AC 2007-2577: TEACHING OF DYNAMIC SYSTEMS WITH INTEGRATED ANALYTICAL AND NUMERICAL TECHNIQUES

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Teaching of Dynamic Systems with Integrated Analytical and Numerical Techniques

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Abstract

Mastering ordinary differential equations is very important and essential to being successful in this course of Dynamics Systems. In order to help the students further explore these new concepts and overcome some of the issues related to these deficiencies in material recall, integrated analytical and numerical techniques are adopted in teaching. One problem can be solved by various different approaches. Analytically, the method of undetermined coefficients and the Laplace transform method are used. Numerically, the transfer function method and the block diagram method in Simulink; LTI models, and symbolic toolbox in Matlab, etc, are used. Numerical approaches, especially with the transfer function method in Simulink, visualize the physics and results behind the seemingly daunting equations. By showing the application of different techniques to the same problem, students are inspired to learn the resulting similarities and differences. The MATLAB graphical user interfaces were developed for second order dynamic systems for both free vibration and forced vibration. The visual interface presents results in a way that students can immediately identify the effects of changing system parameters. Both time response and frequency response are clearly shown in the interface. In the course, a research related project is assigned to identify the dynamic response of a portable telecommunication device. In this project, students are required to use both analytical and numerical approaches to show the insight of the material selection affects the reliability of the portable telecommunication devices.

1. Introduction

A course in system dynamics that deals with mathematical modeling and response analysis of dynamic systems is required in most mechanical and many other engineering curricula. The analysis and design methods in the course cover a wide variety of different systems, such as mechanical, electrical, pneumatic, hydraulic, and thermal systems. Although systems are in various fields, mathematically they all can be simplified and represented by ordinary differential equations. Mastering ordinary differential equations (ODEs) is very important and essential to being successful in this course.

In teaching a Dynamic Systems course, basic concepts of solutions of first and second order differential equations and Laplace transforms are expected to be firmly planted in the students' skill sets. However, the reality is that the students simply do not remember much of former material since these courses were taken a year, or even years earlier. Moreover, new terminology

and concepts, such as transfer function, poles, zeros, stability, and others cause even more confusion for the students [1-3].

In order to help the students further explore these new concepts and overcome some of the issues related to these deficiencies in material recall, integrated analytical and numerical techniques are adopted in teaching. One problem can be solved using various different approaches. Analytically, the method of undetermined coefficients and the Laplace transform method are used. Numerically, one can use the transfer function method and the block diagram method in Simulink, LTI (Linear Time Invariant) functions and symbolic toolbox in Matlab, etc. Numerical approaches, especially with the transfer function method in Simulink, visualize the physics and results incorporated in the seemingly complicated equations. By showing the application of different techniques to the same problem, students are inspired to learn the resulting similarities and differences. These approaches are incorporated in homework and projects in each component of the course for various systems, so that students have chances to practice repeatedly.

The MATLAB graphical user interfaces (GUIs) have also been developed for second order dynamic systems. The visual interfaces present results in a way that students can immediately identify the effects of changing system parameters. Both time response and frequency response are clearly reported in the interfaces.

In the course, a research related project is assigned to investigate the dynamic response of a portable telecommunication device. When a portable electronic product is accidentally dropped on a hard surface, vulnerable elements inside, such as the solder joints, may experience very high accelerations and dynamic stresses. The impacts and shocks, thus, can lead to the failure of the electronic device and the malfunction of the product. Recent research disclosed the close correlation of dynamic performance and the malfunction of the product. In this course project, students are required to establish the mathematical modeling of the product's system, and then use both analytical and numerical approaches to show the effect of the material selection on the reliability of the portable telecommunication device.

In this paper, the various integrated analytical and numerical techniques in solving ODEs are presented first. Two multifunctional GUIs are displayed for the time response and frequency response of both free vibration and forced vibration. An example of a course project is then demonstrated to show the application of system dynamics to a real-world problem that has occurred in the electronic industries. Finally, the summary is given in the last section.

2. Integrated Analytical and Numerical Techniques in Solving ODEs

Figure 1 shows both a mechanical system and an electrical system. For a spring-damper-mass mechanical system in Figure 1(a), mass, damping coefficient, and stiffness are represented by m, b, and k respectively. If the excitation force is $f(t)$, the equation of motion in terms of displacement $x(t)$ is

$$m\ddot{x} + b\dot{x} + kx = f(t) \qquad\qquad (1)$$

For the electrical system in Figure 1(b), L, C, and R represent the inductance, capacitance, and resistance respectively. Mathematical modeling in terms of electrical charge q is as following, when the voltage source $e(t)$ is applied.

$$L\ddot{q} + R\dot{q} + \frac{1}{C}q = e(t) \tag{2}$$

The two physically different systems are analogous, and can be represented by the same type mathematical model, as shown in Equations (1) and (2). They are a 2nd order, linear, time invariant ordinary differential equations. Constants in these equations have different physically meanings, but the equations have essentially the same characteristics.
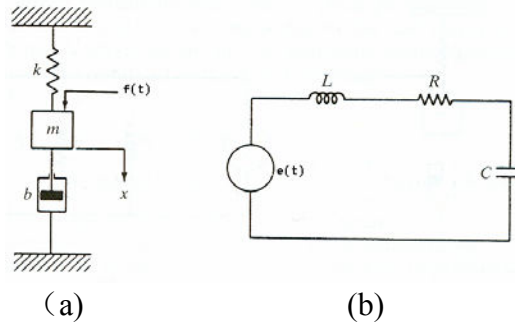

(a)   (b)
Figure 1 – Analogous mechanical and electrical systems

Various different approaches can be used to solve the problem, as summarized in below.
1. Laplace transform method
   a) It is used to solve linear, time invariant ordinary differential equation. Usually, one needs to do the partial fraction expansions and find the inverse Laplace transform to obtain the result [4,5].
2. Matlab method
   a) Find the transfer function and draw the block diagram, then use LTI functions in Matlab, such as 'step', 'lsim', or 'impulse' functions to solve the ODEs numerically for different input functions [4].
   b) Use the Matab Symbolic Toolbox 'dsolve' function to solve the ODEs, and then use 'ezplot' to plot the response.
3. Simulink method
   a) Develop the transfer function to construct the Simulink model.
   b) Use the block diagram based on the equation to construct the Simulink model. In this way, initial conditions can be applied to the integrators directly.

Textbooks in system dynamics usually demonstrate the Laplace Transform method and the use of LTI functions in Matlab [4,5]. In this paper, the focus will be on the Matlab method using Symbolic toolbox and two Simulink approaches.

As an example, for the mechanical system shown in Figure 1(a), assume m = 1 kg, b = 2 N.s/m, k = 5 N/m, with the step force excitation of $f(t)$ = 3 kg and zero initial conditions. The equation of motion and the initial conditions become:

$$\ddot{x} + 2\dot{x} + 5x = 3, \qquad x(0) = 0, \ \dot{x}(0) = 0 \qquad\qquad (3)$$

Using Matlab Symbolic Toolbox, one can find the numerical solution directly from the above equation and initial conditions. Matlab code using Symbolic Toolbox and the response plot are shown in Figure 2.
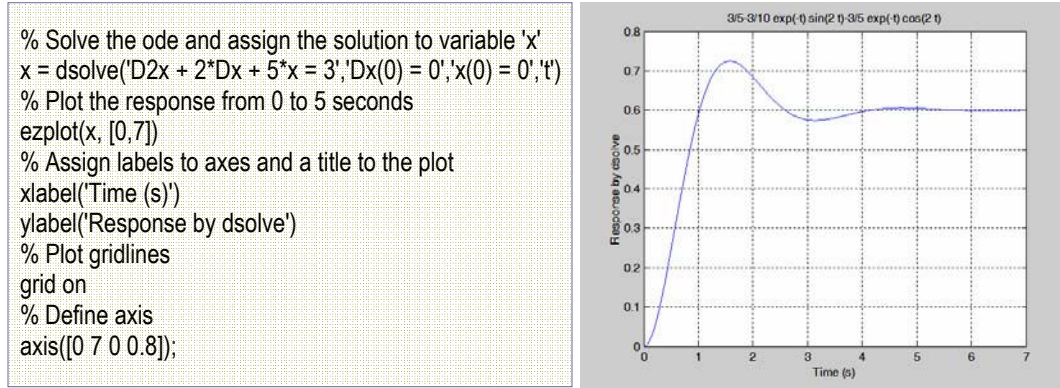
```
% Solve the ode and assign the solution to variable 'x'
x = dsolve('D2x + 2*Dx + 5*x = 3','Dx(0) = 0','x(0) = 0','t')
% Plot the response from 0 to 5 seconds
ezplot(x, [0,7])
% Assign labels to axes and a title to the plot
xlabel('Time (s)')
ylabel('Response by dsolve')
% Plot gridlines
grid on
% Define axis
axis([0 7 0 0.8]);
```

Figure 2 - Matlab code using Symbolic Toolbox and response ploting

Two approaches are introduced to the students to find the solution of an ODE in Simulink. The first one is based on the transfer function. We still use the above example to show the method. In Equation (3), we identify the output of the system as the displacement of x(t), and the input is a step function of f(t) = 3, then, the transfer function of the system is

$$TF = \frac{1}{s^2 + 2s + 5} \qquad\qquad (4)$$

and the block diagram of the system is

input $f(t)$ → $TF = \dfrac{1}{(s^2 + 2s + 5)}$ → output $x(t)$
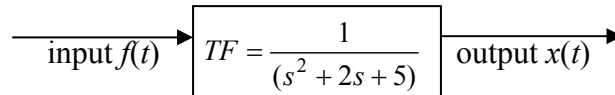
Simulink can directly link the input and transfer function to the system output. The Simulink model constructed from the transfer function is shown in Figure 3(a). The advantage of this method is that it visualizes the physics and results behind the ODE equations.

In the first method in Simulink, one needs to do a mathematical operation to find the transfer function, and then model it in Simulink. Different from the first Simulink method, the second Simulink method constructs the block diagram from equation itself. The Simulink model is shown in Figure 3(b). Initial conditions can be easily applied by double clicking the velocity and displacement integrators. By changing the constants, input, and initial conditions, the model in Figure 3(b) applies to many second order system ODEs.
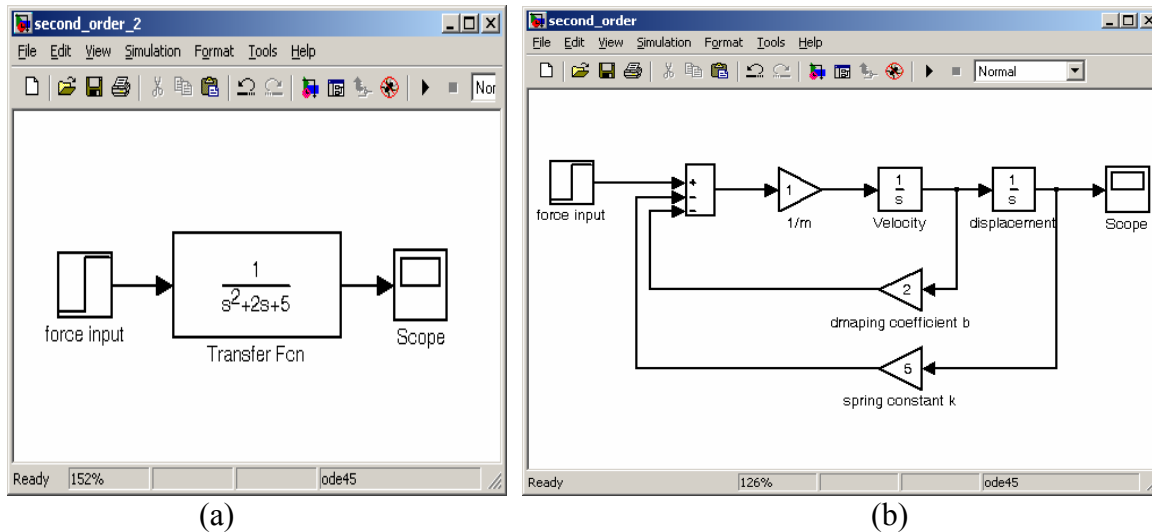
Figure 3 – Two Simulink models: (a) directly from transfer function, (b) from block diagram based on the ODE, initial conditions can be applied directly in the integrators.

By showing the application of different techniques to the same problem, students are inspired to learn the resulting similarities and differences. These approaches are incorporated in homework and projects in each component of the course for various systems, so that students have chances to practice the approaches, and then proficiently mastering the knowledge of solving ODEs.

3. Graphic User Interface

Engineers are primarily visual learners. The literatures has been shown that multimedia content generally enhances student retention and interest [2,3]. A series of Matlab and Simulink-based models with graphical user interfaces have been developed to address various aspects of 1st and 2nd order dynamic systems. The GUIs provide students with an additional tool to understand and visualize mathematically complex concepts covered in typical engineering dynamics systems.

In the paper, two GUIs of single degree of freedom 2nd order systems are displayed. The Matlab GUI for forced vibration is shown in Figure 4. The students are allowed to control the parameters of both system parameters and input forcing functions. System parameters are mass, damping coefficient, and spring constant. Generally used forcing functions, such as sinusoid, step, ramp, and impulse inputs, are available to choose. For any input function chosen, the corresponding parameters can be specified. For example, if the input is sinusoidal function, as shown in Figure 4(a), amplitude, frequency, and phase angle of the input can be input. The GUI reports the time response of the system and the input function as well. The time range of the response plot can be adjusted as desired. Similar to Figure 4(a), Figure 4(b) shows the case of step input with amplitude of 3 units.
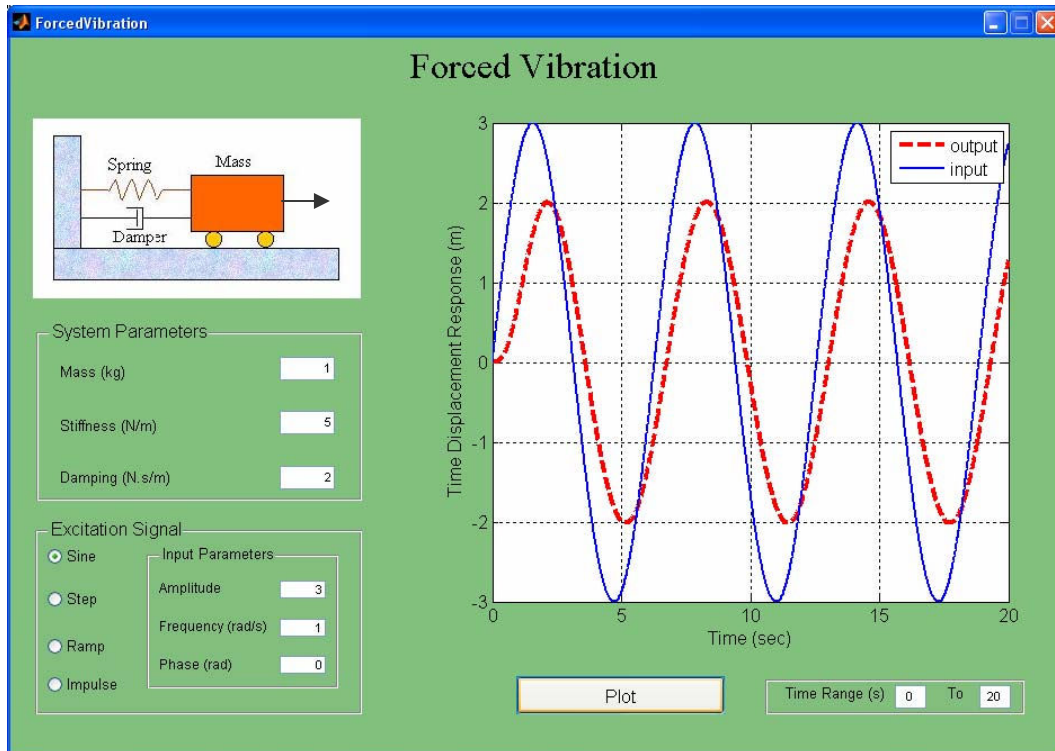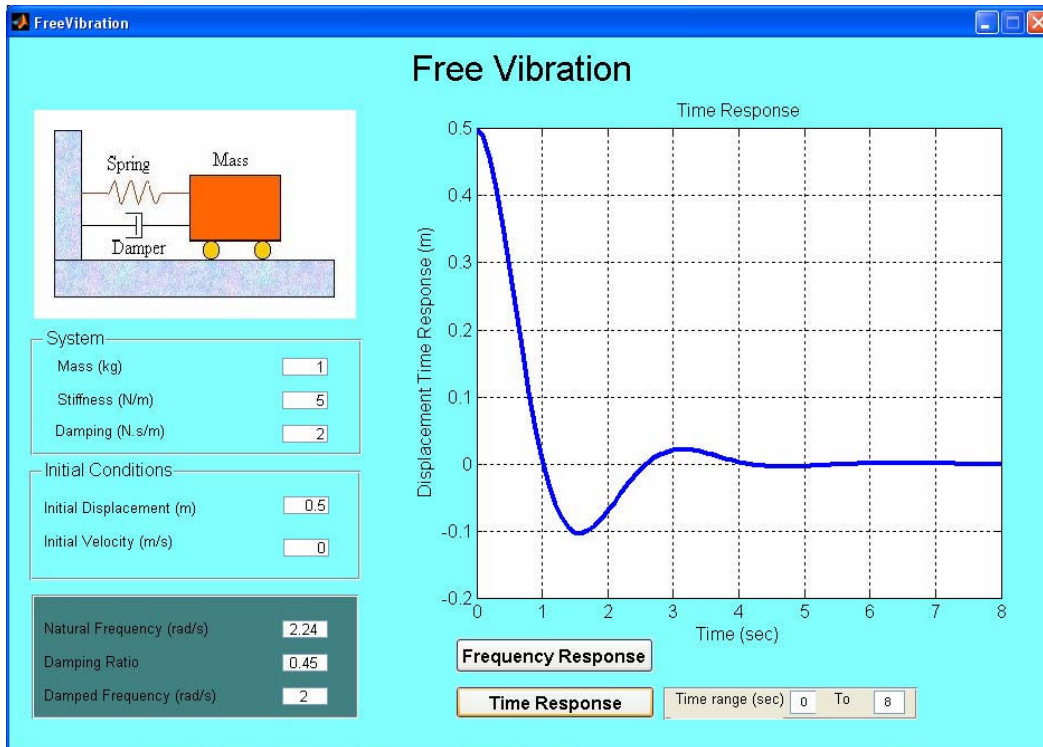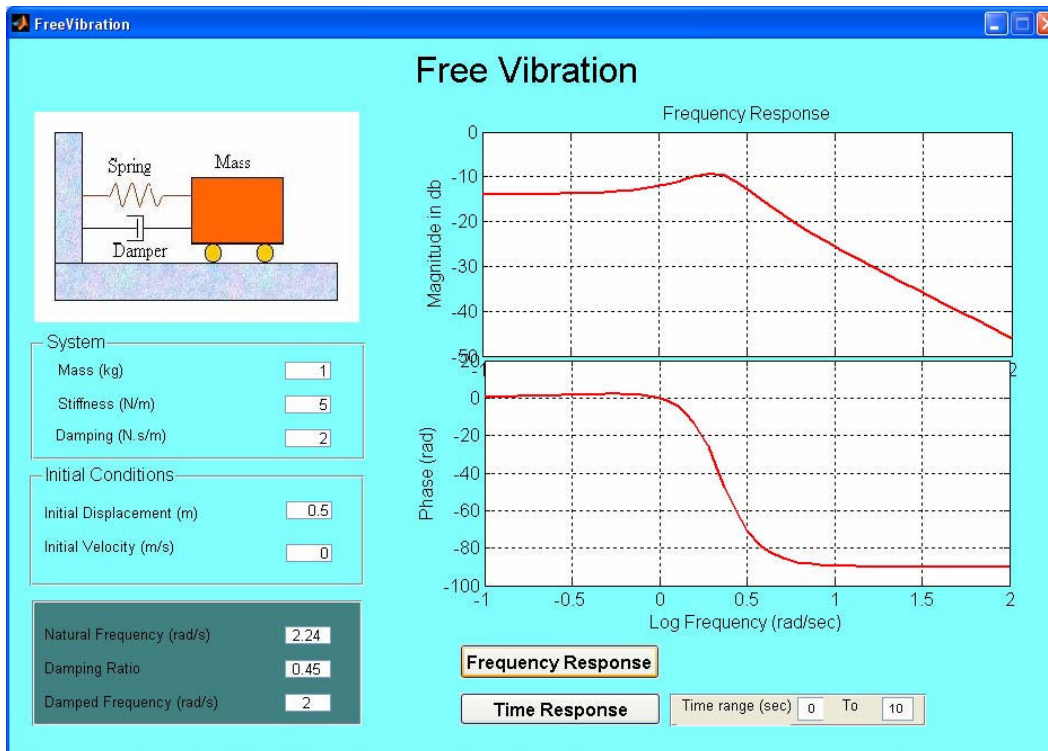
(a)
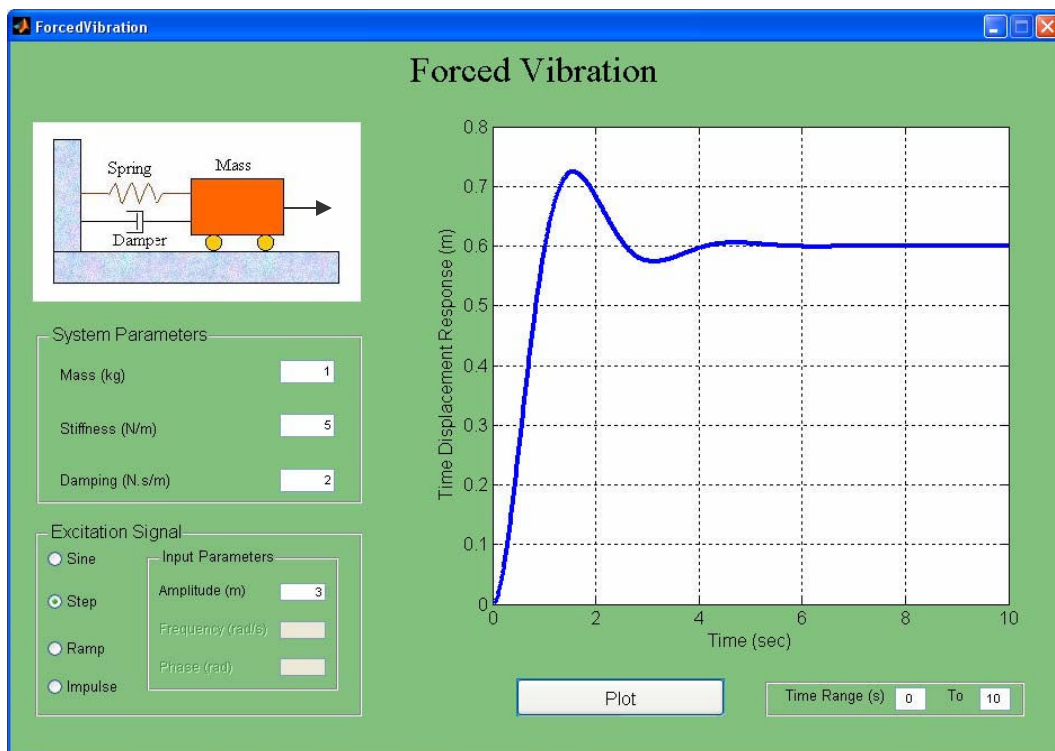
(b)

Figure 4 – Multifunctional GUI for the forced vibration of a 2nd order system: (a) sine force excitation; (b) step input

(a)

(b)

Figure 5 – Multifunctional GUI for the free vibration with initial conditions of a 2nd order
system: (a) time response; (b) frequency response

The Matlab GUI for the initial conditions response of a 2nd order dynamic system is shown in Figure 5. The students are allowed to enter the initial displacement and initial velocity, in addition to the system parameters. Two pushbuttons give the choice for time response plot, as shown in Figure 5(a), or frequency response plots, as shown in Figure 5(b). The GUI reports the natural frequency, damping ratio, and damped frequency, as well.

The visual interface presents results in a way that students can immediately identify the effects of changing system parameters. This enables the student to understand three different modes of the system, i.e., under-damped system, critical damped system and over-damped system. This is where the students often get confused.

4. An Example of a Course Project – Dynamic Response of a Mobile Phone Subjected to Drop Impact

More and more people have been using portable telecommunication devices, such as mobile phones, personal digital assistances, laptop PCs, etc. It is not uncommon for those portable electronic products to be accidentally dropped onto hard surfaces, such as the ground. Weak elements inside such products, such as solder joints connecting the integrated circuit (IC) package to the printed circuit boards (PCB), may experience very high accelerations and dynamic stresses. The impacts and shocks, thus, can lead to the failure of electronic packages and the malfunction of the products. Manufacturers usually determine the fragility of such products by conducting experimental drop tests. This is not only expensive, but time consuming. An alternative approach is to develop analytical dynamics models and to perform numerical simulations [6-11].

In the Dynamic System course, a project is assigned to investigate the dynamic response of a mobile phone. Many mobile phones are composed of a PCB with electronic packages amounted on it and a plastic housing to hold the PCB board. The mobile phone can be simplified as a two-degree-of-freedom system, as shown in Figure 6. In the model, m_1 and k_1 are the mass and spring constant of PCB board; m_2 and k_2 are the mass and spring constant of the housing; x_1 and x_2 are the displacements for the PCB and housing respectively.

Because the deflection associated with PCB bending is the primary driver of solder joint failure during drop impact, the dynamic response of the PCB assembly under impact are important variables to be investigated.
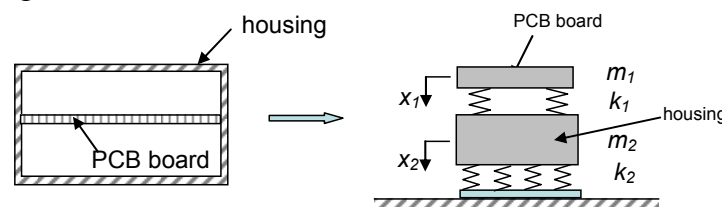

Figure 6 – Simplified model of a mobile phone

In the project, the students are required to finish the following tasks:

1. Derive the system's equations of motion and initial conditions if the mobile phone drops from the height of h.
2. Determine the natural frequencies and vibration modes of the system, for given system parameters.
3. Simulate the displacement response and acceleration response of the PCB board.
4. Discuss the effects of spring constant ratio k_1/k_2 on the maximum displacement and maximum acceleration of the PCB board.

Based on some necessary assumptions [6], The equation of motion for the system in question is,

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = 0 . \qquad (5)$$

The initial conditions are

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = 0, \text{ and } \begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix} = \begin{Bmatrix} \sqrt{2gh} \\ \sqrt{2gh} \end{Bmatrix} \qquad (6)$$

where h is the drop height.

The natural frequencies and vibration modes of the system can be determined based on the above equations of motion. The Laplace transformation method can be used to solve this problem. Assume $X_1(s)$ is the Laplace transform of the PCB displacement $x_1(t)$, and $\ddot{X}_1(s)$ is the Laplace transform of PCB acceleration $\ddot{x}_1(t)$. Based on Eqs. (5) and (6), $X_1(s)$ and $\ddot{X}_1(s)$ can be obtained as,

$$X_1(s) = \frac{\sqrt{2gh}\left(s^2 + \dfrac{k_2}{m_1} + \dfrac{k_1}{m_2} + \dfrac{k_2}{m_2}\right)}{s^4 + \left(\dfrac{k_1}{m_1} + \dfrac{k_1}{m_2} + \dfrac{k_2}{m_2}\right)s^2 + \left(\dfrac{k_1}{m_1}\dfrac{k_2}{m_2}\right)} \qquad (7)$$
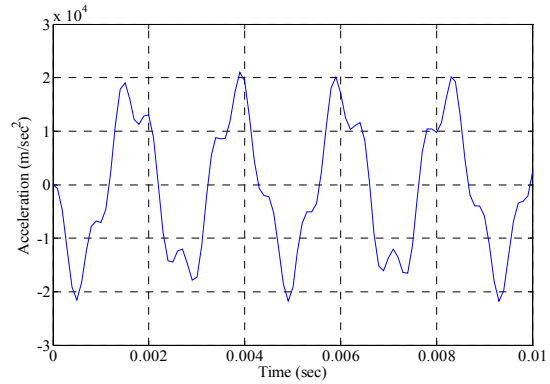
and

$$\ddot{X}_1(s) = s^2 X(s) - \sqrt{2gh} . \qquad (8)$$

Thus, displacement response and acceleration response of the PCB for a examined mobile phone can be easily simulated, as shown in Figure 7.
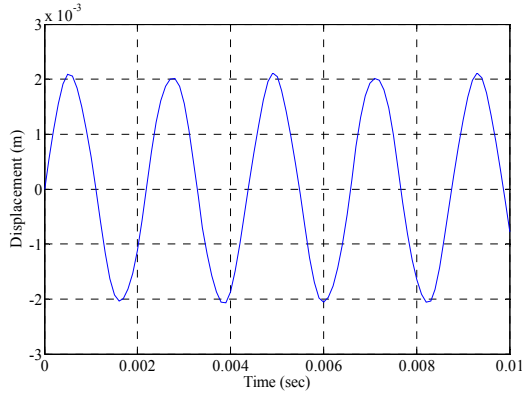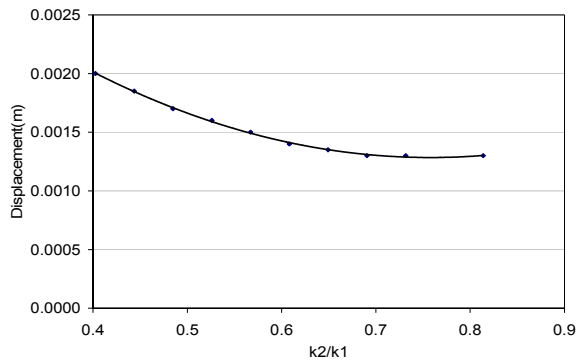
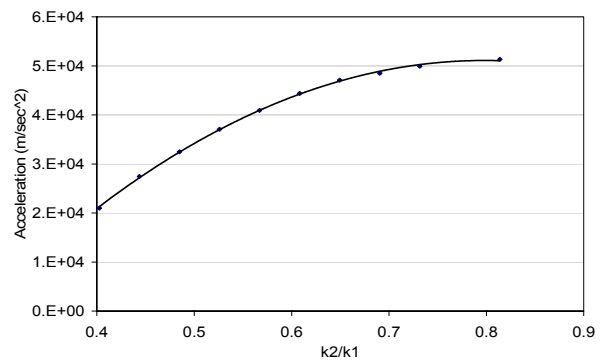(a)   (b)

Figure 7 – (a) Displacement of PCB board in a mobile phone; (b) Acceleration of PCB board in a mobile phone

Example plots are shown in Figure 8 for the effects of spring constant ratio k_1/k_2 on the maximum displacement and maximum acceleration of the PCB board.

(a)   (b)

Figure 8 – (a) Maximum displacement versus stiffness ratio k_2/k_1; (b) Maximum acceleration versus stiffness ratio k_2/k_1

With the increase of stiffness ratio of k_2/k_1, displacement decreases while acceleration increases. Research shows that the displacement plays a more significant role for the failure of the solder joints on PCB board [6,11]. Therefore, the housing should use the materials with relative low stiffness to ensure a low frequency ratio.

This project shows a mechanical application in the electronic industries. The project also demonstrates the direct application of System Dynamics to real world problems. This motivates and retains their interests in learning the subject, and inspires their recognition of the need of life-long learning.

5. Summary

Several approaches using MATLAB and SIMULINK to solve a typical dynamic system ODE have been implemented in teaching a course in System Dynamics. Those approaches include: 1) the symbolic toolbox in MATLAB, in which the ODE can be coded just like the original differential equations to be solved, and the output can be displayed simultaneously; 2) the transfer-function-based block diagram in SIMULINK, which allows one to visualize a dynamic system with clear physical meanings, and obtain the solution at the same time; and 3) the problem-based block diagram in SIMULINK, which are powerful in solving non-standard differential equations with more complex input and initial conditions. Students are encouraged to use all methods for various problems. Those different approaches provide not only flexibility for students to solve the problems differently and efficiently, but also increase students' interests in their involvement with class teaching and enhance hands-on experience in solving problems.

The GUI developments further help students to understand important concepts and theories behind numerical results, such as the relationships between input and output, solutions in time-domain and frequency-domain, three vibration modes for the systems, and the effects of equation constants and input selections on output responses.

The students also learn how to simplify a real-world problem into the standard ODEs, through a project on the dynamic response of a mobile phone subjected to a drop impact. This is a real-world problem that happens in our daily life – dropping a handheld device accidentally. The purpose of the project is to understand how to analyze and design a handheld product based on what has been learned in the class to alleviate the failure due to the mechanical shock.

The students appear to better understand the System Dynamics course material overall through the practice of integrated theoretical and numerical techniques, supplemental GUI interfaces, and the multidisciplinary project.

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