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Measuring the Complexity of Simulated Engineering Design Problems

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Abstract

The ability of tomorrow's engineering professionals to solve complex real-world problems is dependent on their education and training. We posit that engineering education and training in design would be improved by presenting students with design challenges with increasing levels of complexity as they advance in engineering curricula. In order to construct design challenges with increasing levels of complexity, a framework for assessing the complexity of engineering design problems must be developed. As a first step toward this goal, we consider the complexity of *simulated* design problems, which have been previously developed as part of virtual engineering internships and which have the advantages of being well-defined and solvable. In this paper, we present a parameterized, mathematical model to quantify engineering design problem complexity. In particular, we present three functions that model the process by which a student moves from information provided and assumptions to predicting design performance and then to a final design choice. These functions are $\boldsymbol{\hat{F}}$, students' predictions of device performance, $\boldsymbol{V}$, how students value performance criteria, and $\boldsymbol{P}$ how students develop preferences for specific designs. Finally, based on this framework for quantifying simulated design problem complexity, we present a metric of complexity, tractability $\boldsymbol{T}$, supported by data from real student work on a simulated engineering design problem.

Theory

Engineering Design Education

Design is a critical part of the engineering profession [1], [2]. As a result, design is a central focus of engineering education in terms of teaching, learning, and assessment [3], [4]. In a recent study, Sheppard and others [5] interviewed faculty and students about the field of engineering and concluded that design is the most critical component of engineering education. One faculty member asserted that "guiding students to learn 'design thinking' and the design process, so central to professional practice, is the responsibility of engineering education" (p. 98).

Two decades ago, ABET, the accreditation board for engineering programs, developed criteria that included opportunities for design learning. ABET [6] defined engineering design as the "process of devising a system, component, or process to meet desired needs. It is a decision-making process (often iterative), in which the basic sciences, mathematics, and the engineering sciences are applied to convert resources optimally to meet these stated needs" (p. 4). The criteria require that students engage in a major design project.

In response to these requirements, universities developed senior-level *capstone courses*. Either in teams or as individuals, students design a product or a process in these courses. They present their work orally or in a final written report, which the instructor evaluates. The basic purpose of these courses is for students to engage in design activities that are based on real-world engineering practice. Harrisberger and others [7] have categorized real-world experiential learning into two categories: simulated and authentic. Simulations are contrived learning conditions that are carefully designed and controlled by instructors. Authentic learning conditions have students solve real problems in real environments, such as an internship within a company. The majority of capstone courses occur over one or two semesters during the final year of the undergraduate program [8].

In contrast, *cornerstone courses* are design courses for first-year undergraduate students. Many of these courses were developed to reduce attrition and increase persistence in engineering by engaging students in design work early in the curriculum [9], [10]. Dym [11] argues that although freshmen students do not have the engineering technical knowledge to engage in design at the level of professional engineers, freshmen are still able to "take chances at putting together components, matching them in a systems-like approach, recognizing performance characteristics and linking components accordingly" (p. 1). In the cornerstone-capstone model, students engage in a design course in the first and final year of the undergraduate program with little design experience in between.

In recent years, as conceptions of engineering design thinking have broadened and become more complex [3], the capstone-cornerstone curriculum model has been shown to be inadequate [12]. Consequently, there are now programs that are taking an *integrated design* approach where design experiences are incorporated throughout the curriculum [13], [14]. This gives students a more holistic engineering design experience, allows time for design thinking to develop, and exposes students to various design scenarios. For example, the biomedical engineering department at the University of Wisconsin-Madison requires that students enroll in six semesters of design courses. Students work in teams to solve a real-world problem posed by a client, which could be a faculty member, clinician, industry partner, and or a person in the community with a biomedical challenge. Teams are advised by faculty members and in some courses, novice students are mentored by senior students. Several of the students' design work has resulted in winning national competitions, journal publications, and numerous patents for the developed products [15], [16].

Following the University of Wisconsin-Madison example, there is a movement towards greater integration of design throughout undergraduate engineering curricula [4]. If integrated design is an effective and desirable method for exposing engineering students to design learning, then design problem complexity must be adjusted for students at various levels, from the novice first year to the more experienced senior student.

Decision Based Engineering Design

Two obvious aspects of design problem complexity are the number of performance criteria the client has for the final device and the number of design options that the student must consider. The more functions the device must perform and the higher the standards of performance, the more challenging it is to solve; the more open-ended the design problem, the more daunting the task for the student engineer. Independent of the number of input choices and output parameters, a design problem is more difficult to solve if the student has less information about the problem and less knowledge about how performance criteria depend on design choices. The process of selecting a final design based on information, knowledge or assumptions, input choices and output parameters is one aspect of decision-making. In considering the complexity of solving design problems, here we focus on this decision-making aspect.

In the literature, engineering design decision-making has been parameterized and represented mathematically. For example, Hazelrigg [17] argues for a mathematics of design based on decision theory [18], which is now identified as decision-based engineering design. Hazelrigg constructed a set of axioms for designing and formulated two theorems that could be applied to statistical models that account for uncertainty, risk, information, preferences, and external factors. For example, the expected utility theorem states that given a pair of designs, each with a range of possible outcomes and associated probabilities of occurrence, the preferred choice is the alternative that has the highest expected utility. Relatedly, Tian and others (1994) have shown that uncertainty plays a large role in engineering design decision making. They argue that uncertainty occurs in engineering design because in many cases, performance parameters can only be estimated, particularly manufacturing costs. This uncertainty affects the designer's perception of the desirability of choices and a risk analysis may come into play. Building on this work, Thurston [19] developed a model that considers tradeoff decisions under uncertainty and models how the designer may choose to optimize several performance parameters at a time. Radford and Gero [20] describe design as a goal-seeking activity and developed a model that focuses on optimization of design goals. Finally, others have explored pairwise analyses when making design decisions [21], [22]. To date, these models have not been applied to student engineers in a learning environment or studied within a well-defined and solvable design space such as a simulated engineering design problem.

Simulated Engineering Design

In order to develop a mathematical model to quantify engineering design problem complexity for integrating design throughout undergraduate engineering curricula, we considered how students solve *simulated* engineering design problems. An advantage of this approach is that the numbers of input choices and performance parameters, as well as the information provided, are fixed such that student decision-making is the critical factor in design problem complexity.

The simulated design problems considered in this study are within virtual internships that our group has previously developed for first-year introduction to engineering design courses: *Nephrotex* and *RescuShell*. As has been described in detail elsewhere [23], [24], students in *Nephrotex* role-play as interns to design a filtration membrane for a hemodialysis machine and students in *RescuShell* design an exoskeleton to assist rescue workers. In each internship program, students log on to a web interface that simulates a company work portal where they receive tasks from a supervisor. Individually they conduct background research, summarize customer requests and technical constraints, and then, in teams, design and test several devices before deciding on a final prototype. When deciding on a final prototype, students consider conflicting stakeholder requests and choose a design that best meets all of the stated thresholds. For example, the clinical engineer is concerned about blood cell reactivity and flux, and the manufacturing engineer values reliability and cost. At the end of the course, students present their work to their colleagues and instructor.

We use the simulated design problems in *Nephrotex* and *RescuShell* as representations of real design problems and examine how to measure design problem complexity by focusing on student decision-making. Specifically in this study, we (1) describe and define the design parameters and variables related to decision making in *Nephrotex*, (2) mathematically describe the decision making process in *Nephrotex*, (3) define a novel metric for quantifying the difficulty of solving these simulated design problems, and (4) compare the difficulties or complexities of the simulated engineering design problems within *Nephrotex* and *RescuShell* supported by student data.

Framework for Assessing Simulated Design Problem Complexity

Elements in the Design Space

In all design problems there are a set of inputs (design choices, parameters or specifications) and outputs (functions or performance). Mathematically, the space[1] of inputs can be described as a vector $I = [i_1, i_2, \ldots, i_p]$ where each set i represents an input category. In turn, each input category i is composed of a set of choices, $C_i = \{c_1^i, c_2^i, \ldots, c_r^i\}$ where each element c^i within C_i represents a choice within category i. These choices can be either numerical or categorical. Categorical variables may be ordered or unordered. Examples of input categories can be seen in Table 1.

	Ordered	**Unordered**
Numerical	{15, 25, … , 100} {4,16, … ,65536}	---
Categorical	{low, medium, high} {poor, fair, … , excellent}	{red, blue, … , pink} {steel, polymer, … , aluminum}

Table 1. Examples of input categories

For example, in the virtual internship, *Nephrotex*, the inputs space is described as

$$I = [material, surfactant, process, carbon\ nanotube(cnt)].$$

The choices for the first three categories material, surfactant, and process are categorical where

$$C_{material} = \{PMMA, PSF, PRNLT, PESPVP, PAM\}$$

$$C_{surfactant} = \{hydrophillic, negative\ charge, steric\ hindrance, biological, none\}$$

$$C_{process} = \{phase\ inversion, dryjet\ wet, vapor\ deposition\}.$$

The choices for the last category, cnt, are numeric and ordered where

$$C_{cnt} = \{0, 0.5, 1, 1.5, 2, 4, 6, 10, 15, 20\}.$$

[1] Spaces here refer to a set with some structure and are denoted by capital letters (e.g. I). Elements within spaces are denoted by lowercase letters (e.g. $i \in I$).

A potential solution for a design problem, i.e., a set of design choices within the design space, can be described as a vector $x = [x_{i_1}, x_{i_2}, \dots, x_{i_p}]$ for which there is one choice for every input category, i. All solution vectors, x, are within the solution space X, that is $x \in X$ where $X = \times_{i \in I} C_i$, the cross product of all the possible choices in all the categories. That is, X is a space defined by all possible combinations of choices for each input category. There are 4 input categories in *Nephrotex*, so x is always a vector with 4 elements, $[x_{material}, x_{surfactant}, x_{process}, x_{cnt}]$. One example of a solution (or possible device design) in *Nephrotex* is

$$x = [PAM, Hydrophilic, Phase\ Inversion, 2\%].$$

In addition, there is a larger input space, $\dot{X}$, that is defined as the space of *all theoretical* input combinations that may not be available to the designer, potentially because of technical or financial limitations. Thus, the space X is more clearly defined as the set of all input combinations that are available to the designer where $X \subset \dot{X}$.

In design problems, there is also an output space, where $O = [o_1, o_2, \dots, o_n]$, in which each element of o is an aspect of design function or performance. The performance of every solution to the design problem, which we call y, must reside within the output space, that is $y \in O$. In general, the performance is a n dimensional vector $[y_1, y_2, \dots, y_n]$ for which there is one real number value[2] for every output category. In *Nephrotex*, there are five aspects of performance for which the student is designing, so the vector O has five components:

$$O = [marketability, cost, reliability, flux, blood\ cell\ reactivity\ (bcr)].$$

A representative solution to *Neprhotex* is a device with marketability 600,000, cost \$120, reliability 8 hours, flux 23 m^2/day and blood cell reactivity 43.3 nanograms/mL. Thus, the performance of this device can be describe as:

$$y = [600000, 120, 8, 23, 43.4]$$

where

$$y_{marketability} = 600000, y_{cost} = 120, y_{reliability} = 8, y_{flux} = 23, y_{bcr} = 43.4$$

The Design Function, F

In all design problems, the selection of inputs affects the performance of the device. Thus, the design function can be represented as a mapping from the solution space, X, to the performance space, Y, which is a subset of real values in the output space

$$F: X \to Y \subseteq \mathbb{R}^O$$

Or

[2] We understand that performance can be measured qualitatively in some design problems, but this paper examines performance parameters that can be represented quantitatively

$$f(x) = y$$

Where x is a solution vector and y is a performance vector. For example, in *Nephrotex*,

$$F([PAM, Hydrophilic, Phase\ Inversion, 2\%]) = [600000, 120, 8, 23, 43.4].$$

Similar to Thurston (2006), we claim that the performance vector $y = [y_1, y_2, \dots, y_n]$ is defined in terms of the solution vector $x = [x_{i_1}, x_{i_2}, \dots, x_{i_p}]$ as a vector of functions $f = [f_1, f_2, \dots, f_n]$ such that

$$f_1\left(x_{i_1}, x_{i_2}, \dots, x_{i_p}\right) = y_1$$

$$f_2\left(x_{i_1}, x_{i_2}, \dots, x_{i_p}\right) = y_2$$

$$\vdots$$

$$f_n\left(x_{i_1}, x_{i_2}, \dots, x_{i_p}\right) = y_n$$

If we assume a linear model, we can represent the design functions as a matrix equation (Eq. 1) where a_{ij} are coefficients and b_j are coefficients:

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} + \begin{bmatrix} a_{11} & \cdots & a_{1i_p} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{ni_p} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{i_p} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Eq. 1

In real-world complex design scenarios, the engineer may have to decompose Eq. 1 into several smaller equations that do not contain interdependencies and thus are easier to solve. For example, in the filtration membrane design in *Nephrotex*, it's not possible to improve the performance parameter, y_{flux} by increasing x_{cnt} without worsening y_{cost}. Thus, the engineer may have to identify which feasible combinations of performance parameters will best lead to good design choices [19], [25].

To begin solving Eq. 1 (or some subsets of Eq. 1, if it has been separated into several equations), engineers determine the coefficients and constants by collecting information in terms of conducting research, running experiments, and performing analyses.

Approximate Understanding of the Design Function, $\boldsymbol{\hat{F}}$

In an ideal case, the engineer would have enough information and knowledge to determine the true design function F and all possible relationships between the solution space, X and the performance space, Y. Then, she could find an optimum design. However in real-world scenarios, the design space is vast and complex, and the engineer may not have all the information or background necessary to choose one optimum solution. It is quite often the case that the engineer has a partial or approximate understanding of the design function. This approximate understanding of the design function is represented by $\widehat{\boldsymbol{F}}$, and is dependent on I, the information that the engineer has gathered about the design problem, and is dependent on A, the assumptions the engineer is making about the design problem. Thus, $\widehat{\boldsymbol{F}}$ is a function that maps a solution vector x (an element in the solution space X) to $\hat{y}$ (an element in the approximate performance space $\hat{Y}$)

$$\hat{F}_{I,A,P}(x): X \to \hat{Y}$$

or

$$\hat{F}(x, I, A) = \hat{y}$$

where x is a solution vector and $\hat{y}$ is the engineer's approximation of y.

Because $\hat{y}$ is a representation of the engineer's approximation about the performance of the design, we can think of $\hat{y}$ as a vector of probabilities. That is, the engineer is not certain of the value of y for a solution, x, and as a result, has some possible values in mind as to what y could be. Thus, we can think of $\hat{y}$ as a vector of

$$\hat{y} = [\xi_1, \xi_2, \dots, \xi_o]$$

where each ξ_j is a random variable whose distribution represents the predicted values for y_j and their likelihoods based on available information I, assumptions A. In other words, the distribution $\hat{y}_j$ is a distribution of various values of what the engineer thinks y_j could be.

For example, in *Nephrotex*, students receive partial information in the form of experimental reports that contain data about the design function. At one point in the virtual internship, a student may receive information about previous experiments the company has run using input choices that are of interest for the current design. The information in Table 2 contains two choices from the *material* input category, PMMA and PSF, and their performances on one output, *flux*.

	$x \in X$				y_{flux}
Experiment	$i_1 = Material$	$i_2 = Surfactant$	$i_3 = Process$	$i_4 = CNT$	$o_4 = flux$
E_1	PMMA	Hydrophilic	Phase Inversion	0%	10
E_2	PSF	Hydrophilic	Phase Inversion	0%	12
E_3	PMMA	Biological	Phase Inversion	0%	10
E_4	PSF	Biological	Phase Inversion	0%	11
E_5	PMMA	Negative Charge	Phase Inversion	0%	15

E_6	PSF	Negative Charge	Phase Inversion	0%	14

Table 2. Information students receive for six experiments for one output, flux in *Nephrotex*

Each experiment can then be represented as a relationship between x and y. For example, the first two experiments are:

$$E_1: \{x^1 = [PMMA, Hydrophilic, Phase\ Inversion, 0\%], y^1 = [NA, NA, NA, NA, y_{flux} = 10]\}$$

$$E_2: \{x^2 = [PSF, Hydrophilic, Phase\ Inversion, 0\%], y^1 = [NA, NA, NA, NA, y_{flux} = 12]\}$$

And thus, the information that the student has in this case is

$$I = \{E_1, \dots, E_6\}$$

We have also collected data on the assumptions that students are making while solving design problems[3]. One trend that we have seen is that some students assumes separability, that is they assume that the input choices are independent of one another and do not have an interaction effect. Thus, the set of assumptions may be represented as

$$A = \{A_1 = separability, \dots, A_n\}$$

With this information and his assumptions, the student may make inferences about the two materials' performances. The information he has about the material's flux performance is represented in Table 3.

PMMA	PSF
$y^1_{flux} = 10$	$y^2_{flux} = 12$
$y^3_{flux} = 10$	$y^4_{flux} = 11$
$y^5_{flux} = 15$	$y^6_{flux} = 14$

Table 3. Student's understanding of the effects of PMMA and PSF on flux for six experiments for one output, flux, in *Nephrotex*

Thus, the student's understanding of how the materials, PMMA and PSF, perform in terms of flux can be represented as a distribution:

$$\forall\ x_{i_2} \in C_{i_2}, x_{i_3} \in C_{i_3}, x_{i_4} \in C_{i_4}\ \ \hat{f}_{flux}\left([\boldsymbol{PMMA}, x_{i_2}, x_{i_3}, x_{i_4}], I, A\right) = \hat{y}_{flux} = \begin{cases} 10, p = .667 \\ 15, p = .333 \end{cases}$$

$$\forall\ x_{i_2} \in C_{i_2}, x_{i_3} \in C_{i_3}, x_{i_4} \in C_{i_4}\ \hat{f}_{flux}\left([\boldsymbol{PSF}, x_{i_2}, x_{i_3}, x_{i_4}], I, A\right) = \hat{y}_{flux} = \begin{cases} 11, p = .333 \\ 12, p = .333 \\ 14, p = .333 \end{cases}$$

[3] It is beyond the scope of this paper to present the detailed student data on assumptions while designing. This data will be presented in future papers.

The Valuation Function V

The engineer will then compare elements $\hat{y}$ to the original functional requirements to determine if the product is performing according to technical constraints and functional requirements, to the best of the engineer's knowledge. However, because each $\hat{y}_j$ is a probability distribution, an engineer will typically produce a summary statistic to represent the distribution in order to compare to the functional requirements and to compare performances between devices. More specifically, the engineer can calculate one summary statistic for every $\hat{y}_j$ using a valuation function, **V** that maps a vector, $\hat{y}$ (an element within the space $\hat{Y}$) to a set of all real numbers

$$\mathrm{V}: \hat{Y} \rightarrow \mathbb{R}^{O}$$

This function assigns a value $v_j(\hat{y}_j), \forall \hat{y} \epsilon \hat{Y}, j \epsilon O$ which represents a summary statistic of the distribution $\hat{y}_j$. The valuation vector, v, then contains one value for every element in $\hat{y}$.

There are many ways that an engineer might apply a valuation function and calculate the summary statistic, v_j. For example, she could calculate the mean, median, or mode of the distribution or consider the maximum or minimum.

Determining Valuation by Calculating the Mean, Median, Mode, Minimum, or Maximum

In one scenario, an engineer may approach the valuation process by calculating the mean of $\hat{y}_j$. Consider the example from Table 3. If the engineer determines the valuation of PMMA by calculating the mean of the distribution for a solution vector that has PMMA then,

$$v_{flux}(\hat{y}_{flux} = [10,10,15]) = 11.67$$

And for a vector that has PSF,

$$v_{flux}(\hat{y}_{flux} = [12,11,14]) = 12.33$$

That is, based on the information the engineer has about the design problem, she makes an inference that the flux of a device that has PMMA as a material choice may be equal to 11.67 and the flux of a device that has PSF as a material may be equal to 12.33.

Determining Valuation from Stated Constraints or Functional Requirements

Now let's say because of safety concerns, the engineer has a technical constraint of 11 for flux, meaning that device may not have a flux of less than 11. She may apply a binary function where if the device passes the threshold, it receives a 1, and if it is less than the threshold, it receives a 0 (Table 4).

Experiment	PMMA	Passes Threshold
1	10	0
3	10	0
5	15	1

Experiment	PSF	Passes Threshold
2	12	1
4	11	1
6	14	1

Table 4. Whether a material has passed threshold (1) or not passed threshold (0) for flux

Based on this information, she may choose to apply the valuation function by taking the minimum value,

$$v_{flux}(\hat{y}_{flux} = [10,10,15]) = 0$$

$$v_{flux}(\hat{y}_{flux} = [12,11,14]) = 1$$

Of course, if there are various thresholds that the device could meet, the valuation function may become more complicated.

Furthermore, the engineer may use different valuation functions for each output.

The Partial Ordering Function P

The examples above discussed processes for valuing a single output. However, quite often the engineer will be working with two or more outputs during the design process and will have to evaluate potential solutions across several outputs. Let's consider the previous example and assume that the engineer is calculating the mean value in the distribution for v_{flux}. Let's also assume the engineer is now focusing on two outputs, flux and bcr.

Experiment	PMMA		Experiment	PSF	
	Flux	BCR		Flux	BCR
1	10	56	2	12	72
3	10	60	4	11	46
5	15	48	6	14	53

Table 5. Student's understanding of the effects of PMMA and PSF on flux and bcr for six experiments in *Nephrotex*

PMMA		PSF	
v_{flux}	v_{bcr}	v_{flux}	v_{bcr}
11.67	54.67	12.33	57

Table 6. Valuation based on calculating the mean for PMMA and PSF on flux and BCR

The student then continues collecting information about the relationships between inputs and outputs and applying valuation functions until a set of feasible devices is generated for testing. Let's assume that a student has selected a set of feasible devices in *Nephrotex* where

$$x = [x_{material}, x_{surfactant}, x_{process}, x_{cnt}]$$

and

$$\hat{y} = [\hat{y}_{marketability}, \hat{y}_{cost}, \hat{y}_{reliability}, \hat{y}_{flux}, \hat{y}_{bcr}],$$

meaning that for every feasible device, the student has an approximate understanding of what the performance will be. The student also will apply a valuation function to each $\hat{y}_j$ to create a valuation vector for each device such that:

$$x^1 = [PMMA, Hydrophilic, Phase\ Inversion, 10\%]$$

$$v(\hat{y}^1) = [y_{marketability} = 300000, y_{cost} = 150, y_{reliability} = 8, y_{flux} = 11, y_{bcr} = 75]$$

$$x^2 = [PMMA, Biological, Phase\ Inversion, 4\%]$$

$$v(\hat{y}^2) = [y_{marketability} = 400000, y_{cost} = 130, y_{reliability} = 9, y_{flux} = 13, y_{bcr} = 30]$$

$$x^3 = [PSF, Hydrophilic, Vapor\ Deposition, 4\%]$$

$$v(\hat{y}^3) = [y_{marketability} = 700000, y_{cost} = 100, y_{reliability} = 13, y_{flux} = 13, y_{bcr} = 85]$$

At this point, the student must have some way of comparing the devices in order to choose a final device. The comparison between devices is done through a partial ordering of $\hat{Y}$ defined by the function ***P***:

$$P: \mathbb{R}^O \times \mathbb{R}^O \to \{-1,0,1\}$$

where

$$P\big(v(\hat{y}^1), v(\hat{y}^2)\big) = \begin{cases} 1\ \ if\ \hat{y}^1 \succ \hat{y}^2 \\ 0\ \ if\ \hat{y}^1 \sim \hat{y}^2 \\ -1\ if\ \hat{y}^1 \prec \hat{y}^2 \end{cases}$$

That is, the partial ordering of $\hat{Y}$ indicates, for each pair of designs, whether one is preferred over the other. The symbol $\succ$ refers to something being preferred over another, $\prec$ refers to something being less preferred over another, and $\sim$ indicates no preference. The result of **P** then is a square matrix where the rows and columns are all possible $\hat{y}$ vectors consisting of -1, 0, and 1 values.

In the simplest case, P_0, the engineer finds the device that performs better than the other on *all* outputs:

$$P_0: v(\hat{y}^1) \succ_\epsilon v(\hat{y}^2) \Rightarrow \forall j \in O, \hat{y}_j^1 > \big(\hat{y}_j^2 + \epsilon_j\big)$$

where ϵ_j is some allowance for sensitivity[4]. P_0 is a partial ordering such that one device is better than another if and only if it is better on all outputs.

[4] For example, if choosing between two items, if one item costs, .0001 cents less, I will not prefer it over the other.

For example, consider again the three feasible devices selected above from *Nephrotex*. Using P_0 to compare $v(\hat{y}^2) = [400000,130,9,13,30]$ and $v(\hat{y}^1) = [300000,150,8,11,75]$ would yield 1 because $v(\hat{y}^2)$ performs better on every output[5] when compared to $v(\hat{y}^1)$. However, comparing $v(\hat{y}^2) = [400000,130,9,13,30]$ and $v(\hat{y}^3) = [700000,100,13,13,85]$ does not yield a result because there neither device performs better on *all* outputs compared to the other. The entire partial ordering matrix for P_0 for this example would be:

	$v(\hat{y}^1)$	$v(\hat{y}^2)$	$v(\hat{y}^3)$
$v(\hat{y}^1)$	0	-1	0
$v(\hat{y}^2)$	1	0	0
$v(\hat{y}^3)$	0	0	0

Table 7. Pairwise comparison given criteria that a device is preferred if it performs better on all outputs.

As seen by the example above, in many design situations there may be solutions that do not yield a result, and thus this method may not be desirable for many design scenarios.

The next most complex model, P_1, would allow one device to be superior to another on more than half of the outputs

$$P_1: v(\hat{y}^1) \succ_{\epsilon,\delta} v(\hat{y}^2) \Rightarrow \sum_{j \in O} \mathbb{1}_{\left\{v_j(\hat{y}_j^1) > \left(v_j(\hat{y}_j^2) + \epsilon_j\right)\right\}} > \left(\frac{|O|}{2} + \delta_j\right)$$

Using the same example, $v(\hat{y}^3)$ would be preferred over $v(\hat{y}^1)$ as demonstrated in Table 8.

	Marketability	Cost	Reliability	Flux	BCR
$v(\hat{y}^1)$	300000	150	8	11	75
$v(\hat{y}^3)$	700000	100	13	13	85
Preference	$v(\hat{y}^3)$	$v(\hat{y}^3)$	$v(\hat{y}^3)$	$v(\hat{y}^3)$	$v(\hat{y}^1)$

Table 8. Preference of performance vectors given criteria of performing better on more than half of the outputs.

$$\sum_{j \in O} \mathbb{1}\left\{v(\hat{y}_j^3)\right\} = 4 > \sum_{j \in O} \mathbb{1}\left\{v(\hat{y}_j^1)\right\} = 1$$

While this may be a common decision rule where one device is better on more categories than another device, it doesn't account for the fact that there may be certain outputs that may be weighted as more important than other outputs.

In the next more complex scenario, P_2 calculates a linear combination for every v_j for every device with different linear constants, β_j, for each v_j:

[5] A high marketability, reliability, and flux is desirable, and a low cost and bcr is desirable.

$$P_2: v(\hat{y}^1) \succ_\epsilon v(\hat{y}^2) \Rightarrow \sum_{j \in O} \beta_j \, v_j(\hat{y}_j^1) > \sum_{j \in O} \beta_j \, v_j(\hat{y}_j^2) + \epsilon$$

Using the same example, let's assume the engineer considers bcr four times more important than the other outputs and thus, $\beta_{bcr} = 4$ and $\beta_{marketability} = \beta_{flux} = \beta_{cost} = \beta_{reliability} = 1$. However, before the engineer does a partial ordering where she sums across all v_j, she must use another valuation function that converts each v_j so that they are all on the same scale.

Then let's assume she applies a ranking (1 = lowest performing, 2 = highest performing) comparing across devices for each v_j (Table 9).

	Marketability	Cost	Reliability	Flux	BCR
$v(\hat{y}^1)$	1	1	1	1	2
$v(\hat{y}^3)$	2	2	2	2	1

Table 9. Ranking of outputs for device 1 and 3.

Then, she applies the coefficients, β_j and sums across v_j for each device (Table 10).

	Marketability	Cost	Reliability	Flux	BCR	$\sum_{j \in O} \beta_j \, v_j(\hat{y}_j)$
$v(\hat{y}^1)$	1	1	1	1	2*5	14
$v(\hat{y}^3)$	2	2	2	2	1*5	13

Table 10. Ranking outputs for device 1 and 3 with the weighted sum of the rankings.

In this case, the first device has a score of 14 and the second device has a score of 13.

$$\sum_{j \in O} \beta_j \, v_j(\hat{y}_j^1) = 14 > \sum_{j \in O} \beta_j \, v_j(\hat{y}_j^3) = 13$$

And thus, using this method, $v(\hat{y}^1)$ would be preferred over $v(\hat{y}^3)$.

In some cases the engineer may not apply a linear combination model and instead apply a more complex function to the valuation vectors. Model P_3 accounts for this scenario by applying a function, u to the valuation vectors:

$$P_3: v(\hat{y}^1) \succ_\epsilon v(\hat{y}^2) \Rightarrow u(v(\hat{y}^1)) > u(v(\hat{y}^2)) + \epsilon$$

And finally, there are other cases not accounted for here in which P_4 can be other forms of partial ordering of $v(\hat{y})$.

The Tractability Function, T

Development of T

Taken together, the functions $\hat{\boldsymbol{F}}$, $\boldsymbol{V}$, and $\boldsymbol{P}$ describe the engineer's decision making processes. Based on these processes, we can now explore the complexity of a design problem and how solvable the problem is. In particular, we focus on the optimization aspect of measuring the complexity of a design problem—by examining how difficult is to obtain a device that meets as many functional requirements as possible.

We define *tractability* as how solvable the problem is, which is determined by how difficult it is to obtain a device that meets as many functional requirements as possible. The tractability function, $\boldsymbol{T}$, depends on the number of criteria that the engineer is trying to optimize, the type of valuation functions and partial ordering functions applied in the design scenario, as well as how the engineer is determining the quality of devices.

For example, an engineer may use a valuation method where they assign a value based on a device passing a certain threshold (see table 4), and then apply the partial ordering function (P_3),

$$P_3\colon v(\hat{y}^1) \succ_\epsilon v(\hat{y}^2) \Rightarrow u(v(\hat{y}^1)) > u(v(\hat{y}^2)) + \epsilon$$

where

$$u\big(F(x)\big) = \sum_{j \in O} v_j(y_j)$$

That is, the engineer calculates the quality of a device, u, by summing over the valuation vector. The quality score in this case, u, represents the number of thresholds that the device meets. In this scenario, if there are many devices in the design space that satisfy the given thresholds, satisfactory devices are easier to produce, and as a result the problem is relatively simple. If there are very few devices that satisfy the given thresholds, then the problem becomes more difficult because satisfactory devices may be difficult to produce.

Thus, we define tractability, $T(F)$, as the total quality of all devices relative to the maximum quality achievable. Formally, the function is defined as:

$$T(F) = \frac{\sum_{x \in X} u\big(F(x)\big)}{U|X|}$$

and where $u(F(x))$ represents some function that determines quality and where U is the highest quality solution

$$U = \max_{x \in X}(u(F(x)))$$

Essentially, the tractability function finds each device's percentage of the maximum possible quality and then calculates the mean of those percentages. Thus, every design problem has a tractability value that measures how easy it is to obtain a quality solution. These values range from a minimum of 0 to a maximum of 1. In the extreme case where tractability equals 1, then the problem is trivial—that is, every device $x \in X$ achieves the maximum quality, U. In the most difficult case, the tractability equals 0 and there are no devices that meet any thresholds.

Support of the Theory from Student Design Work

Identification of V, U, and P in student decision-making

In *Nephrotex,* we can calculate ***T*** by considering the specific approach that students use to determine valuation and how they calculate a score to represent the quality of the device.

For example, one student in *Nephrotex* executed a ranking system (1=best performing, 3 = worst performing) and rated each manufacturing process choice, $x_{process}$, on four output values, y_{flux}, $y_{reliability}$, y_{bcr}, and y_{cost} (Figure 1). The student then summed across the rankings and determined a quality score (7, 8, or 9) for each device with that particular manufacturing process. Thus, the student used a valuation function that assigned rankings and summed up the rankings (similar to the example given in table 9).

He explained how the ranking method helped him selected a manufacturing process component:

> **I ranked between the 3 processing methods from 1 to 3** how they rank compared to the other ones, and **I totaled up the scores to see which ones were the lowest**, so that would have the most closest to one. I found the dry jet wet, but I wanted to choose the vapor deposition one because it was only one off and it had a higher flux rate and it was more reliable but it was more expensive and had a higher BCR. **I guess the only reason I didn't pick it was for the steric hindrance--it has one of the highest flux and this one has like the lowest flux so it equaled out to not terrible.**

Nephrotex

Product 507-1128

Poly(ethersulfone) – Polyvinylpyrrolidone

Ultra-Flux Model

Manufacturing Process	Phase Inversion	
Flux Rate	13 m^3/m^2day	1
Time to 75% Performance	3 hours	3
Blood Cell Reactivity	78 ng/ml	3
Cost per unit	$60.00	2

9

High-Flux Model

Manufacturing Process	Vapor Deposition Polymerization	
Flux Rate	11 m^3/m^2day	2
Time to 75% Performance	6 hours	1
Blood Cell Reactivity	77 ng/ml	2
Cost per unit	$70.00	3

8

Low-Flux Model

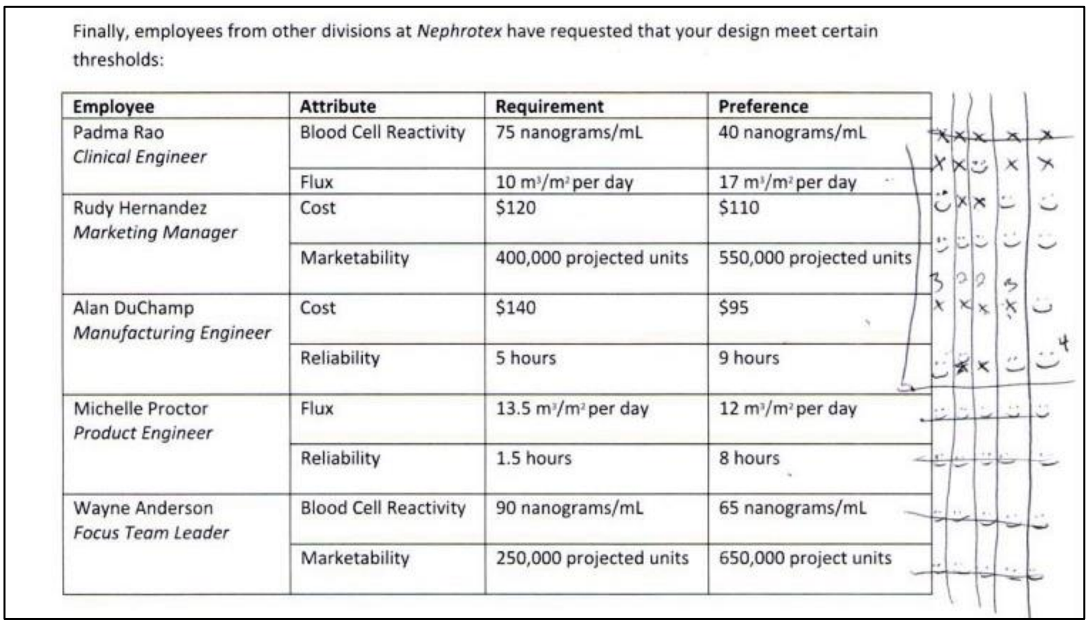
Manufacturing Process	Dry-Jet Wet Printing	
Flux Rate	9 m^3/m^2day	3
Time to 75% Performance	4 hours	2
Blood Cell Reactivity	75 ng/ml	1
Cost per unit	$50.00	1

7

Figure 1. Student's system of ranking devices based on information given.

The student's explanation indicates that the ranking system affected his choice for another input category, a surfactant called steric hindrance. He justified his manufacturing process choice by using a ranking system, but also by thinking about the total impact that all input choices would have on a particular output category, flux.

The same student then compared the devices to the thresholds given by the stakeholders (Figure 2). Each of the 5 hand-written columns represents one device. The student used a system of partial ordering consisting of smile-emoticons (meets the preferred thresholds) and x's (does not meet the preferred threshold). In the middle of the columns, the student summed the number of smile-emoticons for each device and selected the device in fifth column because it had more smile-emoticons than any other device. Thus, the student used a P_3 as a partial ordering function where he summed the valuation rankings to determine a quality value and then compared the quality score, u, for each device.



Finally, employees from other divisions at *Nephrotex* have requested that your design meet certain thresholds:

Employee	Attribute	Requirement	Preference
Padma Rao *Clinical Engineer*	Blood Cell Reactivity	75 nanograms/mL	40 nanograms/mL
	Flux	10 m³/m² per day	17 m³/m² per day
Rudy Hernandez *Marketing Manager*	Cost	$120	$110
	Marketability	400,000 projected units	550,000 projected units
Alan DuChamp *Manufacturing Engineer*	Cost	$140	$95
	Reliability	5 hours	9 hours
Michelle Proctor *Product Engineer*	Flux	13.5 m³/m² per day	12 m³/m² per day
	Reliability	1.5 hours	8 hours
Wayne Anderson *Focus Team Leader*	Blood Cell Reactivity	90 nanograms/mL	65 nanograms/mL
	Marketability	250,000 projected units	650,000 project units

Figure 2. Student's system of ranking devices to determine if devices met thresholds.

Calculation of T in a Simulated Design Problem

The tractability for the design problem in *Nephrotex* is the sum of the quality scores for every possible device divided by the maximum number of thresholds specified by the customer, multiplied by the total number of devices in the design problem, which represents the maximum possible quality. The virtual design problem space has 750 total possible solutions and 20 thresholds identified by the customers and stakeholders. Thus, the tractability for the design problem in *Nephrotex* is:

$$T_{Nephrotex}(F) = \frac{0 + 0 + 0 \ldots + 17 + 17 + 18}{20(750)} = .55$$

In *RescuShell*, the other engineering virtual internship, the virtual design problem space has 810 solutions and 15 thresholds identified by the customers and stakeholders. Thus, the tractability for the design problem in *RescuShell* is:

$$T_{RescuShell}(F) = \frac{4+4+4 \ldots + 11 + 12 + 12}{15(750)} = .50$$

In the case of *Nephrotex* and *RescuShell*, we can see that the tractability of the design problems are similar, but that the design problem in *Nephrotex* is more tractable than *RescuShell*, meaning the problem in *RescuShell* is more difficult.

We then calculated the mean percentage of thresholds that were met for students in *Nephrotex* and *RescuShell* (Figure 3) for ten groups of students from each virtual internship, where each group submitted one final device. On average, *Nephrotex* students' final devices met more thresholds than *RescuShell* students' final devices t(17.1)=12.7, p<.01.

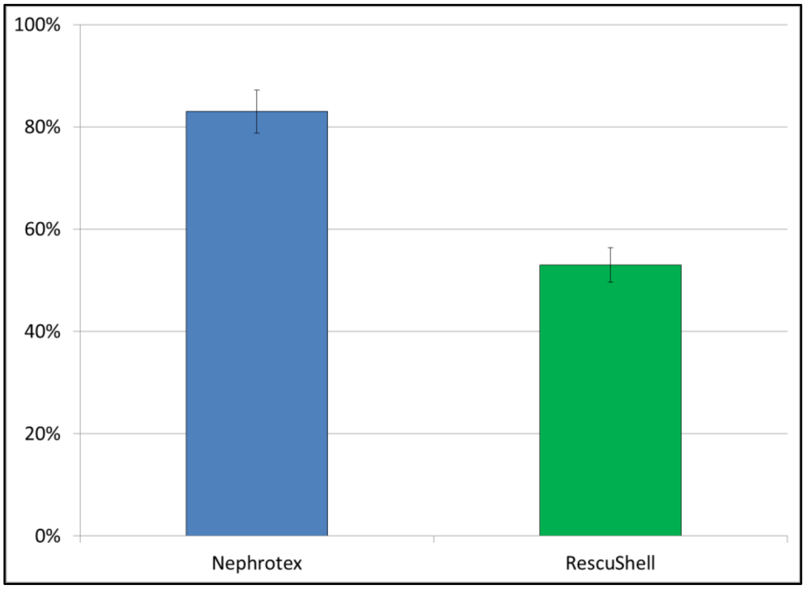


Figure 3. Mean percentage of thresholds met by students' final submitted devices. Error bars are 95% confidence intervals.

Conclusion

The results above show a framework for parameterizing and mathematically modeling student engineers' design processes. This framework informed the development of an analysis of decision-making and a tractability function that measured how difficult it is for a student to optimize over several functional requirements. The tractability function described here is one method of measuring the complexity of simulated design problems in terms of the optimization process. In general, measuring the complexity of design problems allows for instructors to assess the difficulty of problems and in turn, be able to offer design problems of varying complexity to students at a variety of levels.

Limitations

Our work on measuring student design processes and determining tractability has been currently only implemented in virtual internships. Future work includes applying our measurement techniques to other contexts such as other digital design learning environments or design team projects in capstone courses. In addition, this work focuses on the decision-making aspect of engineering design and is suitable for the more structured part of the engineering design process. We also plan to examine other measures of tractability, such as how different amounts of given information about of the design function affects the complexity of the problem. Finally, a larger goal of this research is to develop design problems with various measures of tractability for implementation into courses at different levels, for the novice first year to the more experienced senior student.

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