

Exploring the Effect of Foundation Flexibility on Structural Response

Mr. Alec Roberto Zavala, California Polytechnic State University, San Luis Obispo

Alec Zavala is a Graduate Assistant for the Architectural Engineering Department at California Polytechnic University, San Luis Obispo. He currently conducts research in the field of forced-vibration testing of structures under construction. He will be graduating in June 2016 with the intent of entering the field of structural engineering.

Dr. Peter Laursen, California Polytechnic State University, San Luis Obispo

Dr. Peter Laursen, P.E., is an Associate Professor of Architectural Engineering at the California Polytechnic State University, San Luis Obispo (Cal Poly) where he teaches courses on the analysis and design of structural systems including laboratory courses.

Dr. Cole C. McDaniel, California Polytechnic State University, San Luis Obispo

Dr. Cole McDaniel, P.E., is a Professor of Architectural Engineering at the California Polytechnic State University, San Luis Obispo (Cal Poly) where he teaches courses on the analysis and design of structural systems with a focus on seismic behavior.

Dr. Graham C. Archer P.Eng, California Polytechnic State University, San Luis Obispo

Dr. Graham Archer, P.Eng., is a Professor of Architectural Engineering at the California Polytechnic State University, San Luis Obispo (Cal Poly) where he teaches courses on the analysis and design of structural systems.

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Structural computational models created by Architectural Engineers frequently show an overly simplified representation of the soil-structure interface. Structures are routinely modeled without considering the influence of the structure's foundation and underlying soil resulting in misrepresentation of the actual building response. Critical evaluation of the soil-structure interface was encouraged through a series of dynamic experiments. These experiments not only improved learning of the topics by applying the theory to realistic engineering systems¹, the experiments also served as a point of comparison for the computational models that many students cited as typically lacking and the reason they failed to take the necessary steps to calibrate the models in previous assignments.

In order to expose students to the challenges of accurate foundation modeling, students were asked to determine the appropriate boundary conditions for following three structures:

- 1) A campus structure with a braced frame lateral system founded on embedded concrete pillars.
- 2) An off-campus podium slab structure with concrete columns founded on grade beams. The slab supports an office building currently under construction. In this case, students were invited by the engineer of record to explore the building dynamic characteristics.
- 3) A laboratory two-story steel moment frame bolted to a concrete floor through steel base plates.

The senior level undergraduate students predicted the response of the three structures by computational models and hand calculations. They completed hand calculation estimates first to provide a baseline for the computational models. After predicting the response, the students conducted dynamic experiments to measure the actual response of the structures. Prior to any experimentation, students were surveyed about their choice of boundary condition representing the influence of the structure foundation and surrounding soil. After comparison between the initial computational models and the experiments, the students were, in two cases, challenged to improve their models by refining the ground-structure interface.

This paper presents three case studies based on the structures described above. The case studies demonstrate how students were informed by simple physical experiments. In each case, the initial foundation modeling was overly simplified. Refined models were developed by the students and generally resulted in predictions closer to the experimental results. Descriptions of structure geometry, computer modeling, dynamic experiment and results are given and student learning is discussed. Finally, common conclusions are drawn from the case studies.

Campus structure

The campus structure spans a ravine (Figure 1). Across the ravine the structure acts as a 48 ft. truss, and as a 24 ft. braced frame in the transverse direction. It is founded on concrete pillars with variable soil embedment depths located in the four corners. In this case, the students were asked to create a computational model in order to capture the fundamental frequency and mode shape of the structure based on as-built drawings. A quick hand calculation based on the lateral stiffness of the braces resulted in a frequency of 13.3 Hz, upon comparison with their computer

models the students realized this value was high due to neglect of bending in the Vierendeel truss sections. As for foundation modeling, the students inspected the connection between the lower chord and the concrete pillars and concluded that they were rigidly connected. The students decided to model the foundations with the concrete pillars rigidly fixed at grade level (termed 'original' model). Modal analysis of the original model resulted in a fundamental natural frequency in the vicinity of $f = 7.9$ Hz (Table 1) and the mode shape shown with solid red line in Figure 2. The thin, finely dashed line shows the floor outline at rest.

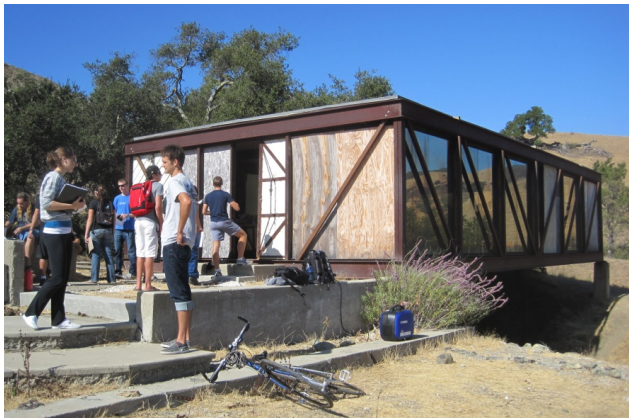
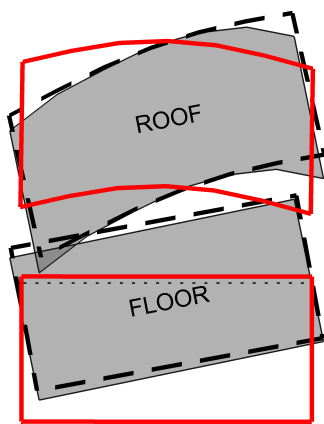


Figure 1: Campus structure: South-West corner

Concrete pillars

Table 1: Model and Experimental natural frequencies, f [Hz]

Model, original rigid foundations	Model, refined flexible foundations	Experimental
7.9	6.7	6.1



Legend:
thin dotted = position at rest
red = rigid foundation model
dashed = flexible foundation model
grey shaded = experimental)

Figure 2: Fundamental mode shapes

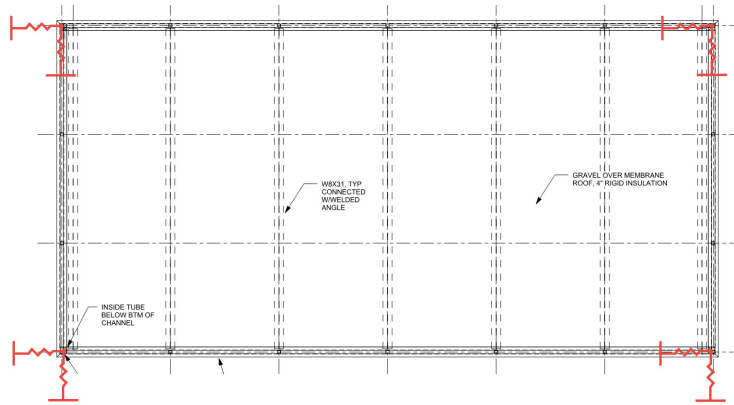


Figure 3-Schematic of floor framing and foundation springs

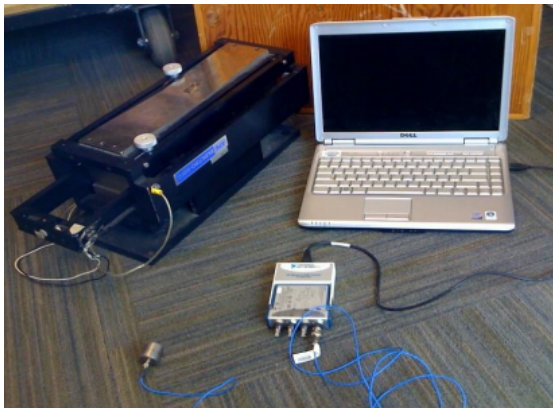


Figure 4: Shaker and accelerometer

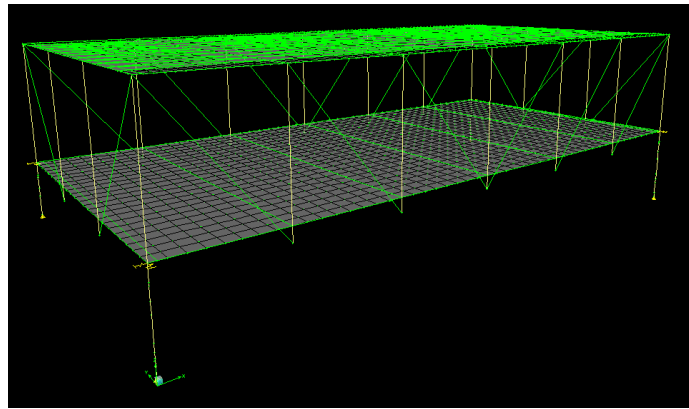


Figure 5: Campus structure, refined model (SAP2000⁴)

Next, students performed forced vibration testing (FVT^{2,3}) to experimentally determine the fundamental natural frequency and corresponding mode shape. The FVT was implemented with a linear shaker that loaded the structure with a dynamic harmonic excitation and a series of accelerometers that measured the response of the floor and the roof. The linear shaker was placed in the middle of the roof to excite the structure at the natural frequency in the transverse direction. Figure 4 shows the FVT setup. The experimental natural frequency was found to be $f = 6.1$ Hz (Table 1) and Figure 2 shows the corresponding mode shape in shaded grey. The students quickly concluded that the original computer model overestimated the system stiffness. Past experience with FVT analysis and computer modeling generally shows that computer models tend to overestimate the system stiffness, so a difference in frequency between the computer model and the experiment of around 30% was not alarming. However, as the students studied the mode shapes, it became clear that the relative movement of the floor slab in the model (solid red in Figure 2) was far too small in comparison with the experimental result. The students concluded that considerable foundation flexibility exists and should be modeled.

In order to quantify the foundation stiffness, the total shear at the top of the foundation pillars, V , at each end of the structure was found as the sum of the story forces from the roof and floor. The story force was calculated using Newton's 2nd Law, $F = ma$, where m is the floor (or roof) mass and a is the measured floor (or roof) acceleration (torsion was also considered). The floor horizontal displacement, U , was calculated by dividing the measured floor accelerations, a , by

the square of, ω , the circular natural frequency⁵. The horizontal stiffness in the transverse direction for each end (each side of ravine) could then be determined as $k = V/U$. Calculations indicated a total stiffness of $k = 600$ kips/in for the West side and 300 kips/in for the East side. Figure 3 suggests that four horizontal springs of 300 k/in incorporated in the model (termed 'refined' model) at the top of the pillars (now pinned at both ends) on the West side. It was assumed that the total transverse stiffness could be divided evenly up between the two pillars and that the springs in the longitudinal direction were of the same magnitude. Similarly four 150 kips/in springs were incorporated on the East side. Figure 5 shows a refined computer model.

Modal analysis of the refined model resulted in a fundamental natural frequency around $f = 6.7$ Hz (Table 1) and the mode shape shown in back dashed line in Figure 2. The students concluded that the refined model remained somewhat on the stiff side, however the prediction was within 10%. More importantly, the refined model mode shape was found to match the experimental mode shapes quite well for both roof and floor as shown in Figure 2. The overall conclusion from the students' side was that careful consideration of the foundation stiffness was warranted in this case because of its significant influence in the structure dynamic response.

Off-campus structure

Construction of a new office building in Berkeley, CA, presented a unique opportunity for student-led experimental exploration of structural dynamic properties. A prestressed concrete podium slab is supported by six large circular concrete filled steel columns founded on grade beams. The office space consists of a two-story framed wood structure constructed on the podium slab. The building owner invited the author and students to shake the building at several construction stages. This is an account of the students' modeling and shaking of the bare podium slab. The owner's motivation was to obtain experimental evidence of the building dynamic properties such as natural frequencies and mode shapes for validation of their design computer models. Pictures of the podium slab and the wooden structure are shown in Figure 6. Prior to the site visit, students first calculated the first mode frequency by hand. They considered two extremes, the columns either in double bending (fully rigid slab and foundation, fix-fix) or in single bending (some flexibility in slab and foundation, fix-pin). Then they were asked to create a computational model of the podium slab structure alone (no timber erected) and predict the first two fundamental frequencies and mode shapes. Most students proposed to fix the columns at the base, a simplistic and rudimentary approach. Others chose to implement grade beams and soil springs. The soil spring properties were based on the site geotechnical rapport. Figure 7 shows student models of the bare podium slab with fixed column bases and the completed structure with flexible grade beams and foundation springs.

Natural frequencies from the hand calculations and models are given in Table 2 and model mode shapes shown Figure 8. Students first observed that the hand calculations provided upper and lower bounds for the model frequencies. It was then observed that the model frequencies dropped about 7% when introducing foundation flexibility. The third observation was that the mode shapes for the two models were virtually identical (only those for the fixed model are shown). The first mode shows almost pure longitudinal translation at a frequencies of 4.95/4.61 Hz (fixed model/flexible model) and the second mode shows 'transverse' translation combined with some rotation at a frequency of 5.08/4.72 Hz. On site, students performed FVT (as

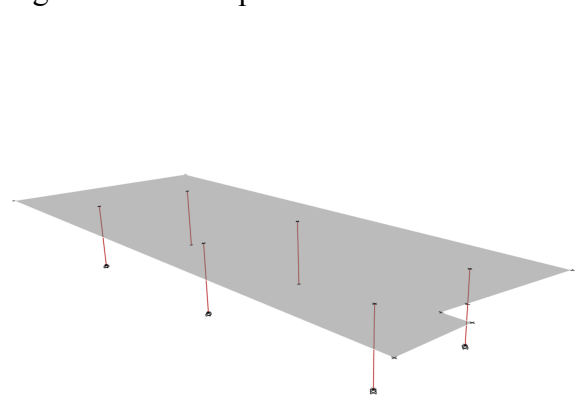
described above) and experimentally determined the natural frequencies and mode shapes. These are given in Table 2 and Figure 9. The experimental frequencies are lower than those from the models by a margin of 22%/14% and 11%/3% for modes 1 and 2 respectively.



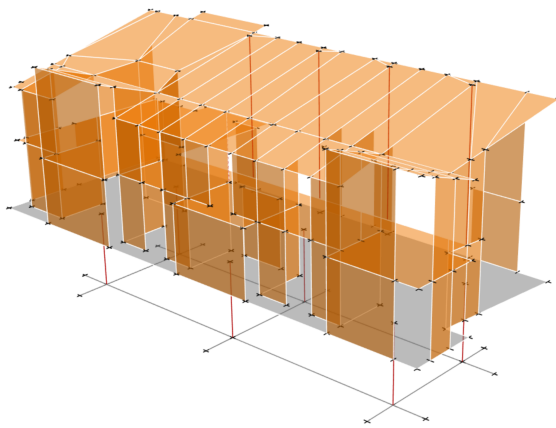
Podium slab from beneath



Timber structure on podium



Fixed foundation model



Flexible foundation model

Figure 7: Student off-campus structure models (ETABS⁴)

Table 2: Off-campus structure

Mode	Periods (s)		Frequencies (Hz)		Frequency % Error*	
	1	2	1	2	1	2
Fix-fix hand calculation	0.137		7.30		80.2%	
Fix-pin hand calculation	0.274		3.65		-9.9%	
Fixed base model	0.202	0.197	4.95	5.08	22.3%	11.2%
Flexible foundation model	0.217	0.212	4.61	4.72	13.8%	3.3%
Experimental	0.247	0.219	4.05	4.57	-	-

*) relative to experiment

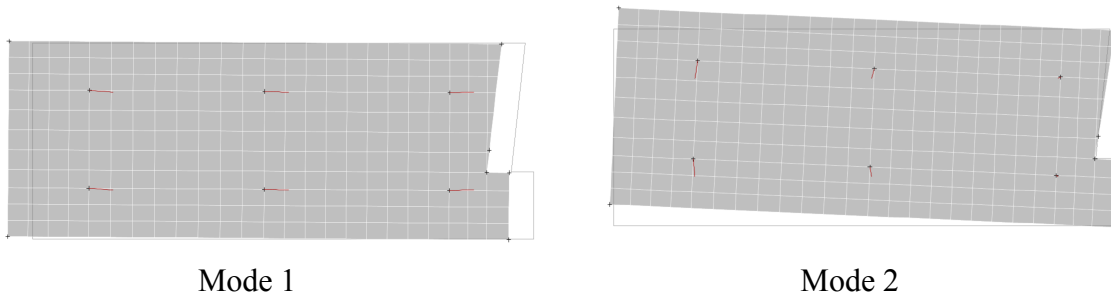


Figure 8: Off-campus structure model, fixed model

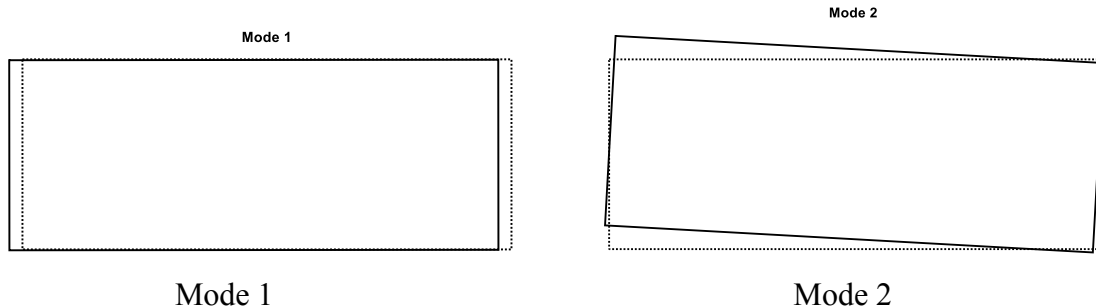


Figure 9: Off-campus structure, experimental

Comparing the experimental and model mode shapes it was found that they were virtually identical and would not reveal any clues as to which analytical model was preferable. The students concluded that the predicted frequencies were affected by the foundation modelling and better results would be obtained with a detailed foundation model. It was however also pointed out that the frequencies improved only moderately and that the extra modeling did not seem worth it.

Laboratory structure

Figure 10 shows the 8½ feet tall, three-dimensional two-story steel moment frame that served as an ideal structure for students to experiment with and model⁶. The frame is composed of W6x9 columns and beams. The 18" thick concrete floor diaphragms were sized to result in realistic natural frequencies for the first few modes of the frame. The columns are connected to the laboratory concrete floor through 1" thick steel base plates and four 5/8" diameter bolts. The weight of each floor, including the beams and columns, is about 6000 lbs. The plan dimensions of the frame are approximately 50" x 50" and the floor heights are 54" and 71" for the 1st and 2nd floors, respectively. The columns are oriented such that there is a strong and a weak axis direction. An advantage of the structure is that it is simple enough to be assessed by hand calculations. It can be analyzed by hand when simplified to a 2D system with 1 translational degree of freedom per floor and used to validate the computer output.



Figure 10: Laboratory structure, Beam/Column Connection and Column Base Connection

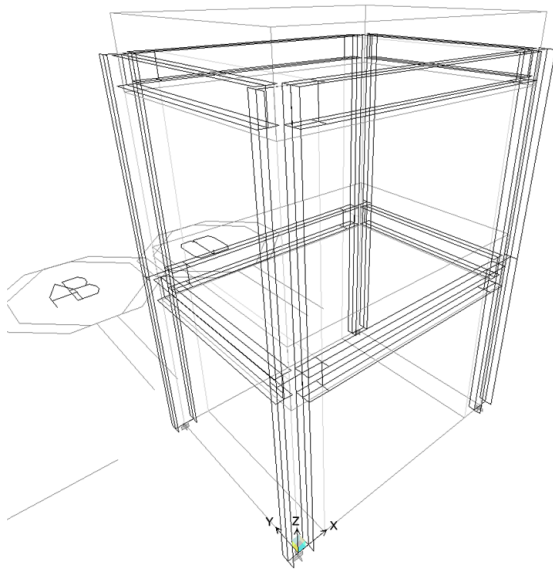


Figure 11: Laboratory structure, Computer model [ETABS⁴]

This exercise started out as an investigation of the foundation stiffness. Prior to conducting the dynamic experiments, students were asked whether they thought the steel column base connection was closer to a fixed connection or closer to a pinned connection, 80% of the class considered a fixed condition to be more realistic and 20% of the class considered a pinned connection to be more realistic. As the initial hand calculations were developed the students discovered that the beam-column stiffness ratio would have a large influence on the structure response, so the question was if the concrete slabs could restrain beam rotation. The majority of the class was in favor of considering the beams rigid based on the argument that each beam was joined to the thick concrete slab along the majority of its length. Considering the base fixed and beams rigid, simplified the lateral stiffness of the columns to $12EI/h^3$ for each column where E is the modulus of elasticity, I is the column moment of inertia, and h is the column centerline height. Figure 11 shows the computer model. The beams were linked to the concrete slabs at their interface leaving only the ends free to deform. Loading the model allowed the students to calculate the column stiffness k_c as the column shear divided by the inter-story displacement. The calculation was done for both fixed and pinned conditions at the base. Table 3 shows the stiffness coefficient $\alpha = k_c/(EI/h^3)$ for the hand calculation and both floors in the strong direction of the frame. It became clear to the students that the computer model predicted a considerable amount of beam flexibility because the fixed base model showed stiffness coefficients of 3.35 and 5.45, thus deflection of about 3 times that of the rigid beam hand calculation. Pinning the base predicted a further drop in lateral stiffness.

Next, students performed forced vibration testing (FVT) to experimentally determine the boundary conditions at the base of the steel frame. The FVT was implemented as detailed above for the Campus Structure. The linear shaker was placed on top of the concrete slab at the second level to excite the structure at the natural frequencies in the column strong axis and column weak axis directions. The story shear, V , was calculated from the story displacement, U , and the story force, F . With the shear force at each level and the displacement of each level, the lateral stiffness of each level was determined through statics. The values for the column stiffness coefficient are shown in the Table 3 and suggest that the structure response is closer to the fixed base than pinned base model, however neither idealized boundary condition is correct. The weak direction response with α near 12 suggests that the beams are relatively rigid in this direction and the base plate provides near full fixity.

Table 3: Laboratory structure, Stiffness Coefficient, α

	2 nd floor	1 st floor
Hand Calculation (strong/weak)	12	12
Fixed Base Model (strong)	3.35	5.45
Pinned Base Model (strong)	2.34	1.50
FVT (strong)	3.19	4.34
FVT (weak)	11.10	10.65
Pinned Base Mod. w/rotational spring (strong)	3.21	4.32

At this point, students were challenged with developing a more accurate computational model to match the experimental results in the strong direction. As a result, the students modeled the column bases in the column strong axis direction with a pinned connection along with a rotational spring. The students converged on a column base rotational spring stiffness of 70,000 K-in/rad. Table 3 presents the stiffness coefficients for the revised model which clearly show that the modified pinned base boundary condition model accurately predicted the two-story frame response.

Conclusion

The students involved in these modelling and experimental studies discovered that the boundary conditions significantly affect the structural response. It was found that the Campus Structure fundamental mode shape and frequency were significantly affected by the foundation flexibility. A laterally fixed foundation model favored by the students proved to be a poor representation of the actual response. The Off-campus Structure experiment showed that considering the flexibility of the seemingly rigid foundation in the calculations improved the predicted fundamental frequency noticeably. Finally, the Laboratory Structure experiment revealed that the beams were relatively flexible (although integrated with the concrete slab) and softened the lateral stiffness by a factor 3 and that the base plate fixity could reasonably be modelled as fixed, although allowing some flexibility in the base plate connection improves the model response.

The opportunity to compare structure response predictions to dynamic experimentation results was well received by the students and led to a deeper understanding of foundation modeling and the associated implications. Several students stated that it was a beneficial way to revisit and reinforce the static and dynamic theory concepts taught in the preceding analysis classes. The students enhanced their ability to assess their own computer models and create more accurate computational models by using simple hand calculations. Comparing response predictions to dynamic experimental results also helped bridge the theories presented in classes with the realities in the workplace. The students learned that a model is only as good as the accuracy of the assumptions. A good model requires a depth of thinking in the input phase as well as a critical review of the results. Lastly, this experimentation supports a main educational goal: critical thinking.

References

1. Campbell, M., *Oh, Now I Get It!*, Frontiers in Education Conference, Tempe, AZ, 1998
2. McDaniel, C. C., Archer, G. C., *Full-scale Mechanical Vibrations Laboratory*, American Society for Engineering Education Conference, Atlanta, 2013
3. McDaniel, C., Archer, G., *Classroom-Based Forced-Vibration Testing*, 15th World Conference on Earthquake Engineering (15th WCEE), Lisbon, Portugal, 2012.
4. *CSI Analysis Reference Manual*. (2013). Rev. 8. Berkeley, California: Computers & Structures Inc.
5. Chopra, A.K., *Dynamics of Structures*, Pearson Prentice-Hall, 2011, ISBN-13: 978-0132858038
6. Raney, J.M., Laursen, P., McDaniel, C., Archer, G., *Influence of Boundary Conditions on Building Behavior*, ASEE Annual Conference and Exposition, Seattle, Washington, June 14-17, 2015, ISBN 978-0-692-50180-1