Using Khan Academy to support students' mathematical skill development in a physics course

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Introduction

This paper reports on a pilot study of using a free online mathematics learning tool, *Khan Academy (KA)*, to strengthen the relevant mathematics skills of pre-service science teachers in their introductory physics course at a large teacher education institution in Norway. The motivation for this project was prior students' struggles with the mathematical aspect of the physics course. By guiding the students to relevant mathematics topics in KA before each physics class, the intention was to improve students' mathematics skills by motivating them to spend more time working on mathematics *and* by letting them work with a pedagogically well-designed mathematics learning tool.

The goal of this research project was to investigate the *value* of integrating KA into the introductory physics course for pre-service science teachers. The value was considered a function of student compliance with the use of KA, the students' mathematics learning gain throughout the course, and the usefulness of the data provided by KA to the lecturer throughout the course.

Background

In 2006 Salman Khan, a financial analyst, began producing short 8- to 10-minute videos on mathematics for his high-school aged cousins, who lived across the country in the US. Khan posted the videos on YouTube as a means of remote tutoring, a format the cousins found particularly helpful, as they could watch, replay and pause the explanations at their own pace. However, the availability on YouTube quickly made the videos a popular resource among thousands of other students searching for online mathematics resources.[1] This was the beginning of Khan Academy, a free online learning resource comprising short videos on a wide range of academic subjects, now ranging from mathematics to economics and history. In 2010, KA received funding from private benefactors, Google and the Bill & Melinda Gates Foundation to form a non-profit organisation with a mission to provide "a free, world-class education for anyone, anywhere."[2]

The mathematics module of KA is particularly well developed, offering practice problems, progress reports and motivational gaming mechanics in addition to the videos. Mathematics from kindergarten to college level is divided into 905 topics (as of fall 2014) and organised in a hierarchical network that visually shows which topics are prerequisites for other topics. Students can achieve varying levels of proficiency in each topic, from basic competency (referred to as 'practiced') via 'level 1' and 'level 2' to 'mastery'. To achieve basic competency, students either need to correctly answer five problems successively or achieve basic competency in a significantly more complex topic that requires competency in the simpler topic. To reach the next three levels, students must successfully answer a problem that appears in a 'mastery challenge', generally comprising problems from multiple topics, which is made available 16 hours after the previous level is achieved to ensure long-term retention of mathematics knowledge. Videos are available on each topic, and as part of the motivational gaming mechanics, points and badges are also awarded for correctly answering problems.

Not much research has been conducted on the use of KA in schools or universities. The most comprehensive research to date is a two-year pilot project conducted from 2011 until 2013 by

KA with the support of the Bill & Melinda Gates Foundation and evaluated by SRI Education.[1] Several volunteer schools were selected for the project, spanning grades 4 to 10. In total, the sample comprised 55 teachers and 1694 students for the 2011-2012 school year and 63 teachers and 2246 students for the 2012-2013 school year, some of which participated both years. The participating teachers were encouraged to use KA in their math classes, but were free as to how and how much they used it. One of the main findings was that there was great variability in how KA was used by teachers and students, both between schools and within schools over time; as teachers and students became more familiar with KA, the nature of how they chose to use KA as a resource changed, but it rarely diminished. Eighty nine percent of the teachers said they intended to use KA in the following school year. The most important role played by KA was in supporting instruction by providing practice problems for students (82% of teachers) as opposed to leaving the introduction of new concepts to KA (20% of teachers). However, a range of uses were found: "teachers used Khan Academy as an intervention for students who had fallen behind their grade-level peers; as an enrichment activity for advanced students, allowing them to explore topics above their grade level; as an accountability tool allowing close monitoring of student progress on problem sets; and as a highly integrated supplemental practice activity, reinforcing skills recently introduced in the classroom."[1]

Average time spent on KA during one academic year ranged widely, from a median of 396 min to 3140 min. The collection of schools with the highest use attributed this to several factors: anytime access to individual computers for students, compulsory completion of prescribed KA topics with consequences for non-compliance, close teacher monitoring of student progress, well-planned integration of KA with curriculum, and extended instructional blocks focusing on mathematics.[1] Exploratory analyses of years 5 and 6 from the largest collection of associated schools found a positive relationship between time spent on KA and number of problem sets completed to proficiency with performance (better than predicted performance in the California Standard Test) and attitudes (lower math anxiety, higher math self-concept and math self-efficacy).

In all cases, KA was used as a supplemental resource in a *blended learning* model. Blended learning as a term emerged in 1999[3] and refers to the blending of "text-based asynchronous Internet technology with face-to-face learning,"[4] where the primary role of ICT is to complement student learning as opposed to replace face-to-face time.[5] One form of blended learning that is becoming increasingly common in tertiary science and engineering education is the combined use of Flipped Classroom (FC) and Just-in-Time Teaching (JiTT). FC refers to a teaching structure where students receive their first exposure to the subject material prior to class so that class time can be freed up to work *with* the material, which is the reverse order to traditional tertiary instruction.[6] The pre-work comprises familiarizing oneself with the subject matter either by reading the textbook or other materials, or by watching online video lectures. FC is often employed in conjunction with JiTT,[7] in which students complete an online pre-test before class. In addition to forcing students to work on some relevant content problems, the pre-test gives feedback to the instructor prior to class about what the student cohort have or have not understood from the pre-work tested. Often, one question in this pre-test will explicitly ask students to indicate what they found difficult in the pre-work. The instructor then uses this feedback to tailor class time to address the particular cohort's educational needs. Because the introduction to the material has been moved out of the classroom, class time can be devoted to student active teaching methods, such as Peer Instruction[8] and collaborative problem solving.

Context

The two main pathways to qualify as a science teacher in Norway are either to complete a three-year Bachelor degree in Science followed by a one-year Graduate Diploma of Teaching (called *'Praktisk-Pedagogisk Utdanning'*) *or* to complete a four-year integrated Bachelor of Teaching. Senior high school teachers (years 12 and 13, ages 16-19) are required to obtain a science degree followed by a Graduate Diploma. However, elementary school (years 1-7, ages 5-13) and junior high school (years 8-10, ages 12-16) teachers typically pursue the integrated Bachelor of Teacher Education. (Both educational routes qualify for teaching year 11.) Elementary school science teachers are required to undertake the equivalent of one semester of science (Science I), whereas junior high school teachers need to complete two semesters of science (Science I and Science II). Science I is a full semester load of courses comprising Biology I, Earth Science I, Physics I, Chemistry I, Science Education I and Technology and Design I.

This project was conducted in the Physics I course, which accounts for 20% of Science I. In fall 2014, the semester lasted 17 weeks, and it had an irregular teaching schedule. The Physics I course was taught in weeks 2-6, and the students had two compulsory physics assignments: one traditional written assignment due two days after the last physics class (in week 6) and one oral group presentation with a teaching focus due in week 8. Teaching stopped at the end of week 15 and the Science I examination (oral) was conducted in week 17.

The Physics I course comprised seven 2 hr 45 min non-compulsory classes, covering thermodynamics, gravity, buoyancy, sound, light, kinematics, forces and energy. A FC and JiTT structure was employed. For each class, the students were given learning goals and pre-work. Between 12 and 22 detailed learning goals, with an average of 17, were ascribed each class, such as 'Explain and perform calculations with the equation $\Delta E = mc\Delta T$', 'Draw and interpret velocity-time graphs' and 'Explain why the sky is blue, the sunset is red and clouds are white'. The pre-work comprised reading relevant sections of the textbook *Complete Physics* by Pople,[9] watching between 6 and 10 short videos (the majority of which were from *Veritasium.com* and *Hewitt Drew-it*), do one or two problems or experiments, and complete an online pre-test with six questions (42 questions in total for the course). Before class, the instructor reviewed the students' performance on the pre-test and used the information to tailor class activities to the particular issues students appeared to be having with the material. Class time was devoted to some JiTT lecturing, Peer Instruction and group work on experiments and problems.

Methodology

The sample for this pilot study was 24 students enrolled in Science I in the fall semester of 2014. The students were all in their fourth and final year, with nine having declared an elementary school major and 15 a junior high school major.

The only change made to the course from the previous time it had been taught (by the same instructor in spring 2014) was that achieving basic competency in four relevant topics in KA was added to the pre-work for each class (see Table 1). To encourage students to do the pre-work, an incentive was given: achieve a specified number of points on the online pre-test and KA topics throughout the course, and be exempt from handing in the fourth and final question on the physics course assignment. Students obtained one point for each satisfactory answer on the pre-test (42 questions in total for the semester) and *lost* one point for each KA topic they had *not* completed. The logic of the point system was that anyone could complete a topic if they worked hard enough at it, as there was no time limit and numerous practice questions, hints and help available. Students who achieved a total pre-work score of 21 points for the semester were exempt from the last assignment question.

Table 1: Overview of KA topics given as class pre-work.

Mathematics category	*KA Topic*
Basic mathematics (grades 5-10)	Units and scale of graphs Graphing proportional relationships Identifying slope of a line Average rate of change Direct and inverse variation Linear and nonlinear functions Constructing linear equations word problems Graphing linear equations Dependent and independent variables Understanding the process for solving linear equations Solving equations in terms of a variable Linear equation word problems Adding and subtracting fractions with unlike denominators
Scientific mathematics (grades 11-13)	Scientific notation intuition Scientific notation Orders of magnitude Positive and negative exponents Multiplying and dividing scientific notation Computing in scientific notation Units Converting units Working with units algebraically Significant figures Understanding linear and exponential models
Vector mathematics (higher education)	Adding and subtracting negative numbers Scaling vectors Adding and subtracting vectors in rectangular form Graphically adding and subtracting vectors

In this particular semester, there was also funding available for five fortnightly two-hour mathematics tutorials, starting in the second week of class. The tutorials focused on basic mathematics relevant to physics such as multiplying fractions, using physics formulae and scientific notation. All students were welcome, but the eight students with the lowest scores on a mathematics pre-test were explicitly encouraged to attend.

The mathematics test was specifically designed to evaluate pre-service science teachers' knowledge of mathematics relevant to the Science I course. It was developed by a Physics Education Researcher (the author, teaching Physics I), a Mathematics Education Researcher (teaching in mathematics teacher education) and a colleague teaching Chemistry I. The test comprised 26 problems, all worth 2 marks, covering mathematics central to Physics I and Chemistry I. The problems were sorted into three broad categories: basic mathematics (covered in grades 5-10), scientific mathematics (covered in grades 11-13) and vectors (covered at the tertiary level) (see Table 2). Ten of the 15 basic mathematics problems were sourced from literature[10-13] and all four problems in vector mathematics were translated from Flores, Kanim and Kautz[14]; the remaining questions were written by the author and her two colleagues. All students were expected to be competent at the mathematics topics covered in grades 5-10, and most, but not all, students were expected to have some level of familiarity with the topics covered in grades 11-13. Vectors is introduced in the introductory physics course, but is not covered in Norwegian high schools, so the students were not expected to be familiar with these four problems prior to Science I.

Table 2: Overview of problems in the mathematics test.

Category	*Type of problem*	*Number of problems*	*Grade level introduced*
Basic mathematics (15 problems)	Numeracy	3	5-10
	Measuring	2	
	Reading tables	2	
	Functions	4	
	Algebra	4	
Scientific mathematics (7 problems)	Logarithms	2	11-13
	Calculations with exponents	2	
	Calculations with scientific notation and units	3	
Vector mathematics (4 problems)	Vectors	4	Higher Education

Eleven problems were limited closed response questions (either multiple choice or ranking questions), 13 problems were unlimited closed response questions (mostly calculations with one correct final answer) and two questions were open response questions (one non-linear graph drawing and one requiring a written explanation). An example of a problem in the category of basic mathematics (numeracy) is the following (problem 1): "10% of the boys and 10% of the girls at school play soccer. How many percent of all students in the school play soccer? A) 5%, B) 10%, C) 15%, D) 20%, E) Cannot answer." A problem from scientific mathematics (calculations with scientific notation and units) is (problem 2) "Complete the calculation: $s = vt = 3.0 \cdot 10^8$ m/s $\cdot\ 2.0 \cdot 10^{-5}$ s = ".

The same mathematics test was used as pre- and post-test. The pre-test was administered in class during the first week of first semester, before the physics course had started, and the post-test was administered in the second week of second semester, which was the first available time. The post-test was done in second semester to ensure that it was given after the final examination in first semester (in case students used KA and worked on their mathematics skills until the examination). As a consequence, the post-test was completed after three of ten lectures in Physics II. However, very little mathematics was used in these three lectures, so it is unlikely to have had much impact on the post-test performance. During the administration, students were informed that the mathematics test was a voluntary diagnostic test comprising problems relevant to Science I. They were told that their participation and score had no bearing on their course mark, and they were informed of their score within two days of the test. Students were given exactly 30 minutes to complete the test, they were not allowed to use any tools other than pen and paper, and the author was present the entire time. No student declined to participate, but one student in the post-test did not appear to make a serious effort by leaving after 14 minutes (as the first student to leave) without having attempted 12 of the 26 problems. This student was the only one to receive a lower score on the post-test than the pre-test by more than one mark, corroborating the assumption of not putting in the same effort in the post-test as the pre-test. This student's post-test result was therefore not included in the analysis.

The pre-test was marked independently by the Physics I and Chemistry I instructors. Using a pre-made marking scheme, marking of the limited closed response questions was unambiguous, whereas a few clarifications of the marking scheme were needed for the unlimited closed response questions (such as whether a unit was required for full marks) and for the open response questions. In the post-test, only four ambiguous responses needed to be discussed with the other instructor and the graph drawing problem was marked independently by both instructors. A marking disagreement only occurred in one case.

The complete data collection comprised time spent on KA, number of prescribed pre-work topics completed, number of topics where basic competence was achieved, physics class attendance, voluntary mathematics tutorial attendance, the mathematics pre- and post-test and an anonymous course evaluation questionnaire. The course evaluation was completed immediately after the compulsory group presentation two weeks after the last physics class.

Results

Voluntary physics class attendance was at 95%. All 24 students made a KA account, but two students never used it. While the physics course was in session, students spent on average 12 hrs 22 mins (SD = 6 hrs 1 min; $N = 22$) on KA. No one spent time on KA after the assigned pre-work topics were completed. Students completed on average 22.3 ($SD = 6.1$; $N = 22$) out of 28 pre-work topics, and achieved basic competency in an average of 148 topics ($SD = 46$; $N = 22$) out of a total of 905 topics currently in KA.

In the physics course evaluation, students were asked three questions relating to KA or the mathematical aspect of the course. When asked for their level of agreement with the statement "It was useful to work on topics in KA before class", two students disagreed, 10 were neutral and 12 either agreed or strongly agreed. In response to "The physics course was mathematically difficult", 13 disagreed or strongly disagreed, five were neutral and six either agreed or strongly agreed. Twelve students responded to the statement "The math tutorials were useful (only respond if you attended at least once)"; one disagreed, two were neutral and nine either agreed or strongly agreed. When asked what worked particularly well in the course in an open response question, seven students mentioned having to do the pre-work. One student said that "the pre-work with a 'carrot' for having done the work was very motivating", whereas another commented "that we had to prepare well for each class, [which] increased the learning gain significantly." KA was explicitly mentioned by one student who thought that what worked particularly well was "KA, [the pre-test] and clear requirements as to pre-work and expected effort."

On the mathematics test, the average score on the pre-test was 21.6 ($SD = 5.7$; $N = 22$) and on the post-test 27.1 ($SD = 6.7$; $N = 21$). Of the students for which both pre- and post-test data were available, this corresponded to a statistically significant improvement of 5.2 marks ($t(19) = 5.5$, $p = 0.000$, $N = 20$). To investigate whether the improvement in mathematics results correlated with KA use, time spent on KA, number of prescribed pre-work topics completed, number of topics in which basic competency was achieved and attendance at the voluntary mathematics tutorials were correlated with pre-test score, post-test score and change in score from pre- to post-test. No statistically significant correlations were found.

Next, the mathematics test scores were analyzed according to basic mathematics (15 problems), scientific mathematics (7 problems), and vector mathematics (4 problems). Comparing the change from pre- to post-tests in these three categories, students showed clear or boarderline statistically significant improvement in all three categories: +1.4 of 30 marks (4.7 percentage points) in basic mathematics (pre: $M = 20.2$, $SD = 4.5$, $N = 20$; post: $M = 21.6$, $SD = 4.1$, $N = 20$; $t(19) = 2.1$, $p = 0.046$); +2.9 of 14 marks (21 percentage points) in scientific mathematics (pre: $M = 1.30$, $SD = 1.78$, $N = 20$; post: $M = 4.20$, $SD = 2.93$, $N = 20$; $t(19) = 5.5$, $p = 0.000$); and +0.90 of 8 marks (11 percentage points) in vector mathematics (pre: $M = 0.40$, $SD = 0.88$, $N = 20$; post: $M = 1.30$, $SD = 1.90$, $N = 20$; $t(19) = 2.02$, $p = 0.058$).

Correlating the change in scores in basic mathematics, scientific mathematics and vector mathematics with time spent on KA, number of prescribed pre-work topics completed, number of topics in which basic competency was achieved and attendance at the voluntary mathematics tutorials, again no statistically significant correlations were identified.

Looking specifically at the two example problems presented earlier, there was no statistically significant improvement in score on the numeracy problem (pre: $M = 0.40$, $SD = 0.82$, $N = 20$; post: $M = 0.60$, $SD = .94$, $N = 20$; $t(19) = 1.00$, $p = 0.330$), whereas a clear and statistically significant improvement was seen in the scientific calculation problem (pre: $M = 0.20$, $SD = 0.62$, $N = 20$; post: $M = 1.00$, $SD = .97$, $N = 20$; $t(19) = 3.39$, $p = 0.003$).

Discussion

The goal of this project was to investigate the value of integrating KA into the introductory physics course for pre-service science teachers. Three different forms of value were considered: student compliance with the use of KA, student learning gain in mathematics throughout the course, and the usefulness of the data provided by KA to the instructor.

Student compliance with the use of KA was high throughout the course, with 22 of 24 students using KA for pre-work, doing on average of 22.3 of 28 prescribed topics. Only two students indicated in the course evaluation questionnaire that they did not find the use of KA before class useful, which may have been the same two students who did not comply with its use, although this is unknown due to the anonymous nature of the evaluation. The reason for the high compliance can likely be contributed to the value students saw in doing carefully structured pre-work and the 'carrot' provided by integrating the use of KA such that those who complied received a significant benefit by being exempt from the fourth and final question in the physics assignment for the course. This interpretation is based both on direct student feedback in the course evaluation and the observation that no one used KA in the time between the final physics class and the examination, a time in which one may have expected that at least some students would have found KA a useful tool. These results are in line with Murphy et al.'s[1] findings that schools with the highest KA use were those where KA had a well-planned integration with the curriculum and compulsory completion of prescribed topics with consequences for non-compliance. Another reason for the successful uptake of KA may be that it served a clear purpose that no other aspects in the course served – a key feature of blended learning – since very little explicit mathematics teaching was done in the course.

The analysis of the mathematics pre- and post-tests showed a marked and statistically significant improvement from 42% to 52%. Particularly in the category of scientific mathematics did the students show a large increase in score from 13% to 30%. Given that 11 of 28 topics in the pre-work addressed scientific mathematics, and that this was an area of mathematics students were relatively unfamiliar with from before, this may indicate that KA use was a contributing factor to a developing understanding of scientific mathematics.

In the post-test, students scored an average of 72% in basic mathematics (grade 5-10), displaying clear difficulties with elementary school topics such as percentage. An average score of 30% in scientific mathematics and a meagre 16% in vector mathematics – after a full semester of science courses where extra measures were taken to help students with mathematics – is disconcerting for a group of students that is qualified to become elementary school science teachers after Science I.

The data generated by students' interaction with KA is valuable to the course instructor in several ways. Firstly, it gives the instructor a good overview of the collective knowledge that the class has of various topics in mathematics. By prescribing certain relevant topics as pre-work – especially topics that are assumed pre-requisite knowledge – the instructor receives feedback on whether the students actually have this knowledge. It also makes clear to the students what the instructor expects them to know or learn. In cases where the students are not as knowledgeable as expected, KA gives the instructor valuable feedback that encourages him or her to adjust the teaching according to what the students have displayed of actual knowledge. Although such an insight may be disappointing to the instructor at times, it most certainly is sobering and valuable to ensure appropriate adjustment of the level of instruction. The data generated by KA is also valuable on an individual student basis. Over time, it gives the instructor an opportunity to compile information on individual students on several independent factors: are they lazy or conscientious, do they have high or low prior knowledge of relevant mathematics topics, are they slow or efficient at working through new topics, and do they display motivation to work with mathematics topics beyond those prescribed by the instructor? Such information is of great value to tailor feedback to students: a student who clearly shows high levels of prior knowledge but rarely complies with tasks will most likely benefit from different motivational feedback than a student who spends a lot of time but is still struggling with some of the basic mathematics concepts in a course. Thus, KA provides the instructor with data that puts him or her in the position to individualize instruction within an existing course structure – particularly when combined with other individual measures from the course – by commencing activities for at-risk students at an early stage when problems can still be addressed in time and by monitoring students' continuous efforts throughout the course.

Although this particular experiment was conducted with a sample of pre-service science teachers, the results are in agreement with previous research findings from different student populations. The findings should therefore be of interest to any tertiary engineering education instructor who expects a certain level of mathematics proficiency of their students, regardless of what level of mathematics is of interest. An added benefit to engineering education – if somewhat peripheral and on a longer time-scale – is that when KA is introduced in pre-service science education, it increases the likelihood that these future science teachers will introduce their students to KA, and thus encourage potentially future engineers to spend more time on mathematics, which is likely to benefit their level of mathematics readiness for their engineering courses a few years down the track.

Conclusion

This pilot project has investigated the value of integrating KA into the introductory physics course for pre-service science teachers at a large teacher education institution in Norway. The value of KA was considered a function of student compliance with the use of KA, the students' mathematics learning gain throughout the course, and the usefulness of the data provided by KA to the lecturer. Findings were that student compliance was high, the majority was positive towards using KA, and a clear and statistically significant learning gain was seen in students' mathematics knowledge. It was not possible to convincingly prove that this learning gain was a consequence of KA use, however, which would be a useful avenue of further research. The data generated by KA was useful for the instructor, both on a class level as well as on an individual student level. The data from KA enabled the instructor to tailor her teaching according to actual as presumed student knowledge, and it also allowed the instructor to identify who were struggling with mathematics so that extra or alternative measures could be offered both in and out of class.

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