

## **Representations in an Integrated Physics and Mathematics Course Based on Models**

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# **Representations in an integrated Physics and Mathematics course based on models**

## **Abstract**

This study took place in an integrated Physics and Mathematics course for first-year engineering students at a private university in northern Mexico. The integrated course is taught based on Modeling Instruction as well as models and modeling perspective in an active learning environment. This innovative approach, combined with a classroom setting that incorporates the use of technology, promotes the connection and application of the contents from both courses. During the semester, students engage in solving problems (individually and collaboratively), present their models on whiteboards, and have discussions in teams and with the rest of the group. The objective of this study is to analyze how the models and representations shown in students' work evolve throughout the semester. Fifty-four first-year engineering students enrolled in the Physics I and Mathematics I integrated course. When students worked collaboratively in groups of three, they took pictures of their whiteboards, and all pictures were stored in a repository for all (students, instructors and researchers) to review. When students worked individually, instructors assessed their learning based on their written solutions. The analysis of the whiteboards confirmed that at the beginning of the semester students used few representations, whereas by the end of it they were incorporating congruently more than ten different representations, making their models more robust.

## **1. Introduction**

In the past four years, a special course has been taught in a private university in northern Mexico. While this is not the first course that has attempted to integrate two different subjects, even within the same university where this study took place, it seems to have been more successful due to its approach on how to integrate the courses. This initiative seeks to create a common ground applicable to most areas to encourage the creation of hybrid courses to benefit the understanding of both students and teachers, by creating an integrated curriculum instead of isolated islands of knowledge. This specific approach centers on the integration of the Physics I and Mathematics I courses, named Fis-Mat, for its name in Spanish. While efforts have been made to create the following courses and some implementations have been made for a Fis-Mat 3 course, the first one is our center of study as it is the most mature.

Different aspects of the course have been reported in previous research<sup>1,2,3</sup>. In this article, the focus is on the use of models, the representations that are part of them and how they provide both teachers and students a common ground to communicate a structure to follow and become the overall objective of the course.

The course focuses on models and Modelling Instruction<sup>4,5,6,7,8,9</sup>. These provide a framework in which Physics and Mathematics not only can coexist, but need each other, as every model incorporates both the physical concepts that provide context and reality to the model while expressing it in representations, and the use of mathematical reasoning to understand the past and future behaviors. The approach is based on general ideas and concepts that are always true (within a certain context), and building the specifics for each situation. The models are created and

improved through cycles of research, testing and improvement, until they are robust enough to be widely used in different situations that satisfy similar constraints.

Since it is impossible to modify directly the students' mental models of how the world works, collaboration is a key element, as it allows students to constantly question themselves and their peers, encouraging them to compare their mental models with what the others have. Students work in groups of three or four students. They then share and present their findings to the whole class with the aid of whiteboards. To make more efficient use of time and to avoid transcribing the data to their notebooks, all their work is digitally photographed, and uploaded to a common place where they can review every team's work and interact with each other. To promote interaction, the course was implemented in a SCALE-UP classroom<sup>10</sup>, which is used mostly as a place to exchange ideas, have access to equipment to perform measurements when they are needed and facilitate the use of technology<sup>11</sup>.

## **2. Models and representations**

The representations can include some information about a specific situation presented in a visual, mathematical, or conceptual form. While usually each representation is useful and appropriate to every problem, it becomes a complement to provide with new information that a previous one might already have not yet fulfilled for that specific situation. Emphasis is made in that the uncertainty of the situation can always be complemented with more representations, therefore increasing the understanding of the situation, providing a better approach to finding a solution.

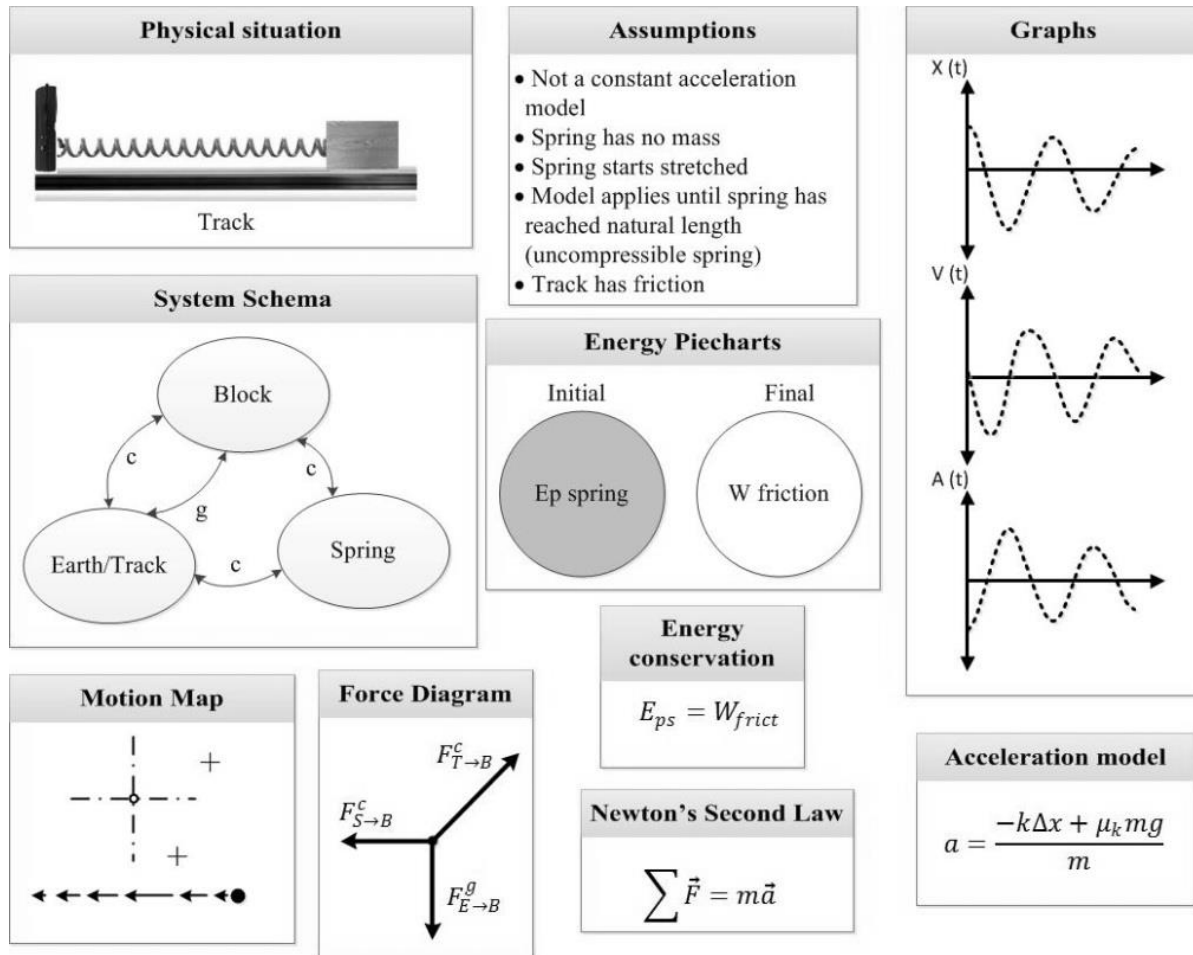
Sequences for model development consist of activities that foster the development of a system of relationships that are generalizable and reusable<sup>12</sup>. A model is normally expressed through a variety of representations; the course focuses on using various representations that create a specific model within the same context and assumptions. The presented proposal argues that representations can be categorized in two different types: descriptive representations, which focus on creating a better understanding of the situation at hand and usually do not include numbers; and the mathematical representations, that have the power of predicting behaviors when applied to a specific situation.

Descriptive representations focus on creating a better understanding of the situation and can become unnecessary once mastery is achieved. However, as novices start their journey, they are very important for students' comprehension. When students are stuck in a problem, typically the first advice to help them advance is asking for a missing representation; this is often help enough for students to finish the path by themselves. Some of the descriptive representations used in the Fis-Mat 1 course are: assumptions, drawing of the physical situation, motion map, system schema, energy pie charts/bar charts and force diagrams. It is possible to attach data or numbers to these representations, but they are not an essential part of them.

Mathematical representations, often preferred by students as they are seen as the key to solving the problem at hand, can be a great tool to better build a situation, connect an idea (like the acceleration being constant), and create a variety of options to solve for anything missing<sup>13</sup>. Some

of the representations used during the semester are: graphs, equations, Newton's second law, energy conservation law and momentum conservation law.

Once all representations are together, it is important to make them consistent. The representations used while building the model relate with the ease of getting the answers right (Dominguez et al. 2015). An example of a model with most of its corresponding representations is shown in Figure 1, which describes (1) physical situation, (2) system schema, (3) motion map, (4) force diagram, (5) assumptions, (6) energy pie charts, (7) energy conservation, (8) newton's second law, (9) graphs, and (10) acceleration model. This particular representation analyzes how a block would move if attached to a spring and was released.



**Figure 1.** Complete model for calculating the block acceleration (c.f., Dominguez et al. 2015, p. 517).

Since the objective was to recreate the actual process that a scientist goes through when creating knowledge, the representations and their relationships are not given to the students; they are asked to build them through three different types of activities: investigation, conceptual, and modeling. Focusing on real life situation and measurements, the investigation activities are used as the foundation of how the representations are built and related in each specific model. Conceptual activities focus on a better understanding of both the concepts and the representations. In modeling activities, models are used to predict and explain future or past behaviors in a specific situation.

### 3. Students and course

Fifty-four students enrolled in a first-semester physics course and first-semester calculus course for engineering majors. They worked in groups of three or four students during the semester. One goal of this course was for students to perceive mathematics as a useful tool to solve problems, and to apply them in physical contexts. Therefore, both instructors (one from the physics department and one from the mathematics department) are present in every class, and the material is designed to foster the integration of both disciplines.

Most of the group collaboration activities are performed using portable whiteboards. This fosters interaction and participation within the small groups, and at the end allows a discussion with the whole class, based on the information written in the whiteboards. Pictures of the whiteboards are shared and organized in a common virtual space. The amount of whiteboards uploaded by activities varies from class to class, depending on the number of attending students. The whiteboard pictures serve as notes for students to go over the class content, material for formative assessment for the instructors, and material for research. Depending on their performance (participation, efficiency, and understanding) some collaborative groups changed every four weeks. Thus, the analysis conducted for this study is about the entire class, rather than a case study of specific groups.

The results are presented comparing how each type of activity performed in the classroom evolved during the semester. Having more representations is not always better; sometimes experts choose only those more efficient or adequate for each problem, striving for mastery and presentation of the representations used. The study centers mostly on the whiteboards since they are used during class time, within a time limit, and with no opportunity for corrections once done.

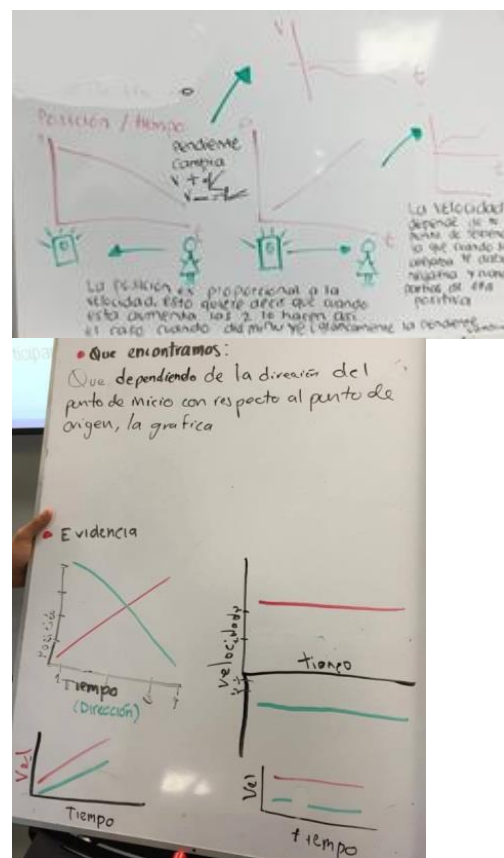
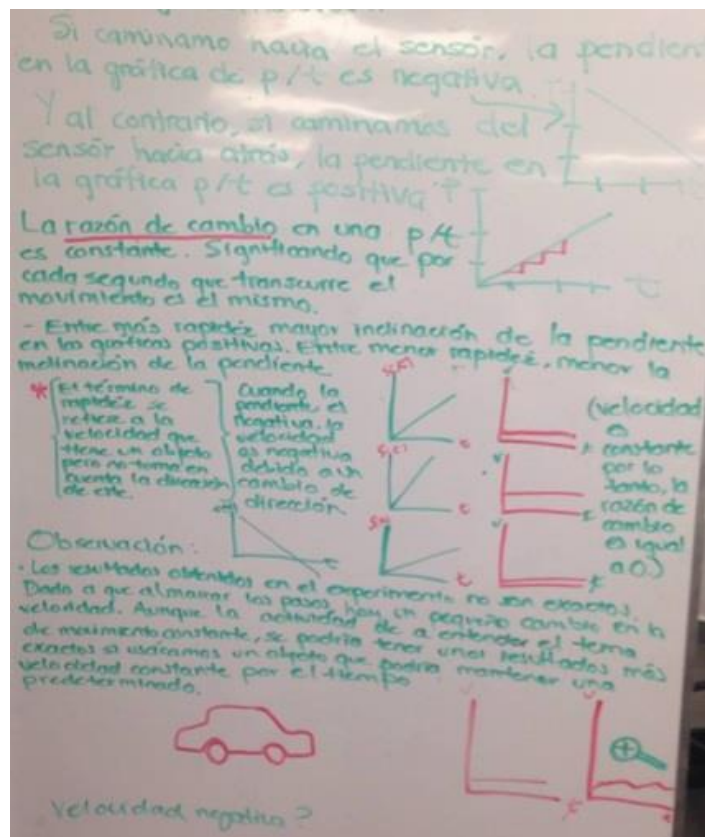
## 4 Results

### 4.1 Investigation activities

Whiteboards from four investigations were selected from different moments during the semester. Those investigations were centered on giving students the opportunity to observe a specific phenomenon to create later a model that incorporated the observed behavior. Only those conclusions supported by data are considered real in the classroom, as anything known previously has not been strictly tested and proven.

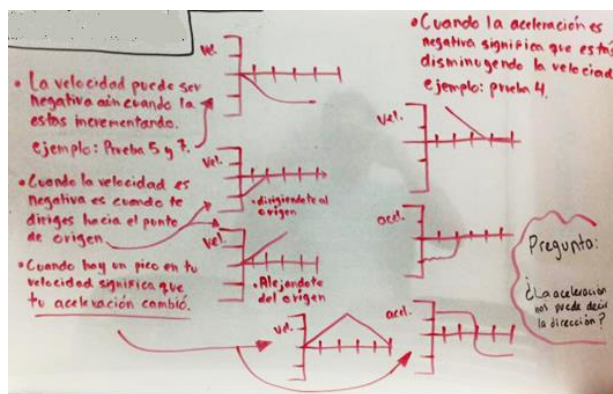
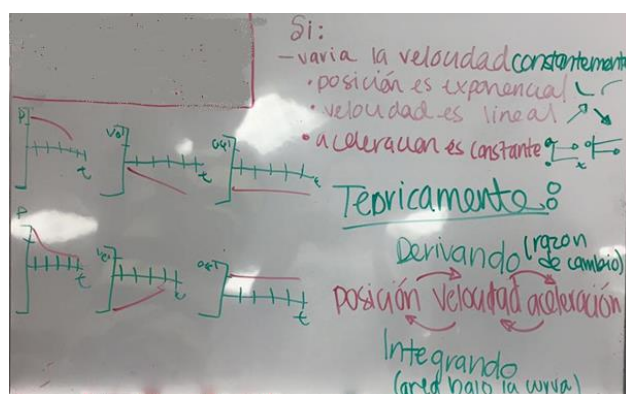
The first Investigation is done the first day of class and consists of walking from and towards a motion sensor at a constant velocity, and analyzing how the Position and Velocity graphs behave, building the relationships between both representations to be used later.

Two main conclusions can be derived from the data on these whiteboards. The first is that 60% of the teams are just copying all the information, and their analysis is mostly crude. The use of representations focuses mainly on graphs and verbal description of the situations, and only 33% of the teams add a drawing. 27% of the teams have some issues with labeling or the consistency of their graphs. Evidence of this can be found at the bottom of the third image in figure 2.



**Figure 2.** Whiteboards from the first investigation.

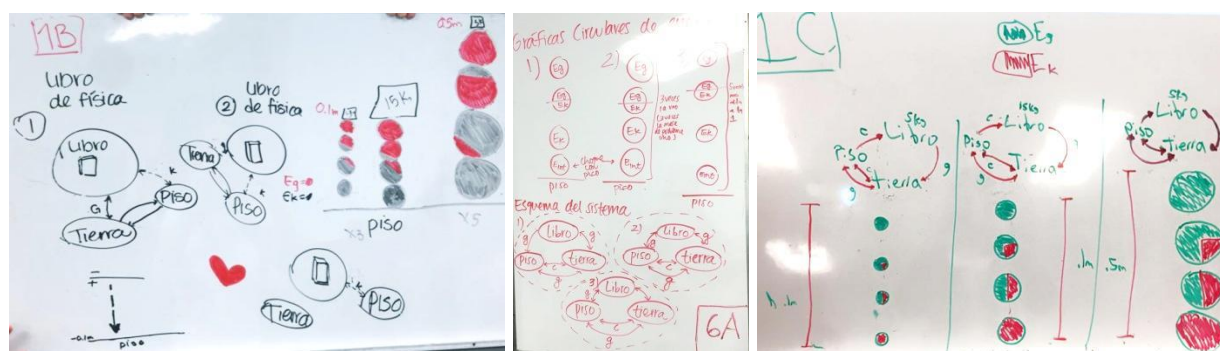
After one more week of practice using graphs and focusing on presenting what they found rather than copying everything done, the whiteboards started to seem better organized. The majority of groups (87%) focused on analyzing. The first picture shown in Figure 3 focused purely on relationships students found, while the one on the right had arrows relating each proposed idea with the graph that held the evidence supporting it. Graphs are more precise, with more detail than before; however, almost no one had used drawings or other representations yet, mainly because they hadn't been formally introduced to them.





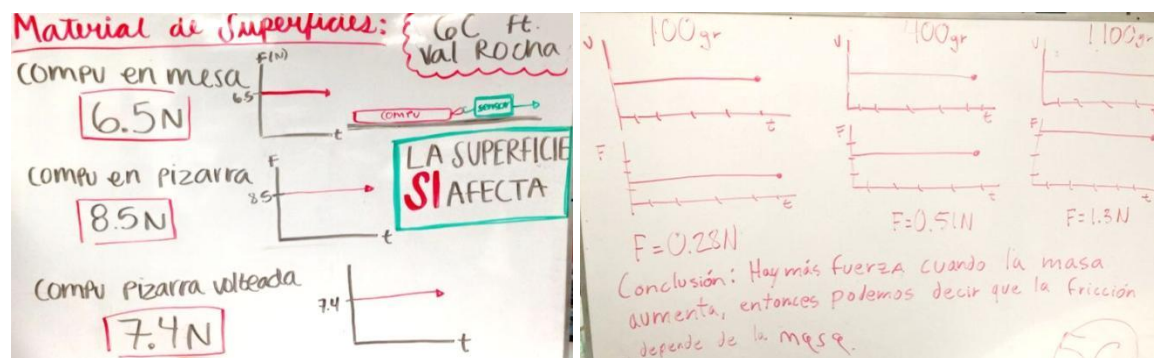
**Figure 3.** Whiteboards from investigating constant acceleration.

By week 8, at least 44% of the teams were using color coding to better express their findings, making them easier to explain and discuss in the circle discussion part of the session. Better scaling could also be observed on some of the whiteboards, while it was still absent on others. As Figure 4 shows, on the representation on the left it is notable that the middle book is bigger and that the third one is dropped from a higher altitude. Other groups, as shown in the middle, had bigger representations making them easier to read from a distance (and from the photograph). It is notable that students did not use graphs at this point. They were analyzing a new movement that at that moment they were unsure of how it related to the one previously seen. It is until the models were used to solve specific situations that students built a bigger model with previous representations.



**Figure 4.** Whiteboards from investigating conservation of energy.

During week 12, each team was tasked with finding if one of four different possible variables affected friction or not. Whiteboards from this session focused greatly on using clear evidence to convince others of their findings. Most students did not have a force diagram on their boards since they had done it for the previous part of the investigation. The use of graphs returned, as 90% of the teams presented them. This could be due to the way force sensors show data (as graphs). Figure 5 shows how students showed what was done in each experiment and capitalized the important data.



## 4.2 Modeling activities

**1B**

$2\pi(2.309)^2 + 2\pi(2.309)h = 32\pi$   
 $10.464162 + 4.618h = 32$   
 $4.618h = 21.535838$   
 $h = 4.6204\text{cm}$

$A = \pi r^2$   
 $V = h\pi r^2$   
 $V = \pi r^2 h$

$V = (4.6204)(2.309)^2 \pi$   
 $V = 77.728\pi\text{cm}^3$   
 $V = 77.728\pi\text{cm}^3$

$16\pi - 3\pi r^2 = 0$   
 $16\pi = 3\pi r^2$   
 $16 = 3r^2$   
 $r^2 = \frac{16}{3}$   
 $r = \sqrt{\frac{16}{3}}$   
 $r = \frac{4}{\sqrt{3}}$   
 $2.3 = r$

$C = \pi d$   
 $C = 2\pi r$

$\text{Area} = 2\pi r^2 + 2\pi rh = 32\pi\text{cm}^2$   
 $V = h\pi r^2$   
 $V = (16 - r^2)\pi r^2$   
 $V'(r) = (16r - 2r^3) + (-2r^2)(\pi r^2)$   
 $V'(r) = 32\pi - 2\pi r^2$   
 $V'(r) = 16\pi - 3\pi r^2$

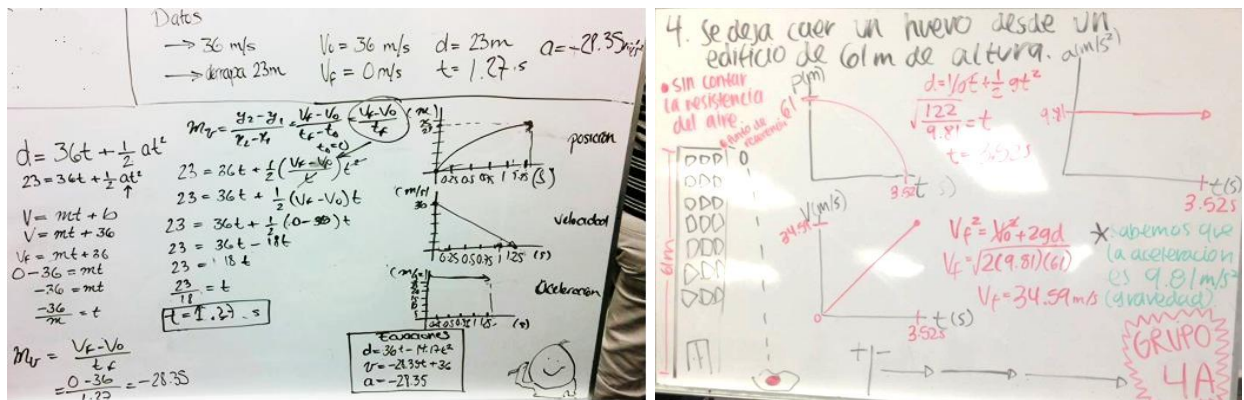
$16 - r^2 = h$   
 $\frac{16 - r^2}{r} = h$

$V = \pi r^2(16 - r^2)$   
 $V = \pi r^2(16 - r^2)$   
 $V = \pi r^2(16 - r^2)$

$V = 77.728\pi\text{cm}^3$

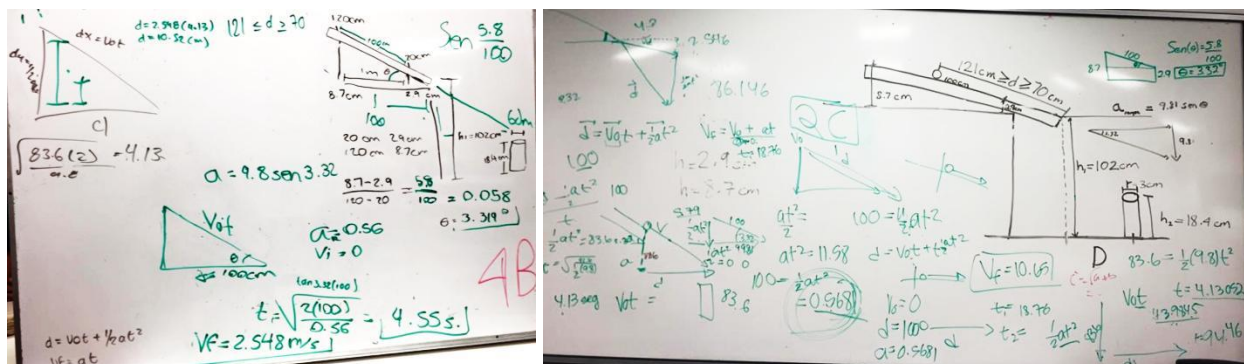
These whiteboards were done right after the constant acceleration investigation, and students finally used all the representations they had been building to solve problems. Half of the groups chose to write the given data like the one on the right of Figure 7, while the other half chose to express such data in a drawing, as it can be seen on the whiteboard to the right. Some groups chose to use the mathematical representations and then copy their results onto the graphs (59%), while the rest chose to use the relationship between graphs to find the solution (area under the curve and slope).





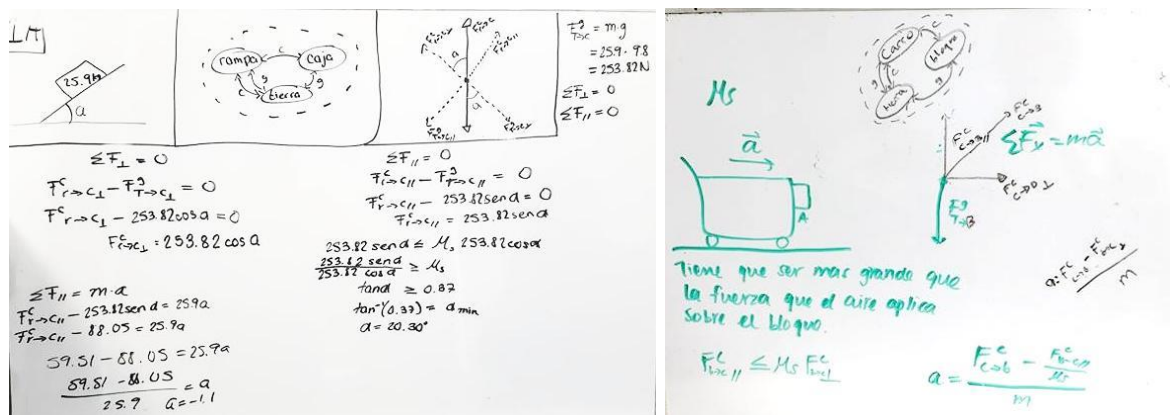
**Figure 7.** Whiteboard of constant acceleration problems.

This situation is a hybrid of investigation and modeling activities. They had a ramp with an actual ball and cup to aim at. They had to use the 2d constant acceleration model that had not been proven to work in real life to predict where to put the cup. After 40 minutes of work they produced their answers, of which examples can be seen in Figure 8, and which were later tested in real life. By not using graphs they fell into simple mistakes, like using units inconsistently. Students' focus was mainly on trigonometry and constant acceleration equations.



**Figure 8.** Whiteboards of a real situation to prove 2d constant acceleration model works.

Close to halfway through the semester students chose only the necessary representations. Figure 9 shows that the whiteboards have drawings (93%), system schema (53%), force diagrams (100%) and Newton's second law (100%). If the problems were to ask questions like how much time would it take to advance certain distance, that would promote the use of graphs in the model.



**Figure 9.** Whiteboards for solving force problems.

## Conclusions

Part of the success of the course was that it was based on the use of models in a concurrent point where both Physics and Mathematics could coexist. By focusing on constructing, improving and using models, the strengths of each science became evident while their relationship emerged naturally.

At the beginning, this approach generated some discomfort in students since they were used to solve problems looking for a specific variable, or doing a similar problem to one previously done by the teacher, therefore, constructing and using a generic model took time to understand. Over time, students realized that the center of each problem was not the answer but the model itself, and that all problems with the same conditions have the same model.

It has not yet been possible to ensure that every whiteboard from every group will be available on the platform and the next steps in these investigations will focus on adding more data sources, like homework and exams.

Finally, both the Physics and Mathematics professors have learned more about their peer's course, often questioning their approach, and creating a better interaction between the concepts of both courses.

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