

An Intervention in Engineering Mathematics: Flipping the Differential Equations Classroom

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Abstract

A flipped classroom design was implemented in the Differential Equations (DE) for Engineering course at the University of Louisville J.B. Speed School of Engineering. Student performance and completion rates were compared to a previous control semester of the course, and student satisfaction was measured. Overall, there were significantly fewer W's (Withdrawals) in the course, meaning a significant improvement in course completion. Additionally, results showed significantly higher performance in non-First Time students, or students that were repeating the course. As DE is a required course for graduation, increasing student performance and decreasing W's will improve graduation rates and is of great value to the university. Student satisfaction with the course format was slightly below average, however as this was the first implementation of the flipped design, it is expected that future semesters with slight modifications will be able to improve on this baseline.

Introduction

Differential Equations (DE) for Engineering is the final course in the required, 4-semester math sequence at the University of Louisville (UofL) Speed School of Engineering. While each course presents new and challenging material, many students struggle most in this final DE course. Some students decide to withdraw from the course mid-semester while others receive a failing grade. Approximately 18 percent of students have to repeat the course, some multiple times, in order to pass. Taking a course multiple times is costly for students; they both pay for additional credit hours as well as delay completion of their required courses, possibly resulting in a delayed graduation date.

This paper assesses an intervention designed to improve math learning and completion of the DE course: a flipped classroom. Flipped classrooms require that students watch lecture material outside of class, and actively work on problems during class time. This method combines active, problem-based learning activities with direct instruction methods, and is seen by many as a

teaching method that results in higher student satisfaction, greater retention of knowledge, and increased depth of knowledge [1].

A review of flipped classroom research was performed by Bishop & Verleger in 2014 [2]. The authors assessed 24 studies, comparing study type, sample size, measurement instruments, theoretical framework, in-class activities, and out-of-class activities. Most of the reviewed studies used subjective opinion surveys for outcome measures. Some conclusions as referenced by the paper were as follows: Students preferred live in-person lectures to video lectures, but also liked interactive class time more than in-person lectures [3]; and shorter, rather than longer videos were preferred [4]. Bishop & Verleger encouraged future research to include performance measures and controlled study designs.

Some more recent studies have shown increased student performance measures when applying this technique to engineering courses (linear algebra [5]; computer science [6]; control systems [7]; chemical and thermal processes [8]). While the 2014 review was inconclusive about the performance benefits of a flipped classroom, these recent studies indicate that flipped courses provide students with better understanding of the material and therefore have better performance in the class.

Other recent studies comparing typical lecture style and flipped classrooms have looked at student performance and learning measures but have not shown statistical differences. Bishop performed his own controlled study in sophomore-level numerical methods for engineers and did not find performance differences but instead reported high student satisfaction [9]. One study reported no significant difference in learning measures for a flipped differential equations classroom [8], however, authors noted that detecting differences in student performance may be difficult due to the high-achieving sample population that was not representative of average undergraduate students.

This paper investigates student performance, satisfaction and course completion in the flipped semester as compared to a control group of students from a previous semester which was taught in the traditional lecture-style. It was hypothesized that student performance would increase due to the flipped classroom design and more students would pass course. It was also hypothesized that increased student satisfaction with the course format would improve completion of the particular course (decreasing the number of withdrawals), thereby decreasing the number of students who need to retake the course. It should be noted that this study was performed on the first implementation of the flipped classroom design, which is significantly different from a traditional design and requires different teaching strategies, so results are preliminary.

Methodology

A flipped classroom design was implemented in the standard DE course required by all engineering students at the University of Louisville J.B. Speed School of Engineering. Student performance in this course was compared to student performance from a previous traditional lecture style semester of the same course that was similar in time of year, performance measures, and instructors. To control for individual differences in the student samples, potential covariates of performance were investigated and significant predictors of performance were included in the

final analysis. A second outcome measure was the percentage of students who received a D, F or W (withdrawal) and additionally, a student survey about the flipped course structure was reviewed.

Participants

In the flipped design semester, 308 students were registered for DE. Fourteen students were in an online section of the course and were excluded from the analyses. An additional 14 students did not have an existing research ID from the university and were also excluded. The remaining 280 students were included in the analysis.

In the traditional lecture style semester, 280 students were registered for DE, 10 of which did not have a research ID from the university. The remaining 270 students were included in the analysis.

The voluntary survey in the flipped design semester had a response rate of 76% (213 responses), 20 of which were incomplete. The remaining 193 responses were included in the analysis.

Materials and Intervention Design

The traditional and flipped DE courses both included instruction in first and higher order DE, systems of DE, partial DE, difference equations, numerical methods, Laplace transforms, and engineering applications. The textbook was Nagle, Saff, & Snider, *Fundamentals of Differential Equations* (8th Ed., 2012). Homework assignments were given weekly and exams were given throughout the semester. In both years, supplemental instruction was available from a campus tutoring organization. Two, hour and a half sessions were given each week. Overall attendance was higher in the flipped classroom semester, however most students attended fewer than 5 times, so it is not considered in this paper as an integral part of the flipped classroom.

In the traditional course design, lectures were given twice a week. Seven exams were given throughout the semester and a final exam was given at the end of the semester. The traditional course semester used in this paper was selected to match the instructors of the flipped classroom semester, thereby controlling for instruction style as well as homework, examinations, and final grading decisions.

In the flipped course design, video lectures were made available to students at the beginning of the year and a video schedule was given. The videos were recorded by the two main professors of the course. In the videos, content and notes would stream across the screen, and professors would stand in “blocked out” locations on either side slightly in front of the display screen. The videos were in the format of a typical lecture, but were able to move faster because the equations were already written down. This reduced the overall video lecture time. Students were provided with a set of notes (using DyKnow classroom management software [10]) associated with each video. Prepared slides matched much of the theoretical development, but students were not given the solutions of all the example problems. Some students simply listened and watched the problems being solved, but those who wanted to write down the example problems would need to pause the video.

Videos were assigned to be watched prior to each course meeting day except for review or exam days. An in-class quiz was given covering video material on all days in which videos were assigned. Lecture time was used to review problems and give students active practice with solving new problems, and a quiz was given at the end of class as well to monitor student participation and measure learning. Five exams were given throughout the semester, and a final at the end of the semester.

DyKnow creates a shared white space between students and instructors, supports digital inking, and during a “session,” students and instructors share a common notebook which students can save. In class, the instructors started a DyKnow session to present exercises, collect student solution attempts and questions, and give live feedback. A typical interactive DyKnow session begins with an instructor presenting a new question to the class displayed on a projector screen, which is also seen by students on their individual computers. Students then attempt to solve the problem in small groups, and an instructor walks around the room answering initial questions. Students submit their answers or questions to the instructor through the software on their individual computers, and an instructor is reviewing the incoming solutions, looking for trending mistakes or errors. If a common error is detected, the instructor displays to the class one representative submission as an example and discusses the error with the class. Correct solutions were also displayed and positive feedback provided. These sessions allowed the instructors to re-inforce thorough and carefully crafted solutions and also identify common errors or misconceptions. Students were always encouraged to work on problems collectively in groups, and to interact with the instructors by asking questions and submitting answers.

In the second to last week of the semester, a voluntary and anonymous survey was given to the students about the course format. The survey was adapted from those used in the flipped classroom literature, specifically [5]. Survey items included preferences about each course aspect including the video lectures, collaboration, DyKnow, review sessions, and the overall course format. Question types included 5-point Likert scale questions and open-ended questions. The survey was intended to both assess student satisfaction with the course format and gather information to improve future iterations of the flipped classroom.

Analysis Procedures

The primary outcome measures were DE Grade and the number of D, F and W (DFW) grades. For DE Grade, an analysis of covariance (ANCOVA) was used to compare the flipped classroom design to the traditional lecture style course (Classroom Design), using covariates that significantly predict DE Grade. For DFW, binary logistic regression was used.

First, DE Grade was converted from letter grades to the following numeric values:

A+	4.3	B+	3.3	C+	2.3	D+	1.3	F	0.0
A	4.0	B	3.0	C	2.0	D	1.0	W	-1.0
A-	3.7	B-	2.7	C-	1.7	D-	0.7		

Analyses were performed both with and without numerical W's included in the analysis.

To identify significant covariates for DE Grade, potential independent variables were investigated using the traditional lecture data set. The potential variables included:

- ACT Math,
- Prior GPA (in previous math courses),
- Number of Repeats (how many times students repeated previous math classes),
- First Time (whether or not this was the first time students had taken DE), and
- Gender.

The backward-elimination method was used to generate the regression model; with DE grade as the dependent variable, all independent variables were added to the model, and step by step, parameters were removed if they were not significant. Additionally, a logistic regression model with the independent variable DFW was also constructed with the backward elimination method. The significant variables that remained were used as covariates in the ANCOVA.

The survey responses were reviewed by question type: (1) open ended questions were read for similarities, and frequent comments were highlighted, (2) the mean and distribution of Likert-scale responses were reviewed.

Results

This study assessed the difference between flipped classroom design and the traditional lecture style classroom in terms of student performance and student satisfaction in the DE classroom.

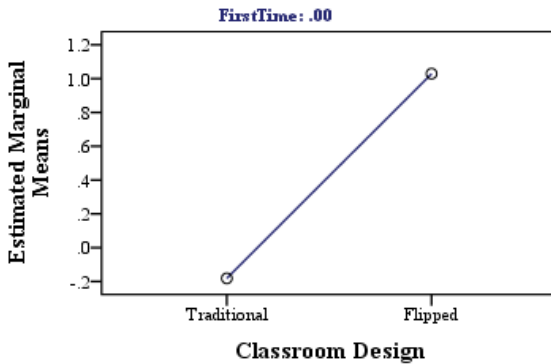
First, potential covariates of DE Grade were investigated. The two variables that were found to significantly predict DE Grade were Prior GPA (an average of the previous math grades from the engineering school) and First Time (whether or not the student was taking the course for the first time), $R^2 = .504$, $F(1, 254) = 128.251$, $p < .001$. These two factors also significantly predicted the number of DFWs received using logistic regression ($X^2 = 86.705$, $p < .001$, $df = 2$), Prior GPA ($p < .001$), and First Time ($p = .036$). Since there were many more students taking the course for the first time ($N = 488$ versus $N = 53$), these two samples were split and analyzed separately. The following analyses were conducted with Prior GPA included as a covariate.

Student Performance

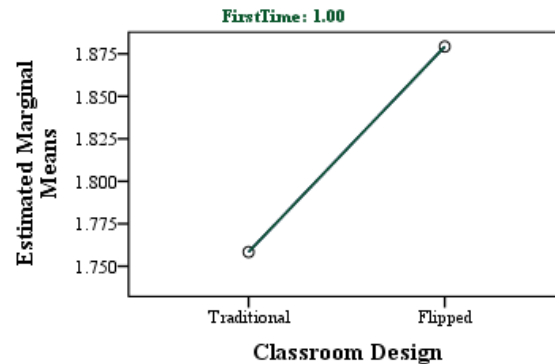
The flipped classroom had a stronger impact on students who were repeating the class than on those in the course for the first time. Means are shown in the charts below.

ANCOVA results showed a significant main effect of Classroom Design on non-First Time students, $F(1, 50) = 14.305$, $p < .001$, $\eta^2 = .222$, and a non-significant effect on First Time students, $p = 0.243$.

Estimated Marginal Means of DEGrade_Number

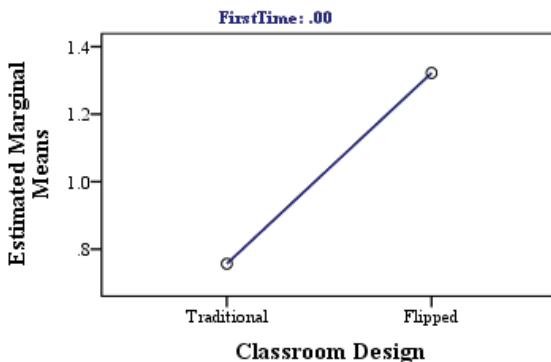


Estimated Marginal Means of DEGrade_Number

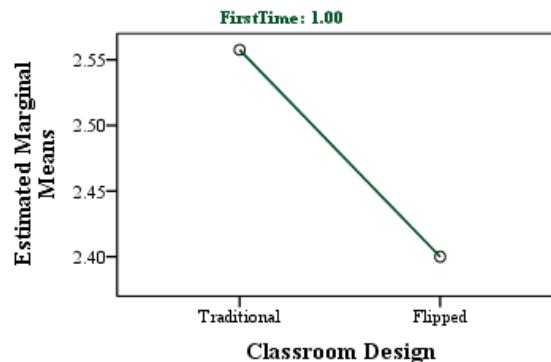


The same analysis was performed with the W (Withdraw) grades removed from the sample. Again, there was a significant interaction between First Time and Classroom Design, $F(1, 428) = 15.622$, $p < .001$, $n^2 = .035$. Means are shown in the chart below.

Estimated Marginal Means of DEGrade_Number



Estimated Marginal Means of DEGrade_Number



In this case, there was only a marginal simple main effect of Classroom Design for non-First Time students, $p = .103$, most likely due to the small and unequal sample sizes ($N_1 = 11$ and $N_2 = 26$). There was a significant simple main effect of Classroom Design for First Time students, $F(1, 393) = 7.706$, $p = .006$, $n^2 = .019$, with students in the flipped classroom performing significantly lower than the students in the traditional classroom.

The difference between these two sets of ANCOVA results indicate that there are W grades that significantly impact the means of both the first-time and non-first-time students, and these W grades lower the traditional lecture group grade means more than the flipped classroom group.

This indicates that there are more W's in the traditional lecture semester. The table below reports the number of W's in both classes which validates the previous results.

CLASSROOM DESIGN	W's	Not W's	Total
Traditional	71	199	270
Flipped	42	238	280
Total	113	437	550

A follow-up logistic regression analysis of number of W's with the independent variable of Classroom Design and covariates of Prior GPA and First Time had significant results, *Nagelkerke* $R^2 = .244$, $X^2 = 90.568$, $p < .001$. The Wald criterion demonstrated that Classroom Design made a significant contribution ($p < .001$) as well as Prior GPA ($p < .001$). First Time was not a significant predictor. The Exp(B) value of Classroom Design indicates that when students are in the flipped classroom, the odds ratio is 2.55 and therefore students are 2.55 times less likely to withdraw from the course.

In summary, Classroom Design improves non-first-time student performance, reduces the number of W's, however lowers the average performance of first-time students due to more people not withdrawing.

The number of DFW's was also analyzed with logistic regression with the same independent variables. The percentage of DFW's was lower in the flipped classroom ($75/280 = 26.8\%$) than the traditional classroom ($84/270 = 31.1\%$). The model was significant, *Nagelkerke* $R^2 = .408$, $X^2 = 180.92$, $p < .001$. The Wald criterion showed marginal significance of Classroom Design, ($p = .053$), a significant effect of Prior GPA ($p < .001$) and no significance of First Time. The Exp(B) value of Classroom Design indicates that when students are in the flipped classroom, the odds ratio is 1.57 and therefore students are 1.57 times less likely to get a DFW in the course.

Student Satisfaction

The survey results showed mixed opinions about the course format and other aspects of the course. Likert-scale responses had a range of 1-5 where 3 indicated a Neutral response.

Students generally enjoyed the interactive part of the class meeting times using DyKnow software. Survey items and results are reported in the following table.

Questions:	I generally participated in the in-class activities.	Real-time feedback helped me learn the material.	Overall I have a positive opinion of DyKnow.
Average:	3.99	4.07	3.99
1 - Strongly Disagree	7	4	7
2	14	9	14
3	22	26	22
4	76	82	76
5 - Strongly Agree	70	69	70
6 (N/A)	4	3	4

Most students reported participating and having a positive opinion of DyKnow. Survey results showed fewer positive responses to the video lectures. Survey items and results are reported in the following table.

Questions:	I generally watched all of the videos before class.	The video lectures helped me learn the material.	Reviewing the video lectures helped me study for exams.	Overall I have a positive opinion of the video lectures.
Average:	4.23	3.22	2.82	2.76
1 - Strongly Disagree	9	27	35	48
2	12	38	48	44
3	5	31	38	35
4	66	57	39	38
5 - Strongly Agree	101	39	23	28
6 (N/A)	0	1	10	0

While students reported that they did watch the videos and that the videos helped them learn the material, the average opinion of the videos was 2.76, slightly below neutral. The open-ended question about the videos was reviewed to determine why the opinion was not positive when the self-reported learning gain was positive. Some recurring complaints included: (1) speed of the videos scrolling through the example problems (too fast), (2) professors being in the way of the notes, and (3) not being able to ask questions.

Additionally, students had a slightly negative overall opinion about the flipped classroom design. Survey items and results are reported in the following table.

	Overall I have a positive opinion of the format of the course (video lectures and in-class activities).	I prefer the format of this course over a traditional lecture format.	Compared to other courses I have taken, I learned more in this course.	Compared to other courses I have taken, I enjoyed this course more.
Average:	2.74	2.51	2.56	2.63
1 - Strongly Disagree	47	66	59	58
2	51	44	37	36
3	29	28	45	40
4	37	28	32	37
5 - Strongly Agree	29	27	19	22
6 (N/A)	0	0	1	0

The table above demonstrates that the overall opinion of the course format was on the negative side, however the results are distributed and there were many students who liked the format. A common point in the open-response portion of the survey was that the course took a lot of time for a 2-credit-hour course.

Discussion

Most importantly, it must be noted that this was the first flipped course implemented by these professors. This study therefore does not directly assess the benefits of a flipped classroom in general, but rather this initial implementation. For example, survey results highlighted some issues

with the course format that could be improved upon. Improving the course structure will likely increase the effectiveness of the flipped course design.

Another shortcoming of this experimental design was a lack of control over the student samples in the two conditions. Covariates helped to control for differences in samples, however it was not a random-assignment design and should not be considered fully controlled. Holding the experiment over multiple years also adds instructor variability; instructors could have spoken differently, given greater encouragement, or given an easier first exam in one year over another. The exams were reviewed and determined to be equivalent in difficulty, however, they were not exactly the same. With those caveats in mind, we discuss the results.

The two covariates found to predict final DE Grade are interesting in themselves. Prior GPA (in previous math courses) was a significant variable in predicting final DE grade, which shows that that performance in DE is similar to performance in previous courses. The number of previous repeats for a student (how many times they repeated previous courses) was not significantly predictive of DE Grade. However, the First Time binary variable (whether or not it was their first time in DE) *was* significant. In other words, those who failed or dropped out of DE previously were more likely to do it again, regardless of whether they have repeated any of the earlier courses in the sequence. These results indicate that the cause of the high repetition rate in DE is not due to a student trait of “Tendency to Repeat” but rather something about the DE course itself that causes students to get stuck. Other differences between DE and previous math courses include the number of credit hours (2 instead of 4), and the timeline of when students end up taking the course. Due to a Co-Op semester in the Spring of students’ sophomore year, it is possible that students take the course after a break of two semesters. This could hinder students in mathematics memory as well as personal responsibility and study habits. It is possible that the flipped design could help get these “gap” students back into the performance mode required by the class.

This raises the question of what in this flipped classroom design was helping the students: the problem-based learning, the collaborative learning, the video lectures, or the in-class quizzes. Daily quizzes covering the video material could have been increased student attendance in class as well as more explicitly shown need or improvement for each individual student. Quizzes also provided retrieval practice for the students, which is shown to benefit learning and memory [10]. However, quizzes at the beginning of class meant that students were required to learn all of the material from the video lectures, as opposed to the ideal flipped classroom design where the topics are taught first with video lecture and then followed by in-class activities. While the instructors originally felt the first quizzes were basic and doable with only a single viewing of the video, this was not what students experienced. In retrospect, this organization overemphasized the video lectures and stressed students, rather than incentivizing them to watch the videos. Identifying this overemphasis helps to understand student feedback in the survey about the videos.

The three major complaints in the survey were the speed of the videos, the inability to ask questions during the videos, and the amount of time required for the course given that it was 2 credit hours. As mentioned in the Materials and Procedures section of this paper, the professors had decided to move quickly through the notes to reduce the length of the videos, and it was intended that students pause the videos if they needed to copy notes. With a quiz at the beginning of class, however, more

time had to be spent with the material over and above watching through the videos. Overemphasis on the videos resulted in students having to take much more time to take notes and practice prior to class, causing them to feel that the course took too much time. The inability to ask questions during a video lecture is a direct result of the flipped classroom design. It is possible however, in future iterations, to include things like a discussion board. Additionally, instead of quizzing students at the beginning of each class on understanding the concepts from the videos, it is possible instead to make the emphasis of the videos exposure to the concepts and quiz students on whether or not they had watched them. This would allow students to become familiar with concepts before putting them into practice in the classroom, and would relieve any anxiety felt by the students when learning new material through the video lectures.

Additionally, some students complained that professors blocked the notes. Upon more careful investigation and interviews of specific students, it was discovered that the speed of the videos and the slight movements of the professors (pointing to key steps) required students to pause the videos at an appropriate moment when the entire problem was visible. Some students did not like this trade-off, but would have preferred the problem appear slowly, much like taking notes in a live lecture. This is an interesting result of several reasonable decisions during the video creation, and will be kept in mind for future recorded lectures. Additionally, a possible solution to this problem would be to provide outlined notes that would help students who would like to stay engaged by writing things down while maintaining the pace of the videos.

Student feedback indicated a positive opinion of in-class activities as opposed to video lectures, as reported in previous studies [3], but it is not possible in this paper to separate the effects of one or the other. In fact, the planned in-class problem-based activities would not be possible without the pre-recorded video lectures.

Conclusions

Results from this study indicate that this initial implementation of the flipped DE course strongly benefitted students who were repeating the course. Additionally, the flipped classroom design reduced the number of DFWs overall, which had been identified as a significant problem. While survey results were slightly below average, the significant results indicate that the design does benefit student completion of the course, and moves them forward towards graduation. The flipped classroom design therefore benefitted the university as well as the students. The fact that these beneficial results were seen from the first implementation of the fully flipped course show great promise for future iterations. We conclude that the flipped course design should continue to be used in the upcoming semesters.

Future work will investigate more refined implementations of the flipped classroom to determine whether it can be more effective for First-Time students and whether student satisfaction will improve. Likely first steps will include a different introduction to and emphasis on the videos, adding a discussion board, and modifying in-class activities that promote watching the videos without anxiety like moving the quizzes to the end of class.

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