

## **AC 2007-2014: A MODEL FOR VERTICAL INTEGRATION OF REAL-WORLD PROBLEMS IN MATHEMATICS**

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# **A MODEL FOR VERTICAL INTEGRATION OF REAL-WORLD PROBLEMS IN MATHEMATICS**

## **Abstract**

Traditionally, mathematics has served as a tool for engineers, entering the undergraduate curriculum as something acquired by engineering students in their university tour outside the engineering units. While this strengthens the engineering student's intellectual arsenal, it makes little impression on the mathematics student not naturally inclined toward engineering.

To stimulate the flow in the opposite direction, WPI has developed opportunities for students and teachers of mathematics to gain experience with the applications of mathematics in real world settings, in particular in industrial problems typical of those in which engineers would depend upon mathematics for solutions. This approach is based upon WPI's project-based style of education, one that employs the engineering approach to problem solving broadly across the sciences and mathematics.

This project oriented curriculum, introduced over 30 years ago at WPI, has facilitated a major change in mathematics education. In the past ten years WPI faculty have developed a successful model that introduces real-world, industrial, projects in mathematics education, at all levels from middle school to the Ph.D. program and faculty research. The faculty and students affiliated with the Center for Industrial Mathematics and Statistics have developed project collaborations with over 30 companies, businesses, and government labs. These projects serve to motivate students to study mathematics and prepare them for interdisciplinary work in their careers. With funding from NSF, SIAM, the GE Foundation, the Alfred P. Sloan Foundation and Intel, several vertically integrated educational programs have been developed.

## **Introduction**

The demand for professionals capable of modeling and simulating emerging technologies has dramatically increased in recent years. As mathematics becomes more complex, and the numerical modeling more sophisticated, the need for mathematically trained professionals is increasing. Modeling, analysis, and computations performed by mathematicians in collaboration with other professionals can provide technical advantages and cost savings, important for a company's competitive edge.

Survey data from the CBMS 2000 report<sup>13</sup> concerning four-year college and university undergraduate mathematics programs describe troublesome findings: since 1995 the number of mathematics bachelor's degrees has dropped by about 14%. And the numbers are more dramatic if we compare them with those of 30 years ago. In today's ever more challenging environment, it is necessary to find new ways to attract students to mathematics. There is a need to prepare them to become successful professionals in a society that is very much different from the one 30 years ago.

Applications can provide motivation; while some students are attracted by the intrinsic beauty of mathematics, with logic and proofs, others find additional appeal in the relevance of mathematics to societal needs. Several authors (Friedman and Littman<sup>8</sup> and MacCluer<sup>3</sup> etc.) show how industrial problems can be used to introduce new topics in mathematics. The industrial projects can provide an additional impact and motivation that cannot be obtained in any standard course or academic experience. Many students are motivated by working with a company on an industrial problem: the problem is real and the company needs a solution. Placing students in a situation where someone is going to make a corporate decision based on their mathematical work, challenges and motivates them. This added motivation can be used to attract more students to mathematics.

From its very beginnings in 1865, Worcester Polytechnic Institute has promoted, at the foundation of its educational philosophy, the balance between theory and practice. The vision of WPI's Founders to emphasize the mutual reinforcement between theory and applications, reflected in the university's motto "Lehr und Kunst", is ubiquitous on campus from the undergraduate and graduate curriculum to the university seal and the architectural details of its buildings.

WPI's undergraduate curriculum affords a seamless transition from courses to real-world projects. In 1971 the WPI Plan pioneered a radical departure from the conventional approaches to engineering education by introducing major projects as degree requirements. All undergraduates, in their junior year, work on projects that focus on the impact of technology on society. In their senior year all the undergraduates work on projects in their own major. Most of these projects are sponsored by corporations and non-profit organizations; corporate partnerships have been a hallmark of WPI's programs, and through the years WPI has enjoyed mutually beneficial relationships with hundreds of corporations, locally, nationally and worldwide.

In the early '90s the Mathematical Sciences Department began developing senior-year projects for its majors that are based on real-world problems. The first corporate-sponsored projects were highly successful, beneficial to both students and sponsors: the students were more motivated by the challenge of open-ended problems and by the corporate interest in their results; the corporate sponsors were pleased to see a new approach to their problems, took pride for having an impact on the educational process, and valued their access to potential hires.

As a result of this initial success, the WPI Center for Industrial Mathematics and Statistics (CIMS) was established in 1997. Faculty affiliated with CIMS have made a concerted effort to integrate industrial applications vertically at all levels in the mathematics curriculum. Each year about half of the mathematics majors' senior-year projects are industrial projects. Since 1998 we have provided research opportunities for undergraduates from other universities through an NSF-sponsored Research Experience for Undergraduates (REU) in Industrial Mathematics and Statistics, the first of its kind, to our knowledge. At the graduate level, with support from the Sloan Foundation, we have introduced a Professional Science Master's degree in Financial Mathematics and one in Industrial Mathematics; the Ph.D. program in mathematics was also adapted to require the completion of a project outside the Mathematics Department. Outreach programs for middle and high school teachers include the Mathematics in Industry Institute for Teachers, funded by the Society for Industrial and Applied Mathematics and the GE Foundation,

and the Math and Science Partnership, funded by NSF. These involve helping teachers develop industrial mini-projects which they can use in their classrooms to motivate their students. This is truly a novel type of professional training opportunity.

These industrial projects in the Mathematics Department, provide students with much needed training for successful careers outside academia. It prepares them to be versatile problem solvers in the wide variety of problems a company faces. It prepares them as well to understand the engineering and business language in which problems will be presented to them, and to communicate their results in the same language.

## **Undergraduate Projects**

**The senior-year projects**, called major qualifying projects (MQP's) are part of the degree requirements of all WPI seniors. Usually a team effort, the MQP is completed in the major field of study, spanning most of the academic year. The purpose of the MQP is to provide a capstone experience in the student's major that develops creativity, instills self-confidence, and enhances the ability to communicate ideas and synthesize fundamental concepts. To complete an MQP successfully, the student must formulate a problem, develop a solution, and implement it competently and professionally; be exposed to interaction with the outside world before starting a career; be able to work in teams; and communicate well orally and in writing. This project activity has become a cornerstone of the WPI education and has been highly successful at involving undergraduates in significant research.

The first MQP's in industrial mathematics were developed in the early '90s and were sponsored by PresMet, a manufacturer of pressed-metal parts for the automotive industry, and by Morgan Construction, the largest US manufacturer of steel rolling mills. Since then, faculty affiliated with the Center for Industrial Mathematics and Statistics have developed over 50 projects in collaboration with over 35 sponsors from industry, business, and government such as: Bose, Compaq, Deka Research, GE, IBM, Procter & Gamble, United Technologies, and Veeder-Root.

The industrial mathematics MQP's start with a problem generated by our industrial partners. In the first stage, the industrial representatives and the faculty advisors work on the initial formulation of the problem, bearing in mind that appropriate projects need to involve, besides the industrial importance, a significant need for mathematical modeling and computations. Once the students start the work, they need to appreciate the engineering significance of the problem and the mathematical tools needed to attack it. Almost always, students need to learn the engineering language, so often foreign to them, and at the same time they usually need to learn more mathematical theory. Students discover that real-life problems are very different from textbook problems, as the former come almost always incompletely formulated, with messy data and are not preceded by a math lesson for which these are the applications. As they learn more about the problems, the students often realize the need to re-formulate them, or the need to ask more questions or obtain more data. They also realize that in industry one needs the best possible answer in the time frame dictated by design and production constraints. Students who are used to working individually on math problems discover the need for teamwork. In meetings with the sponsor, students often learn that their analysis will be used to make a real decision, in some

cases expensive decisions. This has an impact that homework and tests cannot imitate. Finally, they learn to communicate their conclusions in a language accessible to engineers and business majors, when presenting for the sponsors, and in a mathematical language when presenting to their mathematics faculty.

The following are a few examples of past projects.

***Mathematical Modeling in Metal Processing*** (1996 and 1997)

6 students; Advisor: Prof. Bogdan Vernescu; Sponsor: Morgan Construction Co.

These projects developed several methods to simulate the wear and optimal geometry of the Laying Pipe mechanism, used for coiling steel rods in rolling mills. The wear models included inertial effects and Coulomb-type friction. Strong correlation between the computed wear and the measured data was obtained. To improve the wear distribution an optimized shape was obtained using optimal control theory and calculus of variations techniques.

***Mathematical Models of Damage Spread in Networks*** (2002)

1 student; Advisor: Prof. Arthur Heinricher; Sponsor: Lehman Brothers

This project described mathematical models for how damage, measured in lost capacity, can spread through an organization. The model allows a company to simulate damage spread and compare different strategies for allocating repair resources after the initial damage has occurred. This project was completed in collaboration with Lehman Brothers investment firm in the aftermath of the terrorist attacks in September 11, 2001.

***Modeling of Torque for Screw Insertion Processes*** (2006)

3 students; Advisor: Prof. Suzanne Weekes; Sponsor: Bose Corporation

A self-tapping screw is a high-strength one-piece fastener that is driven into preformed holes. Students analyzed and improved a mathematical model of the self-tapping screw insertion process so that it can be used in manufacturing processes at the BOSE Corporation. They built a Graphical User Interface in MATHLAB which allows users to enter fundamental data and produces the corresponding torque curve. The accuracy and robustness of the model was tested by comparing predictions to empirical data collected at BOSE.

**The REU projects.** The first Research Experience for Undergraduates program in industrial mathematics, sponsored by NSF, was started at WPI in 1998 (P. Davis<sup>6</sup>). More recently similar REU programs were funded at UCLA and NCSU. Since 1998, 94 students (including 46 female and 9 minority students) from universities across the United States have participated in the WPI program. REU students have worked on 27 different projects for 14 different companies with 12 different faculty advisors. It is a measure of success that several companies, including Bose, John Hancock Insurance, and DEKA Research and Development, have sponsored multiple projects over the years.

In this eight-week summer program, students have experiences similar to the ones WPI students have in their senior year. The students work closely in teams with a faculty advisor and meet regularly with the industrial sponsor. In addition to project work, the students meet with industrial mathematicians to better understand how mathematics is used in the real world. Mathematicians from companies such as Microsoft, John Hancock, Fidelity Investments and United Technologies are invited to talk with the REU students, and there are visits in industry to companies like GE Plastics, IBM T.J. Watson Research Center, The Mathworks, Bose, and DEKA Research & Development.

The REU program at WPI provides an excellent experience for advanced undergraduate students. The experience is certainly valuable for students interested in following non-academic career paths, but it is just as valuable for students who enter “traditional” graduate programs and go into academic careers in that it broadens their perspectives on the uses of mathematics (A. C. Heinricher and S. L. Weekes<sup>12</sup>, B. Vernescu and A.C. Heinricher<sup>19</sup>)

Here are some sample REU projects from past research summers (more are available on the CIMS web at [www.wpi.edu/+CIMS](http://www.wpi.edu/+CIMS)):

***Optimal Cession Strategies –***

Sponsor: Premier Insurance Co.; Faculty advisor: Arthur Heinricher; Industrial advisors: Richard Welch, CEO, and Martin Couture.

In the state of Massachusetts, the automobile insurance industry is highly regulated. Not only are insurance rates fixed by the state, but no company can refuse insurance to anyone who requests coverage. Companies do have the option to “cede” high-risk policies to a state agency. With every new customer, and with every policy renewal, the company must make a decision: keep or cede the policy? This problem contains many of the interesting/difficult aspects of real decision problems. For example, many of the parameters in the problem must be estimated from data which are often incomplete or flawed. One project team analyzed the “risk” associated with cession strategies. A second project team developed an efficient algorithm for identifying “good” and “bad” agents, and showed that the agent who sold the insurance was one of the better predictors of loss.

***Modeling Fluid Flow in a Positive Displacement Pump***

Sponsor: DEKA Research and Development; Faculty advisor: Suzanne Weekes; Industrial advisor: Dr. Derek Kane.

This was one of four projects completed for DEKA Research and Development. Dean Kamen, the founder and owner of the company, said in a recent interview, “I don't work on a problem unless it is going to make life significantly better for a lot of people.” This philosophy has impressed and inspired four different REU teams at WPI. One particular project focused on modeling flow and phase transition in a simple type of pump. The students developed mathematical models and Matlab code to implement the models. The company was able to use the models to optimize design parameters for the pump and improve efficiency. The pump under study was a crucial component in a new, highly efficient water purification system being developed by DEKA for use in third-world countries. Two years later, a new REU team working on a new project visited DEKA, saw a working prototype, and heard DEKA engineers describe how the system would bring pure water to some of the poorest regions in the world.

***Statistical Procedures for Failure-Mode Testing of Diagnostic Systems*** (2001 and 2002)

Sponsor: Veeder-Root; Advisor: Arthur Heinricher; Industrial advisor: Robert Hart.

In most states, gas stations are required to have equipment that will collect and contain harmful vapors emitted during the refueling process. Stations must also have in place diagnostic systems that continuously monitor key aspects of the vapor recovery systems. The diagnostic system must issue a warning if any of these key components is not operating within required limits. The project goal was to develop statistical tests that would certify that the monitoring systems met the standards set by the California Air Resources Board (CARB). The project team developed failure-mode tests in which artificial failures are created for the system to identify. Since 2007 these tests are part of the certification procedure used by CARB and EPA.

### ***Estimation and Optimization for Constructing Hedge Portfolios***

Sponsor: Deutsche Bank; Faculty advisors: Arthur Heinricher and Carlos Morales; Industrial advisor: Dr. Ara Pehlivanian.

Finance has been one of the fastest growing areas for applications of mathematics for several years. In the summer of 2003, a team of six REU students worked on a portfolio analysis problem for Deutsche Bank in New York. The goal of this project was to develop and test a new method for building a short term hedge portfolio. The team was divided into two groups, one to focus on statistical analysis to build good estimates for the covariance matrix and another to focus on optimization algorithms for identifying (quickly) good hedge pairs and constructing efficient portfolios using the hedge pairs. The students learned about the difficulties of working with “dirty data”. They also learned to trade optimality for efficiency and found that an approximate solution on time is better than an optimal solution too late.

## **Graduate Projects**

In 2000 we developed two Professional Science Masters (PSM) Programs: one in **Financial Mathematics** and one in **Industrial Mathematics**. Funding for the development of these programs was provided by the Alfred P. Sloan Foundation. Without precluding students from continuing in a Ph.D. program, these programs are specifically designed to provide the training that would make graduates successful in industry ( see S. Tobias et al.<sup>18</sup> and L. Sims<sup>17</sup>).

Graduate education in mathematics provides students with the ability to think logically, to analyze complex systems, to identify underlying patterns and structures, to conceptualize and formulate problems, and to create new ideas. Mathematicians preparing to work in industry also need: knowledge and exposure to applications of mathematics in sciences and in engineering, experience in formulating and solving open-ended problems, and computational, communication, and teamwork skills. This is why the Professional Science Master Programs require students to complete courses from other departments, introduce them to professional skills through a special seminar, and provide them with internship opportunities.

The Financial Mathematics graduate program at WPI ([www.wpi.edu/+math/Grad/profms.html](http://www.wpi.edu/+math/Grad/profms.html)) has been designed to lead students to the frontlines of the financial revolution of the new century. Coursework on mathematics and finance, computational laboratories, industrial internships, and project work equip students with the knowledge, skills, and experience necessary for the quantitative positions in investment banks, securities houses, insurance companies, and money management firms. Mastery of mathematical models for risk and its relationship to returns, trading strategies, structured contracts, and derivative securities requires a strong mathematical background. The program is built on collaborations between our faculty and the financial services industry concentrated in the Boston – Hartford corridor. Graduates of the program start their professional careers in jobs involving financial product development and pricing, risk measurement and control, and investment decision support or portfolio management.

The Industrial Mathematics Program ([www.wpi.edu/+math/Grad/profms.html](http://www.wpi.edu/+math/Grad/profms.html)) is aimed at training students for professional careers in industrial environments through developing their analysis, modeling, and computational skills. It gives students a competitive career advantage by providing the breadth required by industrial multidisciplinary team environments through elective coordinated modules of mathematics and engineering/science courses (e.g. physics,

computer science, mechanical engineering, electrical and computer engineering, bioengineering), tailored to individual students' interests and needs. By also developing the students' communication and business skills, the program aims to creating successful professionals for the corporate world.

In both programs, industrial experience is gained through an industrial project sponsored by local industry. Industrial summer internships are encouraged and facilitated by CIMS through its industrial partners.

All the graduates from these programs have been successfully employed. One success story of Simon Donkor '03 was featured in The Wall Street Journal to illustrate the career value of professional science master's degrees. After earning his PSM at WPI, Donkor found an attractive position in financial modeling with Fidelity Investments in Boston.

The industrial experience and project training are also parts of our other graduate programs in mathematical sciences. In 2006 the SPAIG (Statistical Partnerships among Academe, Industry and Government) Award from the American Statistical Association (ASA), which recognizes outstanding partnerships established between academe and business, industry, and government organizations, was awarded to the WPI Mathematical Sciences Department for the partnership established by Professor Balgobin Nandram with the National Center for Health Statistics (NCHS). This partnership supported projects and theses for undergraduate and graduate students, as well as internships. NCHS statisticians worked with students to understand many difficult NCHS survey data problems and co-authored journal papers.

The Ph.D. Program in Mathematical Sciences has also been designed to expose students to the full range of mathematics, from its intellectual challenges, to its usefulness and applications (see Chan1); thus a 9-credit project outside the Department is one of its degree requirements. Ph.D. students have worked on projects for Kodak, Air Force Labs, Sandia, and the Lawrence-Livermore National Laboratories.

## **Workshops for Faculty**

WPI was chosen to be the site for the first **SIAM Mathematics in Industry Regional Workshop**<sup>4</sup>, held in May 18-19, 1998. The purpose of the workshop was to help mathematics faculty study and implement some of changes recommend in the SIAM Report on Mathematics in Industry<sup>16</sup>. The keynote speakers from IBM, Kodak, Lucent and Lehman Brothers discussed the need for sophisticated mathematics modeling in industry, and participants presented successful university-industry project collaborations.

This workshop was the first in a series of six regional workshops that brought together mathematicians from industry and academia to showcase their scientific and educational collaborations. It was followed by five other regional workshops at the University of Illinois in 1998 (Midwest), Claremont Colleges in 1999 (Western), North Carolina State in 1999 (Southeast), University of Washington in 2000 (Northwest) and University of Huston in 2001 (Southwest). These workshops have emphasized the need for better connecting the mathematics



world with the industrial world and are now followed by the biannual SIAM Mathematics for Industry Conferences.

The **Mathematics Problems in Industry Workshops** (MPI) are one-week long meetings dedicated to providing new insights and research directions on industrial mathematics problems. First organized at Oxford University in England, this type of hands-on workshop is now organized in several countries around the world (Australia, Canada, Germany, India etc.). In the U.S.A. the workshops are organized yearly and have rotated among RPI, U. Delaware and WPI. WPI hosted the 2003 and 2005 editions (<http://www.wpi.edu/+Math/News/MPI2003/MPIsite/>) and the latest 2006 edition was hosted by Olin College.

The workshops attract leading academic mathematicians and graduate students who work for the week on problems posed by engineers and scientists from industry. In the past, these problems have included, but were not limited to, engineering and product design, process design and control, environmental remediation, scheduling and optimization, and financial modeling. As outcomes, these workshops provide the industrial sponsors a better understanding of existing models and methods, access to advanced computing solutions, and a dialog with mathematicians in academia and government labs that provides a new look and fresh ideas.

The problems presented in 2003 and 2005 at WPI were:

- Safe Fuel/Air Slow Compression (Gilbarco/Veeder-Root, Simsbury, CT)
- Optimal Wear for a Laying Pipe (Morgan Construction, Worcester, MA)
- Need a Lift?: An Elevator Queuing Problem (UTRC, East Hartford, CT)
- Enhanced Leak Detection in Fuel Tanks (Gilbarco/Veeder-Root, Simsbury, CT)
- Stability of the Oil-Air Boundary in Fluid Dynamic Bearings of Hard Disk Drives (Hitachi GST, San Jose, CA)
- Lubrication Layer Perturbations in Chemical-Mechanical Polishing (Araca, Inc., Mesa, AZ)

### **Middle and High School Outreach Program**

Business and industrial applications of mathematics are clearly a valuable but underutilized means of both teaching and motivating underserved middle and high school students in mathematics. Too often, mathematics is taught disconnected from applications and the real-world problems it is so well equipped to solve. As a result, it may seem irrelevant to the career interests of many students. Thus, it should come as no surprise that most students, particularly women and minority students, lose interest in mathematics. Mathematics becomes a barrier to be overcome or, more often, simply avoided.

Research reveals (e.g. E. Fennema and G. Leder<sup>9</sup> and R. Moses<sup>14</sup>) that young women and underrepresented minority children do not persist in mathematics because they appreciate neither its connection to careers nor its utility in helping to solve society's problems. Research has also shown that one-time visit by a working scientist or engineer is not always the best way to encourage students to continue their studies in mathematics and science (P. Campbell et al.<sup>2</sup>).

Professional development that relates to the content and materials teachers are using in the classroom has been shown to be more beneficial for teaching and learning than professional development on abstract concepts. It has also been shown that student performance is greatly improved when teaching concepts and applying them to concrete problems as opposed to only drill and practice (H. Wenglinsky<sup>20</sup>). Unfortunately, only three out of five teachers use concrete applications in class and only 13% of homework involves such activities.

In June 2000, with support from the NSF through the SIAM, we organized our first Mathematics in Industry Institute (MII) for teachers. Thirty-six teachers from New England spent five days at WPI working on industrial projects. They attended presentations from industrial mathematicians and participated in a field-trip at the United Technologies Research Center and the Pratt & Whitney jet engine plant in Hartford, CT, where they learned about the mathematics used in developing some of the products.

This pilot work was continued and expanded into the Mathematics in Industry Institutes for Teachers, with support from the GE Foundation, for four consecutive years: 2001-2005. The Institutes consisted of: workshops actively involving teachers in developing industrial projects suitable for use in the classroom; development of dissemination activities to bring these idea and resources to entire school districts; and incorporation of these projects into their curriculum. A follow-up workshop in the spring allowed teachers to share “best practices”. With the completion of the 2005 MII, more than 240 teachers from 18 states and Canada have come to WPI to work on industrial mathematics projects. There are now 23 industrial mathematics projects on the CIMS web site ([www.wpi.edu/+CIMS](http://www.wpi.edu/+CIMS)), with extensive background and supporting materials designed to make the projects easy for classroom use.

We have received significant positive feedback on the impact of the institutes. Teachers report that the institutes have changed the way that they think about mathematics, talk about applications for mathematics, and discuss quantitative careers in the classroom. For example, we saw an increase from 25% to 58% in the percentage of teachers who regularly use engineering and quantitative careers in homework and examples. Approximately 50% of all participating teachers reported an increase in the use of industrial math examples in the classroom.

There is also additional evidence for the broader impact of the Industrial Math REU and the RET program.

- A new course for seniors was introduced at Wachusett High School in Holden, MA entitled “Mathematics for Decision Making in Business, Industry, and Government.” The core of the course is a sequence of team project derived from our industrial projects.
- A Mathematics in Industry Competition for middle and high school students was held in Fairfax County, Virginia in the fall of 2002. Students from 10 schools worked on versions of the REU projects and presented their work at the Thomas A. Edison High School in Fairfax.
- A teacher from the Higgins Middle School in Peabody, Massachusetts developed industrial connections with Eastman Gel, and her students worked as “mathematical consultants” for engineers at Eastman Gel. She is continuing to work with WPI to help other teachers build similar partnerships.

- Versions of the WPI projects have been included in the Benchmarks for High School Mathematics Education developed at the American Diploma Project (an affiliate of the Achieve Organization [www.achieve.org](http://www.achieve.org) ).
- The industrial projects for teachers, developed by CIMS, form a key part of our contribution to the NSF-sponsored Mathematics and Sciences Partnership project connecting WPI, Boston University, University of Massachusetts Lowell, the Educational Development Center, and five Boston-area school districts (Arlington, Chelsea, Lawrence, Waltham, and Watertown).

With additional funding from NSF, through Research Experience for Teachers (RET), and with corporate support from the Exxon-Mobil Research Foundation, a group of Massachusetts teachers were funded for the past six summers to develop industrial projects for middle and high school. These teachers served as project guides and mentors during the MII. After the Institute they continued at WPI to develop the industrial mathematics projects, ready for classroom use.

## Conclusions

The real-world problems have an educational impact that cannot be replicated in the traditional mathematics classroom setting. They can offer, at every level, a view on the importance and on the impact mathematics can have in providing solutions for real problems in industry and business. At the middle and high school levels, these projects provide the motivation some students need in their mathematics classes and the connections to concepts from their science classes; for mathematics undergraduate and graduate students it provides them the training they need for a more diverse set of career options and brings them closer to the engineering and scientific campus community; for mathematics faculty these projects provide a different type of challenges, a new tool for training their students, and a way to use their knowledge for a broader good; for corporations they provide new approaches and solutions to their problems, and access to students and faculty.

The models developed at WPI form a vertical integration of innovative industrial projects in the mathematics curriculum at all levels, from middle and high school, through undergraduate programs and up to graduate programs and faculty research.

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