixated on an example designs and underlying principles [4, 5]. Additionally, they may exhibit fixation as an attachment to their own early mathematical or design concepts, ideas or solutions.

For this paper, video data was segmented in a manner which represents the breadth of the data available. The analysis of the first hour of the first-year engineering students design tasks work was coded and was visualized using a tool in the NVIVO software. From here the diverse ways that students used their time, during the first hour, were compared. Further analysis of distinct codes within the mathematical thinking and design thinking parent codes was also completed. The findings from this analysis are presented in narrative form.

Findings

The findings from this analysis are represented in the form of nine analytical narratives which provide insights into the diverse ways that the students engaged in mathematical and design thinking behaviors. These narratives will also give the reader insight into how the students featured in the narrative used the first hour as they were completing the design task. The analytical narratives were created from memos written by the research team during data collection and after video coding.

Although all 14 first-year engineering students used unique process, there were some similarities (i.e. brainstorming first, not asking for community preferences). Nine analytical narratives were chosen to represent some of the different ways that students chose to work towards completing the playground design task, during the first hour. Two students, Sabrina and Mark, were selected to highlight the difference in design process used by students with different precollege experiences and perceptions of their mathematical and design thinking abilities.

Peter: Iterative Idea Generation using Information Gathered from Multiple Sources

Figure 2: Peter sketching a design solution

During Peter's design time, he brainstormed and developed a list of potential equipment. Then he determined if the equipment that was currently on the list would accommodate the given constraints. Peter seemed to make quick decisions about materials as he was gathering information online about them. The idea and selection codes seem as if they might be merged, eventually. There is evidence that the information that Peter gathered directly impacted his design. Initially, his slide was going to be straight and flat. After learning that a slide can have a bent end, Peter adjusted his design. See figure 2 for a frame of Peter sketching a design solution. He initially generated a list of ideas and in general did not discard his idea. He often modified his current idea to meet a newly found or newly understood constraint.

Tabitha: Methodical Process which led to Early Integration of Mathematical and Design Thinking

Tabitha began the playground task by stating assumptions about the layout of the lot and the regional location. She reflected on her childhood and remembered playing on the merry-go-round. She did not spend time generating ideas rather she first completely designed and thought about what would be required to build the merry-go-round. Coincidently, she also exhibited mathematical thinking behaviors earlier than most of the participants. In figure 3, Tabitha can be seen using the ruler to provide her a visual aid and context for the dimensions she was creating. An example of feasibility analysis occurred when she considered the implications of using a specific material which will accommodate the weight of both children and adults. Her mathematical language and thinking included: radius, degree, tangent and applying that mathematical concept knowledge the given situation.

Figure 3: Tabitha using the ruler as a visual aid

Sarah: Early Focus on Design Thinking and Inarticulate Decision Making

During Sarah's first hour she spent a majority of her time using design thinking. Early in the task work, she looked online for designs, made a list and selected from that created list. She soon began to communicate her design through written instructions and sketching. It seemed that as she wrote, she made decisions about the size, material and construction. Mathematical thinking was shown when she began to look for the costs associated with specific materials. Her design work is based on the information that she gathered from internet resources and a written but unarticulated thought. It is apparent in many segments that there were quick decisions made as she communicated her design through writing.

Kasira: Questioning Design Decisions through Feasibility Analysis

Kasira often commented: "I want to make sure that it will fit in this space." Variants of this phrase were coded as both feasibility and mathematizing. Kasira spent the first 30 minutes cycling through design ideas. She developed a list and then thought about the feasibility of each item on the list. At times, for example with the swing, she thought about different variants of the equipment before deciding on a specific design. With respect to mathematical thinking, she seemed to most use it when she thought about dimensions of the equipment compared to the dimensions of the lot. Several times she mentioned, "I want to make sure that it will fit in this space." During coding, this was considered mathematizing from the mathematical thinking perspective and feasibility from the design thinking perspective. She spent more of her first hour using design thinking than she did mathematical thinking. She participated in this study two months into her first semester as an engineering student.

April: Design Process with a Focus on the Needs of the Client

As April worked on her playground design, she thought about an urban city and how the needs of the city might be different than a non-urban area. Specifically she thought about a park in New York City. April also asked for information about what the community –the adults and children- wanted on the lot. This is interesting because most participants have not asked for this information. April created a simple ranking system for each of the wants of the parents and children, in order to determine which pieces of equipment that to select. The system is based on the provided percentage ranking of what each group wanted. (This information was provided on the community information sheet, which she requested) She then compared the results and considered what she could actually make. She spent time looking for trends in the community wants and needs. For April there was also some obvious overlap in design and mathematical thinking. In her case, the overlap occurred when she was looking at cost and determining feasibility.

Andrew: Considering the Alternatives throughout the Design Process

Andrew also spent much of his first hour generating ideas and evaluating alternatives. During his first hour, Andrew wrote down his assumptions and decisions made as he made them. There were instances when he made a decision then he discovered other options for approaching the problem. This led him to re-evaluate his decisions. With respect to mathematical thinking, Andrew searched online for the cost of the materials needed to build the playground. For each piece of equipment that he decided to build Andrew developed a material and costs list. It was during this process what he thought about alternatives and evaluated aspects of his design.

Ashley: Identifying "Prior Art" and Screening Ideas

Ashley began her first hour by creating a list of potential pieces of equipment for the playground. Her initial pieces of equipment included: a slide, a see saw and monkey bars. Ashley spent a lot of her time looking for what she considered "prior art", which are existing example drawings and models for the pieces of equipment that she generated in her list. She also spent a lot of time in the idea screening stage of idea generation. She exhibited very little mathematical thinking. All her time is spend generating, screening and selecting and eliminating ideas. The first instance of mathematical thinking occurred during the second hour of participation.

Unexpected Finding: Considering an Engineering Fit vs. Confident about Engineering

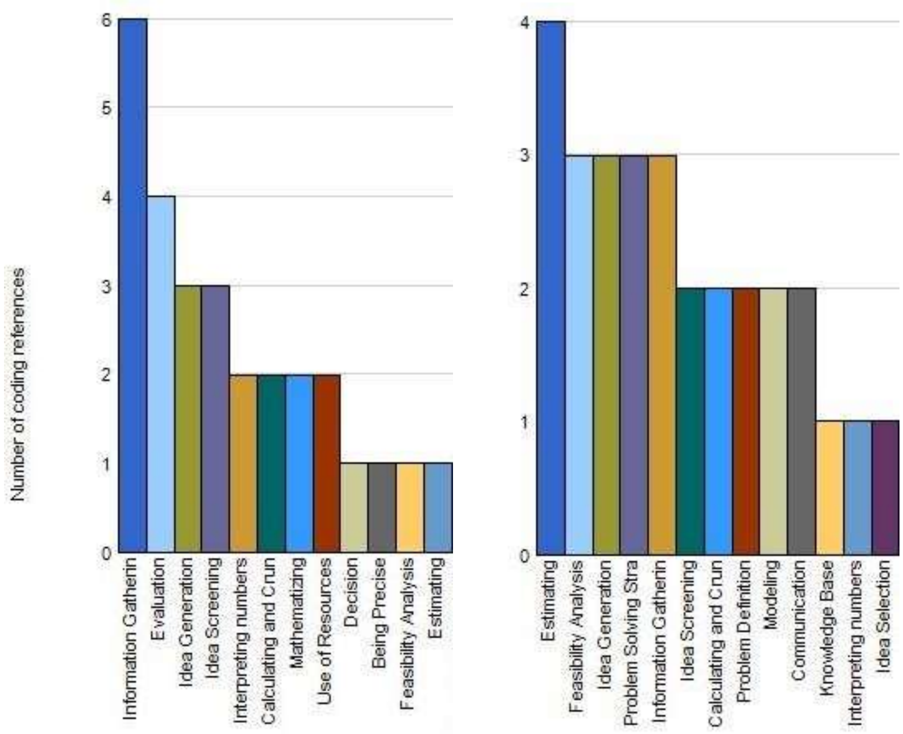

Figure 4: Mark (Left) and Sabrina's (Right) use of Mathematical and Design Thinking

The focus of this paper is to present findings and discussion related to the investigating how students respond to open-ended, ambiguous design tasks (*research question one*). The investigators also explored that data to do a preliminary investigation on how students' mathematical thinking activities impacted their design thinking activities (*research question two*). In the cases of Sabrina and Mark, not only did we identify evidence of how these students' mathematical thinking impacted their design thinking activities, but we also found that mathematical thinking can impact the more affective sense of belonging in engineering. Figure 4 represents the most frequently used coding references for both Mark and Sabrina. One of the more pronounced differences occurred with the estimation code. There were six stances of estimation in Sabrina's first hour as opposed to Mark's single instance. With respect to codes related to developing ideas, Marks spent more of his time generating, screening and evaluating ideas then Sabrina did. One of the unique characteristics of Sabrina's design process was the way and the frequency that she used resources as aids to visualize and determine dimensions. She often used dimensions she was familiar with (i.e. her forearm, the length from her knee cap to the floor) to justify decisions about dimensions made for the playground equipment. During her follow-up interview Sabrina, also expressed frustration with not being really good at mathematics and not having a wealth of design experiences. However, Mark was very confident in his ability to apply his mathematical and design knowledge to this experience. During the follow-up interview, he shared that he had a diverse set of precollege engineering experiences to draw knowledge from. In contrast to Mark's confidence, Sabrina was still questioning if engineering was the right fit for her. She had limited precollege engineering exposure and was already experiencing challenges, which led her to questioning her fit in engineering.

Discussion

The results from this study helped the research team to understand how students respond to open-ended, ambiguous design tasks. Using the first hour of the 14 first-year engineering students' video data the following objectives guided the exploration of the data: (a) How do students use their first hour of problem solving in this design task? (b) How do students score their design and mathematical thinking abilities? and (c) What are their pre-college mathematical and design experiences?

These objectives one provided structure for this work. The research team anticipated observing: how students experienced design fixation, the ways that students engaged in specific mathematical thinking behaviors and the students' use of diverse design processes. In addition, we thought that the using the second author's modified version of Schoenfeld's mathematical thinking framework would allow the researchers to investigate mathematical thinking behaviors beyond the use of equations and language. We also believed that the design thinking framework, would allow the researchers to create a summary of major activities and the overall structure of each student's design process.

The findings support that each student utilizes different mathematical and design strategies. Some students acknowledge that they have a perceived weakness in one or both of these areas, while others exhibit confidence in their own abilities. These varying levels of confidence are often observed by the facilitator during study and validated during the follow-up interview. With respect to design strategies, students participate in this study at different points in the semester. Students who participate after learning new design thinking skills articulate that there is a tension between the design process taught before college and that being taught in their introductory engineering course. This finding resonates with Salzman's [1] study exploring the different ways that students experience the transition of moving from precollege engineering experiences to first-year engineering experiences.

With respect to mathematical thinking, students often commented that they used very simple mathematics but fail to recognize the diverse types of mathematical knowledge they are accessing and applying, to develop a solution to this design task. This finding resonates with the second author's earlier work. Design fixation occurs in most of the participants' data but it does not occur in the same form across the data. From the students investigated in this study it seems that there is a relationship between the mathematical and design strategies exercised by students during this task and their previous engineering, design and mathematics experiences.

Implications for First-Year Program Instructors

Help Students Reconcile Previous and Recently Learning Design Process Knowledge

As seen in this study, first-year engineers are trying to reconcile their previously learned design process knowledge as they are transitioning into their programs and learning new knowledge. We must recognize that students do not enter the classroom as blank slates. They have previous knowledge and experiences which may conflict or support the new engineering design ways of thinking that are being introduced in the first-year engineering curriculum. It is important for students to have an opportunity to investigate how the ways they have previously approached design tasks may support or hinder future problems solving.

Help Students Recognize the Diverse Ways that Mathematical Thinking Can be Applied in Design Tasks

A student's perception of applying mathematical thinking is often limited to using calculations. However, in order to apply a skill most effectively, the students should know what skills they can draw from and how to use them. The data provided instances of students who were aware of the diverse ways of implementing mathematical thinking, also exhibiting different design approaches. It seems that is would be valuable to students if they understood and valued the diverse ways that mathematical thinking can be used in their design processes.

Future Work

Recruitment, data collection and analysis are still underway. The coding scheme used to analyze the data in this study, will be refined. To this end, the research team will meet with other scholars, whom are familiar with mathematical and design thinking, to identify themes that might subsume the current codes. The updated coding scheme will be shared in future publications. In addition, using the new coding scheme, the researchers anticipate that a contribution of this work will be an enhanced characterization of engineering and non-engineering students' design processes. Beyond the intended scope of this publication, the cases of Sabrina and Mark provide a powerful motivator for further investigation of how pre-college mathematics and design preparedness can impact students' self-efficacy, confidence and sense of belonging.

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