

## **Demonstration and Simulation of Dispersion in Coaxial Cables with Low Pass** Filters - A Teaching Laboratory Experiment

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#### Background

Data suggest that continuous time signals and systems (CTSS), and its underlying concepts, can be difficult for students to grasp. CTSS and Electromagnetics courses at certain universities experience drop/failure rates 2-3 times higher than other required courses [1]. Likewise, the phenomenon of attenuation and dispersion is only briefly discussed in most undergraduate Electromagnetic and CTSS textbooks. This paper proposes a new teaching technique of simulating and modeling the attenuation and dispersion in a communication cable using low pass filters.

One goal in this paper is to provide a hands-on learning experience for students in order to reinforce their understanding of attenuation and dispersion. Another is to reinforce the concept of Fourier analysis in modeling the input and output signals, and gain an appreciation for how a linear system modifies the amplitude and phase of a signal to produce an output signal. In this case, the dispersion and attenuation are undesired effects of the coaxial cable which can distort the output signal at higher frequencies. Lastly, the construction of the filters allows students to see how two different systems with the same transfer function can produce a closely similar output signal from an input signal.

### Theory of Dispersion and Attenuation in Coaxial Cables

The coaxial cable can be modeled as a linear system. The model is widely used, and is essentially a transmission line comprised of an infinite series elementary components. Each elementary component consists of a resistor and inductor in series to represent the resistance and inductance of the conducting material. Each component also has a shunt capacitor and shunt resistor in parallel with each other to represent the capacitance between the inner and outer conductor and the resistance of the intervening dielectric material [2].

Attenuation and dispersion in coaxial cables are more significant in coaxial cables at higher frequencies. In general, the resulting high frequency distortion is predominantly due to the frequency dependence of both the resistance of the conductor and of the conductance of the intervening dielectric material [3].

In our experiment we used an RG-59 coaxial cable, which is similar to the RG-58 cable. The excess delay, phase shift, and attenuation due to dispersion have been studied for a RG-58 cable [4]. These all contribute to the overall signal distortion and arise from a combination of the resistance R, the total series inductance L, and the shunt capacitance C of the cable. The resistance is frequency dependent whereas the capacitance is frequency independent. The total series inductance *L* is the sum of the ideal inductance  $L_0$ , and the inductance of the inner and outer conductors  $L_a$  and  $L_b$  respectively.  $L_0$  is determined by the physical parameters of the inner and outer conductors as well as the intervening material, and is thus independent of frequency.  $L_a$  and  $L_b$  arise from the fact that the inner and outer conductors are not ideal conductors, thus there is some penetration of the fields into their interior. Once the skin depth of both conductors is taken into account the frequency dependence becomes apparent. Further theoretical calculations show, for a cable length of *l* at frequency *f*, the excess time delay due to dispersion can be represented by:

$$\tau_d = 4.78 \, ns \, \left(\frac{l}{100m}\right) \left(\frac{f}{10 \, MHz}\right)^{-\frac{1}{2}} \tag{1}$$

Also of interest to us for measuring and modeling dispersion, and the eventual design of a low pass filter to simulate it, is the total phase difference as a function of frequency. The total phase shift is represented theoretically by:

$$\varphi = 2\pi t_0 \left( f + \frac{1}{2} \frac{L_{s0}}{L_0} f^{1/2} \right) \tag{2}$$

*f* represents frequency and  $t_0$  represents the signal propagation time from one end of the cable to the other.  $L_0$  is the ideal inductance mentioned above and  $L_{s0}$  is a constant determined by the physical dimensions of the inner and outer conductors as well as their conductivity.

Likewise, similar theoretical calculations show that the attenuation, in dB, can be expressed as:

$$A = -20\log_{10}e^{\alpha l} \tag{3}$$

*l* represents the length of the cable.  $\alpha$  represents the attenuation constant which is frequency dependent, and is determined from the physical parameters of the inner conductor, outer conductor, and the intervening material. Figure 1 shows the theoretical attenuation.



Fig. 1. Theoretical attenuation of a 70 meter RG-58 coaxial cable

For the purposes of our experiment, there is sufficient agreement between the data and the theory such that it validates treating the coaxial cable as a linear system. This can be seen when comparing Figure 1 with Figure 5. This theory and data could be presented to the students or there could be some activity during which they derive certain aspects of it and/or conduct the measurements themselves, depending how deeply the instructor wants to delve into it.

### Measurements

In our experiment, we used two different lengths of RG-59 coaxial cable for our measurements. A 1 m cable served as an effective way to observe the output signal with negligible distortion from the input signal. And a 70 m cable was used to observe the distortion of the output signal from its input signal. Using a function generator and an oscilloscope, we were able to qualitatively observe dispersion of a square pulse with an amplitude of 2V and a pulse width of 0.1µs. Figure 2 shows a square pulse output using the 1 m cable. Notice there is very little distortion for the 1 m length of cable. Figure 3 shows the output signal for the 70 m cable. The distortion of the square pulse and the increased rise time can be seen, and is consistent with the expected behavior at high frequencies and increased cable length [5].



Fig. 2. Square pulse output from 1 m cable (without dispersion)



Fig. 3. Square pulse output from 70 m cable (with dispersion)

For a more quantitative validation of frequency dependence of attenuation and dispersion, we then used a gain-phase analyzer to measure the amplitude change and phase shift produced by the 70 m cable as a function of frequency. Figures 4 and 5 show plots of phase shift and gain in dB from the gain-phase analyzer. They show frequency dependency of the attenuation and phase shift contributing to the overall dispersion of the pulse. Our results suggest the frequency response of the cable is very much like a low pass filter. The students can now use the measured amplitude and phase response to mathematically model dispersion in the cable. They can also construct a set of filters experimentally to approximate the effects of dispersion observed in the cable, and thus gain a better understanding of the concept.



Fig. 4. Expanded phase vs. frequency result from the gain phase analyzer



Fig. 5. Gain vs. frequency result from the gain phase analyzer

### Modeling

The dispersion in the cable can be modeled by students using a Fourier Series. At the input, any periodic square pulse signal can be expressed as the weighted sum of an infinite number of sine and cosine functions. At the output, the individual frequency components are modified with a new amplitude and phase term based on the measured frequency response of the cable, and

combined to give a distorted signal. We were able to use MATLAB and model the distorted pulse using measured amplitude and phase. The results are shown in Figures 6 and 7 respectively. Figure 6 shows the MATLAB simulation of a square pulse, with a pulse width of 0.1  $\mu$ s, using a finite number of sine and cosine functions. Figure 7 shows the reconstructed square pulse after dispersion. The dispersion was simulated by changing the amplitude and phase term of each frequency components based on the measured attenuation and phase shift.



Fig. 6. MATLAB simulation of a square pulse (Amplitude=2V, Pulse width= 0.1 µs).



Fig. 7. MATLAB simulation of reconstructed pulse due to dispersion.

#### Low Pass Filter

A low pass filter circuit can be constructed to simulate the behavior of dispersion in the cable. In constructing the filter to match the phase shift, we have used the time delay in the theoretical equation (1) as our reference, and converting from time delay to phase shift. This phase shift only accounts for the dispersion delay. Thus, it is much smaller than the measured phase shift seen in Fig. 4, since the measured phase shift is the total delay including the propagation delay and the dispersion delay. In addition, the standard Operational Amplifier (OpAmp) based filter circuits cannot be used here because of the limited bandwidth that an OpAmp possesses. From the theoretical and measured results, we can see the attenuation and phase shift start to have significant effects after 100 MHz. Therefore, we only used passive elements in the filter circuit.

Figure 8 shows the RC circuit is used to simulate the dispersion in the cable. A Multisim circuit simulation is performed. The source of the circuit is a square pulse with a pulse width of  $0.1 \,\mu$ s (10% duty cycle of a 1MHz square wave). The input and the distorted output pulse is displayed in Figure 9. The frequency response of the circuit is shown in Figure 10. The circuit produces a similar frequency dependent gain and phase delay as the theory predicted. Figures 11 & 12 depict the comparisons between the simulated and theoretically calculated gain and phase shift, respectively.



Fig. 8. RC circuit diagram used to simulate dispersion in the cable.



Fig. 9. Multisim simulation of the input and output pulse from the RC filter circuit.





Fig. 11. A comparison of gain curve between the simulated values and theoretical calculated values



Fig. 12. A comparison of phase delay due to dispersion between the simulated values and theoretical calculated values

The circuit was built and tested experimentally with the same input pulse signal. The measured output signal of the filter is shown in Fig 13. Its shape is very similar to the output of the cable due to dispersion. There is some noise introduced in the measurement due to exposed terminal connectors of the coaxial cable.



Fig. 13. Measurement of the output pulse from the RC circuit.

#### Conclusion

In conclusion, the agreement between the theory and our data validated our modeling of the coaxial cable as a linear system. This would then allow students to model the dispersion in the cable using a Fourier Series. We found that the distortion of the output signal can be effectively modeled by modifying the individual frequency components with a new amplitude and phase term based on the measured frequency response of the cable. The distortion in the cable can be simulated by a low pass filter circuit, in this case an RC circuit. The students can thus understand how two different physical systems with the same transfer function can reproduce a closely similar output signal.

At this point, we were unable to attempt this teaching experiment in a classroom environment. The next step is to effectively incorporate the theory, modeling, simulation, and filter design and construction into a lesson or laboratory experiment. This could be challenging as it would necessitate students have familiarity with electromagnetic theory, Fourier Series, computer simulation, and filter design. One possibility is to have the experiment be instructor led with student input at key points. Regardless of the challenges, we feel it has value as the experiment brings together concepts from different electrical engineering and physics courses. Likewise, coaxial cables are ubiquitous in today's world and this experiment can illuminate student understanding of this important technology and its limitations.

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