



Revisiting Graphical Statics

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Introduction

Up until the 1950's, a significant part of static analysis and design was done using the tools of Graphical Statics. Graphical Statics is based on the graphical method of adding vectors; briefly, when vectors are drawn to scale, the sum of the vectors, a resultant, can be measured on the drawing. The roots of using graphical methods to solve engineering problems can be traced back to Leonardo da Vinci and Galileo Galilei. German engineer Karl Culmann, however, is generally considered the father of graphical statics.¹ Early works formalized these methods for engineering,² and the topic is of on-going interest in teaching structural design to architecture students,^{3,4} and in emphasizing the significance of form in design to civil engineering students.^{5,6,7}

The methods existed as a pedagogical technique in engineering and architecture programs until the 1990's when, with the accessibility of desktop computing and relatively inexpensive software, computer-aided drawing began to dominate and manual drafting classes disappeared from the engineering curriculum. Visualization and analysis can now be done more quickly and accurately using CAD programs. In addition, CAD addresses a more diverse range of problems, including those in three-dimensions. As a result, returning to hand drawing in order to solve statics problems is not a choice anyone would make for efficiency.

Visualization skills are thought to be fundamental to spatial thinking, as it is used to represent and manipulate information, and as it contributes to the reflective component of the design thought process.⁸ However, it is clear from the literature⁹ that visualization skills have suffered as computer drawing has increasingly replaced manual drafting. From another perspective, studies point to the strong connection between conceptual understanding and hand sketching in engineering education.¹⁰ While there are no absolute proofs that CAD hinders visualization skills or that hand drawing enhances conceptual understanding, the strong interest of engineering educators in these topics suggests that the optimal approach to strengthening both may need to include a combination of computer and hand-drawing skills.^{11,12}

In this work, the goal is to explore the introduction and use of Graphical Statics modules into sophomore Engineering Statics classes. Because this is preliminary work, the underlying motivations are diverse. At a very fundamental level, students often learn how to draw in perspective and construct scaled drawings in freshmen engineering classes but are almost never asked to use these skills in the next series of classes. Yet, students often have difficulty reading and interpreting textbook drawings and "seeing" in three-dimensions. Additionally, these introductory classes often focus on design - the perspective of solving a problem given some requirements - but for the next several years, classwork focuses on analysis and evaluation of someone else's design. The working hypothesis of this project is that a more hands on, active learning component in Statics, based on Graphical Statics, can be used to address these motivations/issues and has the potential to improve student visualization capabilities, enhance their ability to think critically about whether an answer makes sense, and provide a framework for design at the sophomore level.

Overview of the Work

Two Graphical Statics modules were inserted into two Statics classes taught to sophomore engineering students; one at the University of St. Thomas, in St. Paul, MN, and the second at the University of South Carolina, Aiken. These modules were used in combination with the standard analytic tools taught in the class, not as a replacement. The first module illustrated the solution to a two-dimensional particle equilibrium problems using force polygons. The second module used force polygons, and their extension to funicular polygons, to determine the magnitudes of reaction forces in a beam problem. As a result of the first semester's work, an additional design based module is planned for the Spring semester. Descriptions of model problems, on which classroom work was based, are presented here.

An end-of-course Statics Concept Inventory Exam, was given to the two Statics classes, in which Graphical Statics modules were presented, as well as to two other sections of Statics, at the University of St. Thomas, that did not include the graphics modules. The results of this assessment are discussed.

The paper concludes with a discussion of plans to continue the process of refining the modules, adapting their inclusion more seamlessly into the Statics curriculum, and investigating assessment tools that could be used to evaluate the visualization or critical thinking skills that might result from including Graphical Statics.

Applications

There are three concepts, the understanding of, on which student success in Statics seem to rest. The first is that vectors are defined by two things, magnitude and direction. As a result, they can be described with respect to any coordinate system without changing their meaning; specific coordinates might change but magnitude and direction remain the same. Putting in an arbitrary coordinate system, and moving force vectors along lines of action are challenging concepts to students. The second concept is that vectors can be added graphically. Students are taught this in calculus, but since their calculators can do trigonometry, they often do not see it as useful or practical. The third concept is that drawing or sketching, and in particular scaled drawings, play a significant role in engineering. The most obvious manifestation that this concept needs reinforcing can be found in student resistance to drawing free body diagrams.

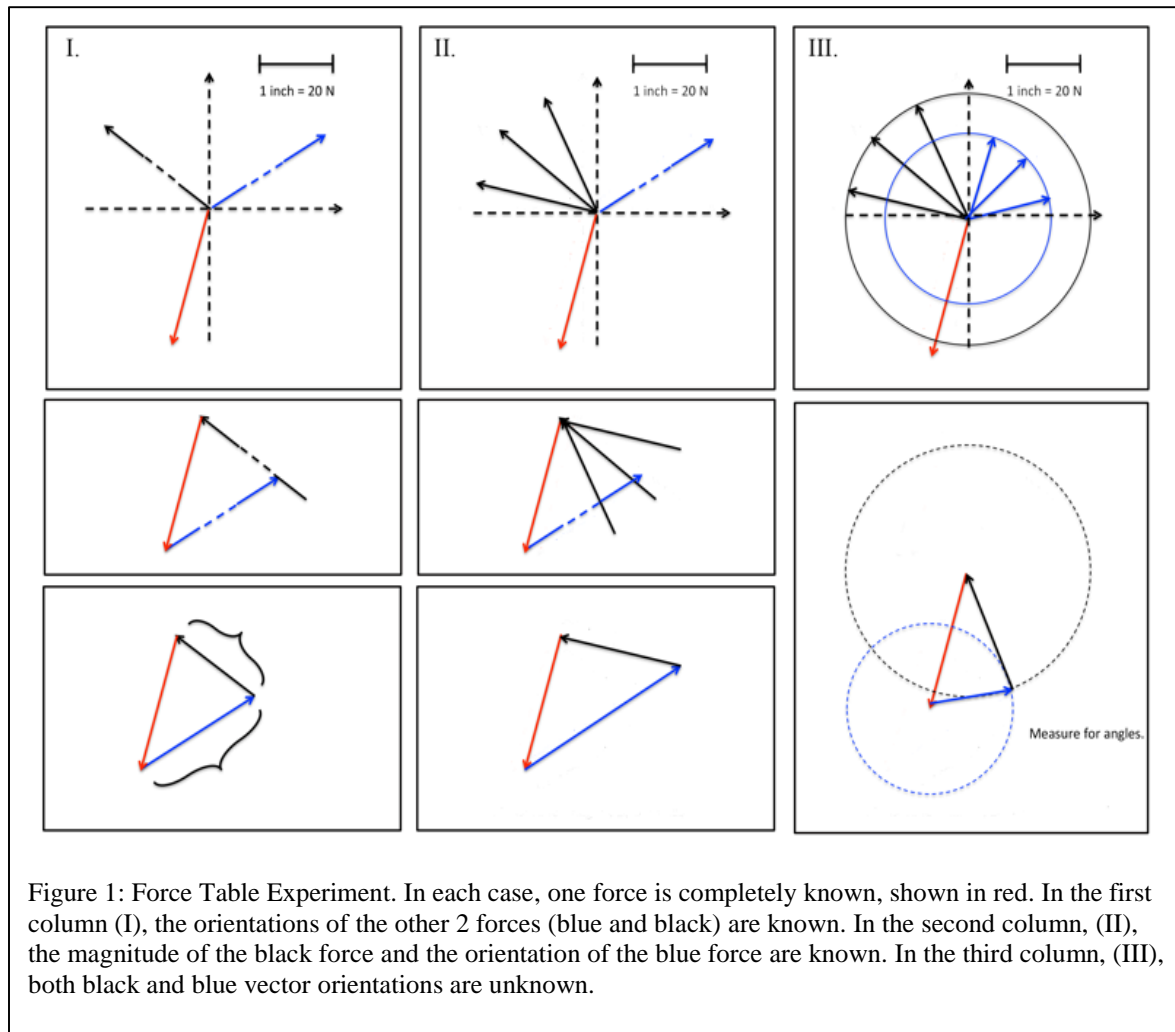
The first two applications presented in this paper are Statics problems that are representative of the type of problem that the instructors used in the classroom. The third is a more novel application for Statics; it is based on a truss analysis, which is standard, but extends the problem to asking how the initial truss could be redesigned to meet different criteria. In each case, the practical application, the “how-to” of the technique is presented. Details of the underlying mathematical/ geometrical theory are beyond the scope of this paper but are available in the literature.^{5,15,16.}

Terms used in Graphical Statics

The main element in a graphical analysis is the construction of a force triangle or force polygon. This is the descriptive name for graphical vector addition where vector **A** is added to vector **B** by putting the tail of **B** at the head of **A** and recognizing that the resultant vector, $\mathbf{C} = \mathbf{A} + \mathbf{B}$ is the one that starts at the tail of **A** and goes to the head of **B**. When a system is in equilibrium, all of

the forces assembled in this way form a closed loop, a polygon or triangle, and the resultant is zero.

For engineering applications, funicular polygons, or equilibrium polygons, come into play. The name, funicular polygon, is based on the shape that a weightless cord would



take if it were hung along its length with weights. This provides a geometric interpretation of equilibrium in non-concurrent forces. Funicular polygons can be used to find resultant forces, shears, and moments of systems of forces that are not concurrent.

2D Equilibrium

The first application is a two-dimensional, point equilibrium problem. The problem generally consists of three forces, coplanar and concurrent (all lines of action all go through the same point). This problem is the basis for the classic Force Table Lab. The standard textbook problem provides students with complete information about one force, magnitude and direction, but only the angles of orientation of the other two, (column I in fig. 1) The problem is solved by having students decompose the vectors into orthogonal parts and then writing out the equilibrium

equations, summing the forces in the x direction, and summing the forces in the y direction. These equations reduce to two linear equations for the magnitudes of the two remaining forces, easily solved using linear algebra or back-substitution.

Column I. illustrates the graphical solution of this first case: the angles are known, a scaled drawing of the force triangle is made, and the point where the lines of action of the two unknown forces meet defines the lengths of the two unknown forces. Measuring the lengths on the scaled drawing gives the unknown magnitudes. There are two other versions of the problem: students are given the magnitude of one of the unknown forces and the orientation of the other or they are given the orientations of the two unknown forces but not their magnitudes. Using this information in the equilibrium equations results in two nonlinear equations, respectively, that are more difficult to solve.

Column II. illustrates the case where one angle is known and one length is known. The line of action of one force is added to the force triangle and the other force arranged at an angle that intersects the line of action, defining the length and angle of the unknown forces, respectively. This problem is of additional interest in that there may be two distinct solutions, one solution, or no solution.

Column III. is the case where both magnitudes are known. In this case, the known magnitudes can be represented by circles with radius equal to the magnitudes (column III, top). One force will begin at the head of the known force, so the circle center is moved to this point to represent all possible orientations of that magnitude. The final unknown force will intersect the tail of the known force and the magnitude circle can be centered there. The intersection of the two unknown forces is where the circles intersect. The angles can be measured from the scaled drawing.

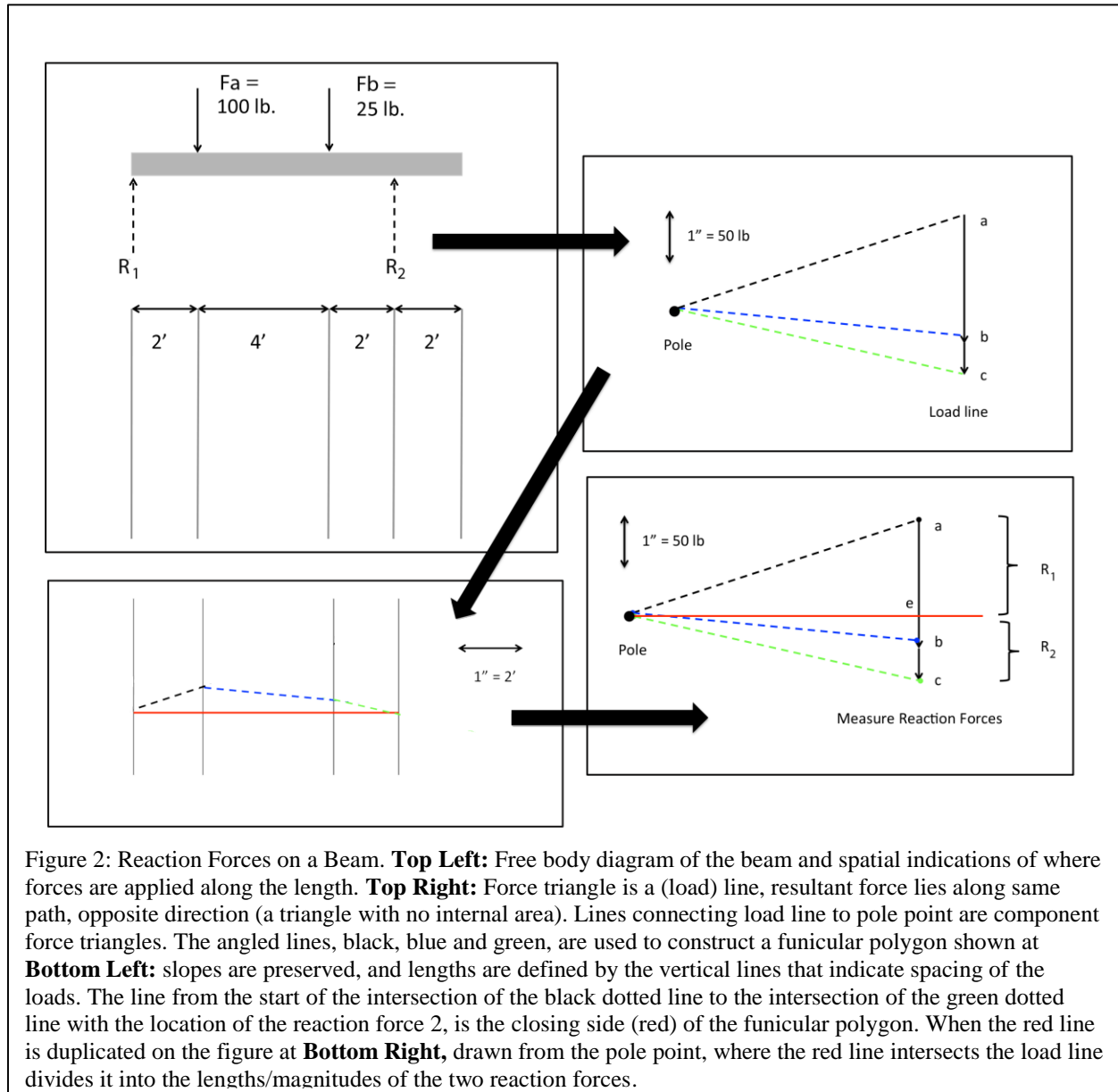
Once a solution is determined from the graphical analysis for any of the cases, it can be 'tested' in the analytic solution, the equilibrium equations, for accuracy. This case reinforces the connections that exist between measurement and analysis, and highlights the impact of uncertainty in measurement.

Reaction Forces: Shear and Moment Diagrams

The use of Graphical Statics to solve for reaction forces in a simple beam problem is shown in fig. 2. A beam is loaded with parallel forces of known magnitude and location (top left). The problem asks students to determine the reaction forces. The force triangle is again drawn to scale. In this case it is a straight line, known in these problems as a load line; the beginning and end of each **applied** force is indicated on the line (top right). The resultant force would be a line of exactly the same length overlapping the load line. This resultant force is the sum of the two reaction forces.

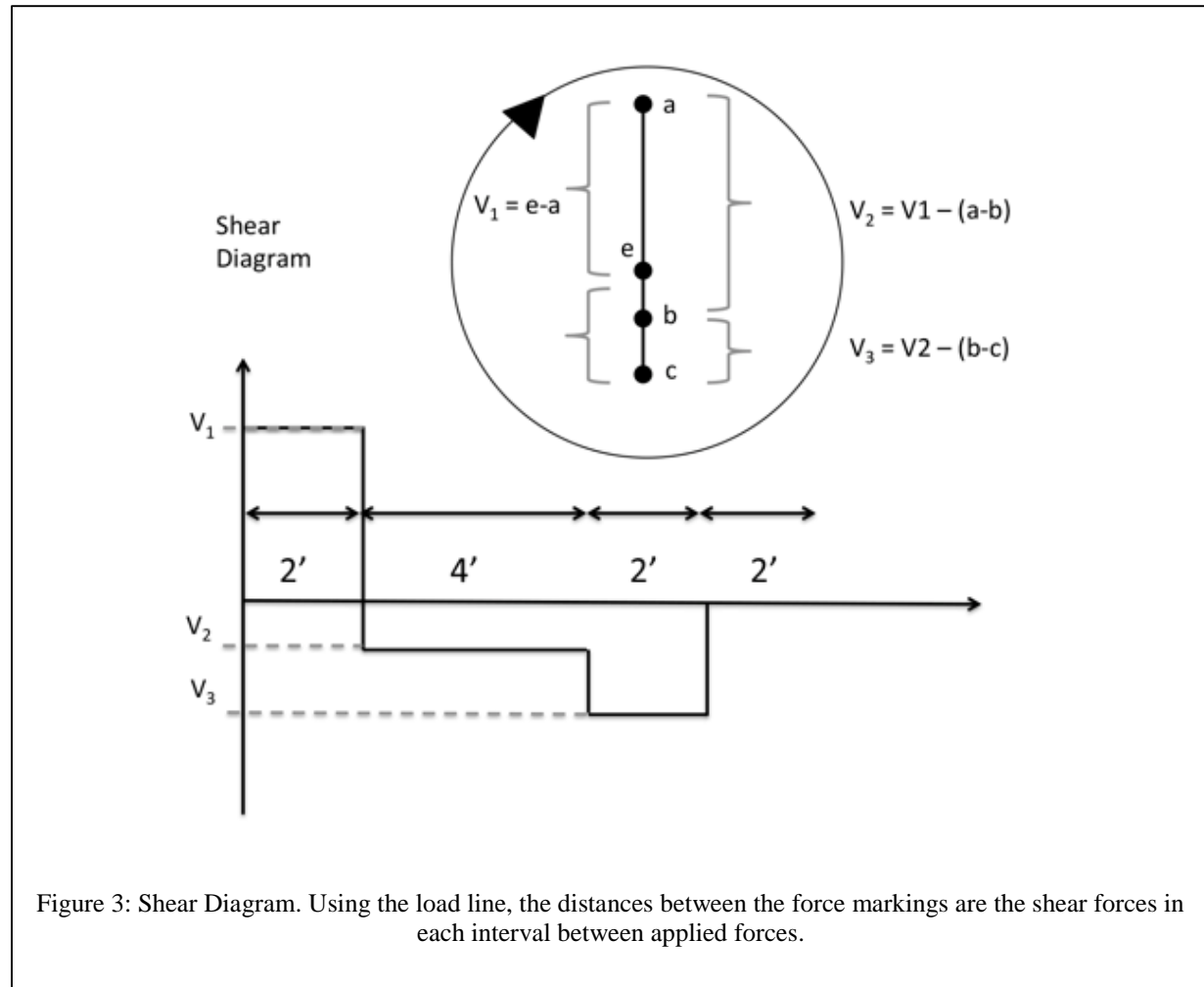
A pole point, O , is placed at an arbitrary distance away from the load line and rays drawn from the pole to each of the ends of the forces in the load line. These lines, with the same slope are duplicated across lines marking the spatial positions corresponding to each **applied** force (bottom left). Each of these lines begins and ends at the spatial position line. A final ray is drawn (solid red) from the first position to the last forming the funicular polygon. This line is

duplicated, at the same slope, on the load line/pole drawing (bottom right). The point where it intersects the load line divides the load line into the magnitudes of the reaction forces.



To construct the shear diagram from the scaled drawing of the load line in fig. 3, one starts at the reaction force intersection point (e) and measures to the point (a), i.e. this distance represents the force from the reaction at A to the point of application of the first applied force, the initial shear, V_1 , in fig. 3. The distance from (a) to (b) is the magnitude of the shear in the next interval, V_2 ; from (b) to (c) is the magnitude of the shear in the third interval, V_3 , and the distance from (c) back to (e) completes diagram.

Determining the shape of the moment diagram based on the Graphical analysis is straightforward: it has the same shape as the funicular polygon at bottom left of fig. 2. The moment at any point along the beam is proportional to the height of the funicular polygon. The magnitudes of the moments are scaled by the distance of the pole.¹⁵

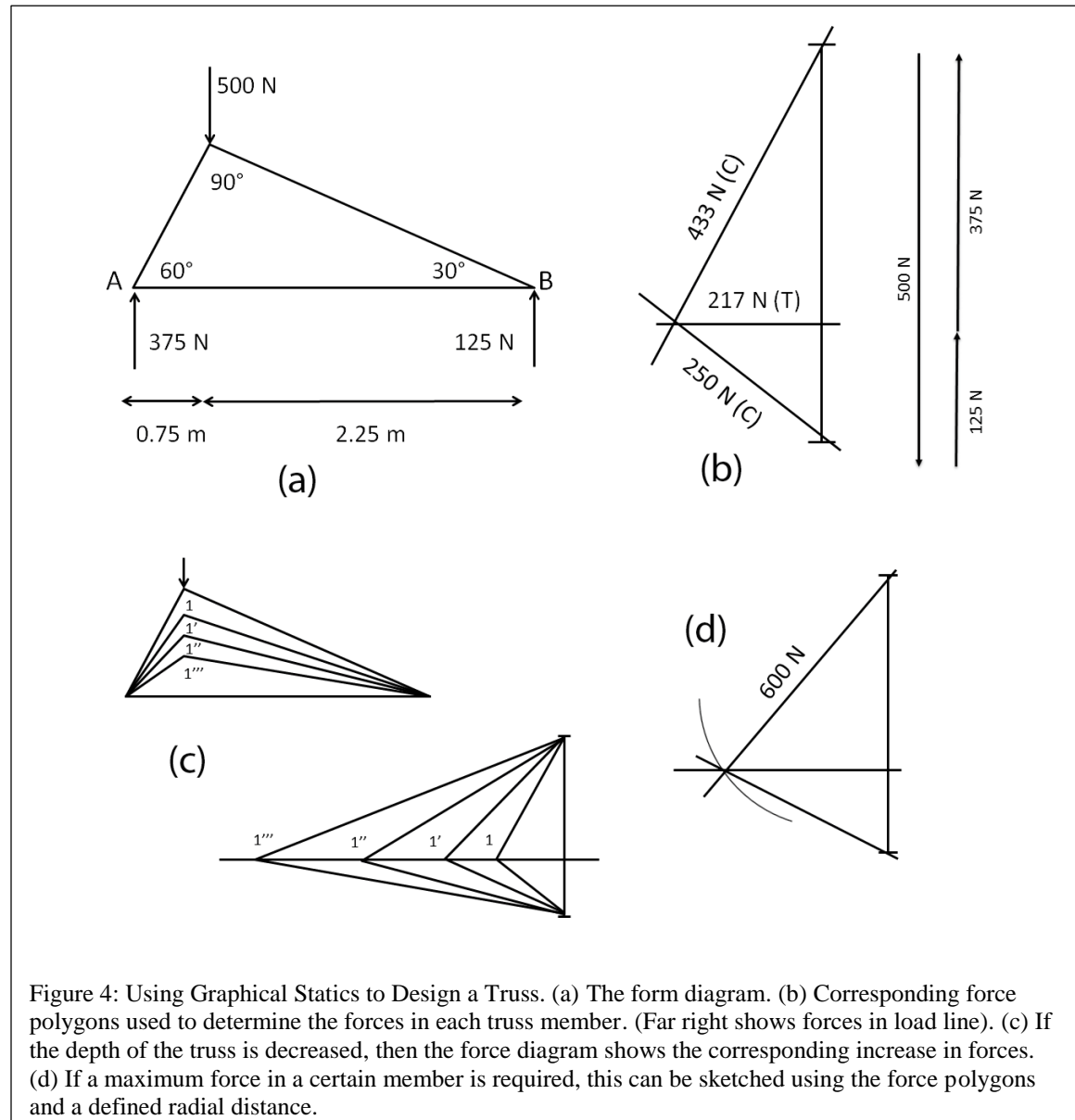


A Design Problem

Graphical statics can also be used in the design process to examine how form can change function in order to meet specific criteria. This can be illustrated with a simple truss example. In fig. 4 (a), a triangular truss is shown, drawn as a form diagram.

The reaction forces can be determined analytically and the forces in each element can be constructed using a load line shown in fig. 4(b). The full length represents the external load, the division indicates the magnitudes of the two reaction forces. Lines are drawn with the same slopes as the three truss elements and their point of intersection defines the length of each force, measurable on a scaled drawing. To illustrate the effect of the depth of the truss, the force polygons are drawn for each different height, i.e. lines at the new angles of the elements. The results clearly show that decreasing the height of the truss increases the forces in the elements; in

the lower image of fig. 4(c), the force lines grow longer with the shallower slopes of the truss with less depth, upper image of fig. 4(c). In fig. 4(d), this idea is extended one step further. If one wanted to choose an optimal depth, balancing minimum forces with more material required for longer lengths of the truss elements, a maximum load for the longest member could be specified,



here indicated by a radial arc (d), that prescribes the maximum force. The other components, depth of the truss, and forces in the other elements, are then defined by this length.

Assessment

The Statics Concept Inventory Exam, developed by Steif, Hansen and Dantzler^{13,14}, was given to the four Statics classes; two who had done the Graphical statics based modules in class and two that had not. One of the classes with Graphical Statics was of sophomore Engineering students, a

mix of Civil and Mechanical, from the University of South Carolina, Aiken, (USCA). The other three classes were sophomores Mechanical Engineering students at the University of St. Thomas, in St. Paul MN, (UST). This concept exam was chosen for an assessment because it is a highly visual test, the fundamental concepts have to be interpreted from drawings rather than verbal descriptions. A statistical t-test was done comparing the grades of the students in the classes with Graphical Statics to the students in classes without Graphical Statics and there was no statistically significant difference. There were also no statistically significant differences between the three classes at St. Thomas, with and without Graphical Statics. There was however, a statistically significant difference between the two Graphical Statics; one at UST compared to one at USCA. This is likely due to the differences in the demographics of the student populations; a private school, with traditionally-aged, full-time students versus a public school with a population that includes non-traditionally aged students, and students with full-time employment.

It is important to note that the modules presented here are 'first drafts' of this project and refining the content and polishing the presentation will change the results of the assessments. A major challenge both instructors encountered was a time constraint; it was difficult to fit the additional work in effectively and efficiently into the already topic dense Statics class. A future goal is to work for more consistency in presentation and content, as well as look for ways to include smaller components over more of the course material. The Concept exam still seems like an appropriate assessment tool for this project and the plan is to continue to use it to compare the classes with and without a graphical statics component and to begin to determine if the graphical component increases student understanding of the fundamental concepts of Statics. This Concept exam can also be used to generate longitudinal data, tracking individual classes with a graphical component, as the course material is refined.

At a very basic level, the instructors to critically examine graphical skills as a topic and determine that it, in and of itself, is worth inclusion in the course. Research into other educational applications, in architectural design classes, for example, will help the authors make a case for this and help identify appropriate applications for the subject itself.

An initial, qualitative assessment is that student accuracy in drawing free-body diagrams, (FBDs) an immediate visual component of the class, has improved. One hypothesis is that the graphical statics modules help students realize the power and usefulness of FBDs. They begin to understand that engineering analysis comes directly from the drawing and that FBDs are not just an after thought in the solution process.

Since free-body diagrams can be tracked consistently through the mechanics curriculum, Mechanics of Solids and Dynamics, the authors propose to develop a set of level appropriate surveys/tests, to be used to assess the skillful drawing and automatic use of FBDs through this sequence. Additionally, a level appropriate version of the survey could be administered in the capstone design course to assess the persistence of FBDs in students' skill set. This quantitative assessment of student visual communication skills, based on clarity and accuracy of free body diagrams, could provide evidence of the impact of the graphical modules and a method of tracking its effect through other classes.

Future Work

This project will be continued, at a minimum, in the Spring and Fall semesters of 2015. Major goals will be to develop, with each application, a more unified approach and more consistent course materials. Emphasis will be on incorporating this material more seamlessly, and potentially integrating it into other topics, for example calculating moments of inertial, using it to analyze simple machines and solve friction problems. A version of the design problem will be introduced, and an assessment tool based on free body diagrams will be developed and administered in the next course in the sequence, Mechanics.

References

- [1] Culmann, Karl, "Die graphische Statik" Zürich: Verlag van Meyer & Zeller, 1864/1866.
- [2] Wolfe, W. S. Graphical Analysis a Text Book on Graphic Statics McGraw-Hill Book Co., Inc. NY1921.
- [3] Dermody, R. J. "Embracing the Past: Using Historical structures to Teach Engineering Fundamentals" ASEE Annual Conference Proceedings, 2012.
- [4] Dermody, R.J. "Get the Form Right!!" Proceedings of the ASEE Annual Conference Proceedings, 2010.
- [5] Allen, E. and Zalewski, W. (Boston Structures Group), Form and Forces Wiley and Sons, Inc. 2010.
- [6] Liem, Yannick "Graphic statics in funicular design calculating force equilibrium through complementary energy", Graduate Thesis TU Delft, Faculty of Architecture.
- [7] Gerhardt, R., Kurrer, K.-E., Pichler, G. "The methods of graphical statics and their relation to the structural form" Proceedings of the First International Congress on Construction History, Madrid, 20th-24th January 2003, ed. S. Huerta, Madrid: I. Juan de Herrera, SEDHC, ETSAM, A. E. Benvenuto, COAM, F. Dragados, 2003.
- [8] Olkun, S. "Making Connections: Improving Spatial Abilities with Engineering Drawing Activities" International Journal of Mathematics Teaching and Learning, April 2003.
- [9] Sorby, Sheryl A., and Beverly J. Baartmans. "The Development and Assessment of a Course for Enhancing the 3-D Spatial Visualization Skills of First Year Engineering Students." *Journal of Engineering Education* 89.3 (2000): 301-307.
- [10] Westmoreland, S., Grenier, A., Schmidt, L., "Sketching During Mechanical Design: Studying Sketching at the University of Maryland" ASEE Annual Conference Proceedings, 2009.
- [11] Robertson, B.F., D.F. Radcliffe, D. F., "Impact of CAD tools on creative problem solving in engineering design" *Computer-Aided Design*, 41(3), 136–146, 2009.
- [12] Jianping Yue, "Does CAD Improve Spatial Visualization Ability?" ASEE Annual Conference Proceedings, Albuquerque, NM, 2001.
- [13] Steif, Paul. S and Hansen, M. "Comparisons between Performances in a Statics Concept Inventory and Course Examinations" *Int. J. Engng Ed*, 22(5), 1070-1076, 2006.
- [14] Steif, Paul S. and Dantzler, John, A. "A Statics Concept Inventory: Development and Psychometric Analysis", *ASEE Journal of Engineering Education*, 363-371, 2007.
- [15] Cathcart, W.L. and Chafee, J.I. "A Short Course in Graphic Statics for Students of Mechanical Engineering" D. Van Nostand Co., N.Y., 1911. (BiblioBazaar Reproductions, LLC).
- [16] Zalewski, W. and Allen, E. Shaping Structures: Statics Wiley and Sons, Inc., 1998.