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BYOE: Introducing the Time and Frequency Domain Relationship in an Introductory Circuits Course

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BYOE: Introducing The Time and Frequency Domain Relationship in an Introductory Circuits Course

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Abstract

Electrical engineering students should understand the relationship between the time domain and the frequency domain. Many students learn to apply analysis techniques in the respective domains without understanding the relationship between the domains. To address this and other challenges to learning, we have changed the content and format of several introductory courses so that more abstract concepts are first introduced earlier in the program in a practical context. This paper describes our new approach to introduce the relationship between the time domain and the frequency domain to students as they are just being introduced to circuits.

Introduction

We have changed the content and format of our introductory electrical engineering courses to emphasize the relationships among the different concepts. Some topics that were first presented in the signals theory course of the old curriculum are now introduced in the initial circuits course of the new curriculum. The earlier introduction and subsequent repetition and enhancement of the abstract concepts help students to understand the theoretical basis for practical ideas. The course formats were also changed to a studio form in which experiential learning was coupled with discussions. This paper describes an instructional module to introduce the relationship between the time domain and the frequency domain.

Background

Balancing discussion with practice increases the effectiveness of technical courses when compared to the traditional methods that rely on lectures and fixed lab experiences.[1] Additional benefits can be gained by using the discussions to guide students to discover concepts through the practical experiences.[2] In turn, these discoveries reinforce important results from the discussions. An objective is to help the students to develop conceptual knowledge.[3] In addition, careful topic sequencing helps student to view multiple representations of a concept in a coherent fashion.[4]

The relationship between the time domain and frequency domain is central to much of electrical engineering, but many students have difficulty understanding the relationship between these domains. The Fourier series can be used to bind these domains together,[5] but the binding can appear as abstract and irrelevant mathematics in the absence of practical illustrations. This paper describes a sequence of studio experiences that teach concepts from both domains while also guiding the student to discover the relationship between the domains.

A Circuit Viewed in the Time Domain

The first part of the course explores the typical techniques for analysis of resistive circuits subject to constant (DC) excitation. The transition from DC analysis to circuits having state starts with an introduction to the capacitor as an energy storage device. This involves a typical development leading to the resulting differential relationship between voltage and current for the ideal capacitor.

This time-dependent relationship is used to convince the students that the applied excitation must change with time. A step change is used for the excitation to the circuit shown in Figure 1, and the circuit is solved to find the natural and forced responses to the step. This development of the time response of an RC circuit to a step input is typical for most textbooks on circuits.

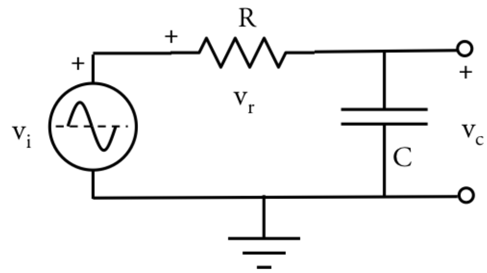

Figure 1 Simple RC Circuit

The studio approach to teaching allows the student to transition immediately from discourse on a topic to practical experience with the topic. The practical experience for this topic requires the students to assemble the circuit shown in Figure 1. A function generator is used for the voltage source, v_i, and the voltage across the capacitor, v_c, is observed using an oscilloscope.

The first part of this assignment involves confirmation of the expected time-domain behavior. Students are instructed to use 10 kΩ for R and 0.1 μF for C. This produces a time constant of 1 ms. They are told to set the function generator to produce a square wave with zero DC offset, 4 V_{PP} amplitude, 50% duty cycle, and a frequency of 10 Hz. They are instructed to observe the function generator output using one oscilloscope trace while also observing the voltage across the capacitor using the other oscilloscope trace.

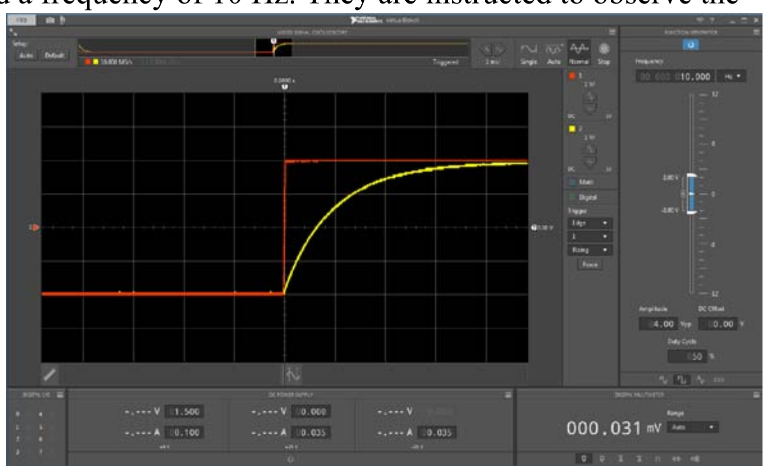

Figure 2 Time Domain Response to Step

This experiment yields the results shown in Figure 2. This shows the rising edge of the square wave input causing the exponential response at the output. The students are instructed to determine the time constant of the circuit from the observed response and compare this time constant with the calculated RC time constant.

They are taught to measure the time constant by referring to the solution to the differential equation, $v_c = v_{i+} + \left(v_{i-} - v_{i+}\right) e^{-t/RC}$, with $t = RC$ and the initial and final values taken from the applied square wave. They observe that $v_c \approx 0.5$ volts when $t = RC$. The output reaches this voltage at approximately 1 ms after the input changes. This observation matches the predicted value.

Transitioning to the Frequency Domain

Students were assigned a similar laboratory experiment in the previous circuits class, but the time-domain experience and time constant measurement were at the end of the assignment. This assignment continues on a path to introduce frequency domain behavior.

The next part of the assignment asks the students to increase the frequency of the applied square wave until the square wave period is approximately ten times the circuit time constant. This task sets the square wave period to 10 ms, and they must determine the frequency to be 100 Hz. This exercise confirms that the exponential response of the circuit is independent of the excitation period. It also introduces the notion of changing the excitation frequency as they review the relationship between frequency and period for a periodic waveform.

The students are next told to change the excitation input from a square wave to a sine wave without making any other changes. They are told to observe the relationships between the input and output waves and note similarities and differences. Figure 3 shows the result of this exercise. The students should note that both waves have the same frequency, but the output has slightly smaller amplitude than the input. Also, the output appears to be shifted with respect to the input. This exercise introduces the concept of circuit behavior in response to sinusoidal excitation.

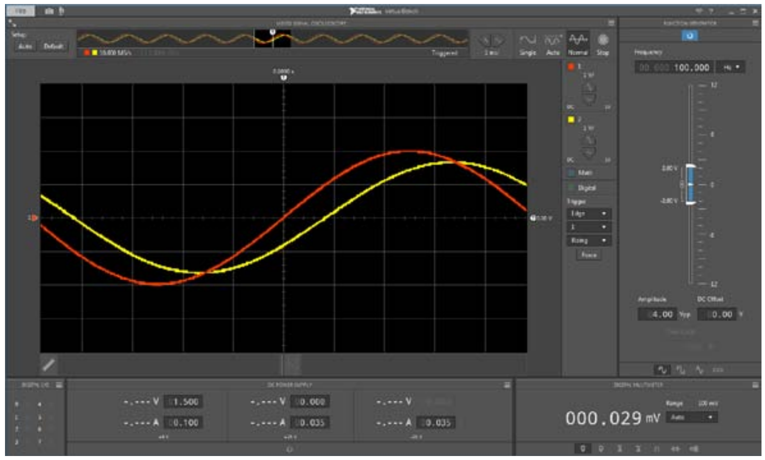

Figure 3 Time Domain Response to 100 Hz Sine Wave

The Theory

Switching the excitation to a sine wave starts to connect the time domain and the frequency domain for the students. The students rarely have trouble accepting a square wave as a repetitive step excitation. Increasing the square wave frequency then helps them to understand the time domain concept that the circuit determines the step response. Finally, the switch to a sine wave moves them away from thinking of the time domain step and toward an excitation that has a more ambiguous place in the time domain than a step that clearly occurs at a specific time.

The sine wave excitation and response invites a discussion of the sine wave characteristics: frequency, amplitude, and phase. Students enter this class with knowledge of the mathematics, and they rarely have difficulties with these sine wave concepts. However, many do have trouble connecting the concepts to the physical world. The next step in the lab establishes this connection.

Sinusoids in Time

The students are to measure the differences in amplitude and phase between the excitation and response sinusoids shown in Figure 3. The measurements using the example components show that the maximum voltage amplitude across the capacitor is about 83% of the maximum excitation voltage amplitude. The voltage sinusoid across the capacitor is measured to be about 860 μs behind the excitation sinusoid. The period of both sinusoids is 10 ms, and the measured phase shift is therefore about -31°.

The students are then asked to repeat the experiment using five specific additional sinusoidal excitations. All of the additional sinusoids have the same 4 V_{PP} amplitudes and differ only in frequencies. The frequencies assigned are 3 kHz, 9 kHz, 15 kHz, 21 kHz, and 27 kHz. They are told to complete a table of sinusoid amplitude ratios and phases. The results should be similar to those given in Table 1.

Table 1 Excitation and Response Relationships

Frequency (kHz)	Amplitude Ratio	Phase (degrees)
3	0.053	-87
9	0.018	-89
15	0.011	-89
21	0.008	-90
27	0.006	-90

Upon completion of this exercise, the students should understand basic sinusoidal steady state measurement.

At this point, the students have observed the time domain step response of the RC circuit. They have observed the time response of this circuit to a slow square wave and to a slow sine wave. Further, they have measured the frequency domain response of the circuit to faster sine waves. The frequency domain responses were measured as amplitude and phase transfer relationships as a function of frequency.

Fourier Series

While not previously included in our traditional circuits course, the Fourier series is introduced at this point in our new curriculum. After brief comments about constraints on the periodic function, $f(t)$, the Fourier series is described as:

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left[A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)\right]$$

The students are shown how to calculate the coefficients for a square wave and for a triangle wave. Both waves are centered vertically about the origin with maximum amplitude excursions between -1 and +1. Both have periods of 2π. The square wave transitions from -1 to 1 at the horizontal origin. The triangle wave is at its minimum at the horizontal origin.

For the square wave $\forall n : A_n = 0$, $\forall n : n$ is even: $B_n = 0$, B_1 = 1.273, B_3 = 0.424, B_5 = 0.255, B_7 = 0.182, B_9 = 0.141. These coefficients appear in the Fourier series equation as:

$$f(t) = 1.273\sin(\omega_0 t) + 0.424\sin(3\omega_0 t) + 0.255\sin(5\omega_0 t) + 0.182\sin(7\omega_0 t) + \ldots$$

For the triangle wave $\forall n : B_n = 0$, $\forall n : n$ is even: $A_n = 0$, A_1 = -0.811, A_3 = -0.090, A_5 = -0.032, A_7 = -0.017, A_9 = -0.010. These coefficients appear in the Fourier series equation as:

$$f(t) = -0.811\cos(\omega_0 t) - 0.090\cos(3\omega_0 t) - 0.032\cos(5\omega_0 t) - 0.017\cos(7\omega_0 t) - \ldots$$

The Fourier series representation for the triangle wave is a sum of cosines rather than a sum of sines as was the case for the square wave. The Fourier coefficients for the triangle wave expansion are all negative while the Fourier coefficients for the square wave are all positive.

These differences can be eliminated by adding a phase adjustment to each of the sinusoids in the triangle wave Fourier series representation.

$$f(t) = 0.811\sin\left(\omega_0 t - \tfrac{\pi}{2}\right) + 0.090\sin\left(3\omega_0 t - \tfrac{\pi}{2}\right) + 0.032\sin\left(5\omega_0 t - \tfrac{\pi}{2}\right) + \ldots$$

With this change, both the square wave and the triangle wave examples are composed from a sine wave at the fundamental frequency plus sine waves at odd harmonics of the fundamental frequency. The Fourier expansions of the square wave and triangle wave differ in the relative amplitudes and phases of the constituent sine waves.

Complex Waveform Transformation

The students are asked to normalize the results from their frequency domain measurements so that the normalized amplitude ratio for the 3 kHz sinusoid is 1. The phase angles for all of the sinusoids are rounded to -90° (-π/2 radians). The results are shown in Table 2.

Table 2 Normalized Measured Sine Wave Transfer Characteristics

Frequency (kHz)	Normalized Amplitude Ratio	Phase (degrees)
3	1	-90
9	0.34	-90
15	0.21	-90
21	0.15	-90
27	0.11	-90

Next, the students are asked to normalize the calculated Fourier coefficients for the square wave and triangle wave so that the coefficient of each fundamental is 1. The results should appear as shown in Table 3.

Table 3 Normalized Fourier Coefficients for Square and Triangle Waves

n	Square Wave	Triangle Wave
1	1	1
3	0.33	0.11
5	0.20	0.04
7	0.14	0.02
9	0.11	0.01

The students are then asked to multiply each normalized square wave Fourier coefficient by the corresponding normalized amplitude ratio measured during the previous practical experience. The students are to compare each of these products with the corresponding triangle wave Fourier coefficient. The results should appear as shown in Table 4.

Table 4 Comparison of Measured Sine Wave Results with Fourier Series

n	Frequency (kHz)	Square Wave Fourier Coefficient	Normalized Measured Amplitude Ratio	Product	Triangle Wave Fourier Coefficient
1	3	1	1	1	1
3	9	0.33	0.34	0.11	0.11
5	15	0.20	0.21	0.04	0.04
7	21	0.14	0.15	0.02	0.02
9	21	0.11	0.11	0.01	0.01

These results indicate that applying a square wave input having a 3 kHz fundamental frequency to the RC circuit of Figure 1 should produce a triangle wave across the capacitor. The students are told to perform this test and observe the result.

The results that the students should see are shown in Figure 4. The square wave with 4 V_{PP} at 3 kHz is presented as the excitation to the RC circuit of Figure 1. The triangle wave is observed across the capacitor. The input is presented with 1 V/division. The output is presented with 100 mV/division. Thus, the amplitude of the triangle wave is much smaller than the amplitude of the square wave. The students should note this, but the more important result of this exercise is the change in the shape of the complex waveform when processed by this simple RC circuit.

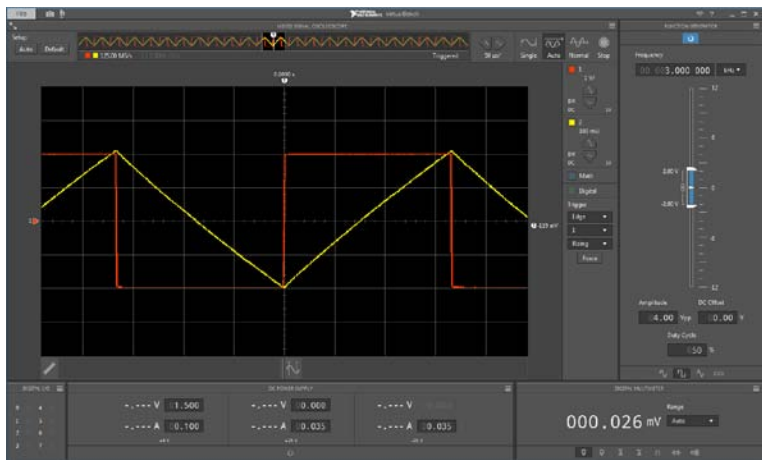

Figure 4 Results of a 3 kHz Square Wave Excitation

As a final exercise, the students are asked to decrease the square wave frequency from 3 kHz back down to 100 Hz by steps of 1 kHz down to 1 kHz and then by 100 Hz steps down to 100 Hz. They are asked to observe how the output waveform changes as the input fundamental frequency is decreased.

The output waveform exhibits a trend toward the exponential shape when the input frequency is 1 kHz. An input frequency of 500 Hz produces an output that is clearly exponential in shape with peak amplitude that is about half of the input square wave peak amplitude. At 100 Hz, the output flattens to approach the input amplitude just before the next transition.

At this point, the students have analyzed, seen and experienced the behavior of the RC circuit of Figure 1 starting from its time-domain exponential response, through its amplitude and phase changing sinusoidal response, to its modification of a complex waveform in the frequency domain – and back. Each of the elements in this instruction module was important as an isolated topic, and the module endeavored to achieve the broader goal of tying these topics together.

Bibliography

1. Hake, Richard R. "Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses." *American Journal of Physics* 66, no. 1 (January 1998): 64 - 74.

2. Hammer, David. "Discovery Learning and Discovery Teaching." *Cognition and Instruction* 15, no. 4 (1997): 485 - 529.

3. Streveler, Ruth A, Thomas A Litzinger, Ronald L Miller, and Paul S Steif. "Learning Conceptual Knowledge in the Engineering Sciences: Overview and Future Research Directions." *Journal of Engineering Education* 97, no. 3 (July 2008): 279 - 294.

4. Danielak, Brian A, Ayush Gupta, and Andrew Elby. "Marginalized Identities of Sense-Makers: Reframing Engineering Student Retention." *Journal of Engineering Education* 103, no. 1 (January 2014): 8 - 44.

5. Kraftmakher, Yaakov. "Some applications of Fourier's great discovery for beginners." *European Journal of Physics* 33 (July 2012): 1249 - 1258.