AC 2007-2853: ENGINEERING STUDENTS' MATHEMATICAL THINKING: IN THE WILD AND WITH A LAB-BASED TASK

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Abstract

Although mathematics is considered to be a fundamental element of engineering education, little empirical research has been conducted to understand how engineering students actually use mathematics. This project takes a research-informed approach towards understanding the role of mathematics in engineering design by combining two studies of engineering students' use of mathematical thinking: a study of engineering students' use of mathematics during an industry-based senior design project and a study of engineering students' use of mathematics during a laboratory based design problem.

The capstone study used a combination of qualitative methodologies to investigate engineering students' use of mathematics during one of their first real-world design projects. For this study, a team of five industrial engineering students agreed to allow the investigator to observe their team meetings, individually interview each team member and analyze their work related to their capstone project. For the laboratory based study, eight industrial engineering seniors were asked to think aloud while completing a three-hour design problem. The findings from the capstone study guided the analysis of the data from the laboratory based study.

Mathematical thinking behavior was investigated using Schoenfeld's five fundamental aspects of mathematical thinking: knowledge base, problem solving strategies or heuristics, effective use of resources, beliefs and affects and mathematical practices[1]. Additionally, Atman and Bursic's design process coding scheme[2] was used to investigate the engineering students' design behavior, and identify relationships between mathematical thinking and engineering design behavior.

In both contexts the engineering students engaged in mathematical thinking throughout their design processes. This paper presents: 1) a summary of the different mathematical thinking activities that the students engaged in during the capstone study, and 2) a summary of the mathematical thinking activities the students engaged in during the laboratory based study, and 3) some insights from the laboratory study into how the students engaged in mathematical thinking during specific design activities.

The results of this study provide insights into how engineering students actually use mathematics, which can inform the way that mathematics is taught to engineering students as well as students at the pre-college level.

Motivation from the literature

Mathematics has been a central part of engineering throughout the history of the profession[3], as well as a typical part of an engineering curriculum. The reasons for its inclusion, however, and stakeholders' perceptions on why it is included, are varied. Some students believe that they take mathematics courses simply because the mathematics courses act as a gatekeeper, ensuring that only the brightest students are able to take engineering courses, while other engineering students view mathematics as one of the many tools that are at an engineer's disposal[4].

Most members of the engineering education community believe that mathematics is both important and helpful for students in designing and developing systems (e.g. Moussavi[5]), as "a medium of knowledge representation"[6] and in developing the ability to reason (e.g. Underwood[7]). Many educators continue to devote attention to how to structure classes to teach mathematics engineering students (e.g. Venable, McConnell and Stiller[8],Aroshas, Verner and Berman[9]; McKenna, McMartin and Agogino[10]).

At the same time, however, there is a prevalent belief among practicing engineers that the mathematics they learned in college is not applicable to their daily work. Pearson[11] estimates that of the thousands of engineers he knew, only three out of ten actually use calculus/differential equations and that "there were/are extremely competent engineers/scientists who never had a course in calculus" (p.8). He also found that many engineers get their mathematical needs met by specialists or mathematicians, and also often just look up the formulas in books. Pearson asks, "why do we continue to teach what the mathematics professors think the engineer/scientist needs, as contrasted with what is actually needed in industry and commerce?" (p. 8).

In looking at engineering students' use of mathematics, there is more to consider than just the mathematical content knowledge taught in mathematics courses. Indeed, mathematics courses may transform a students' way of thinking and approaching problems; students have the opportunity to learn how to engage in mathematical thinking. Schoenfeld[1] identifies five aspects that are part of the process of thinking mathematically: Knowledge Base, Heuristics (problem solving strategies), Monitoring and Control (cognitive mechanisms that the problem solver uses to monitor their progress and their use of cognitive resources), Beliefs and Affects (regarding mathematics) and Practices (context for teaching and learning mathematics). For the purposes of this research project, we consider an amended set of five aspects of mathematical thinking: knowledge base, problem solving strategies, use of resources, beliefs and affects, mathematical practices. These are defined in Table 1.

Table 1: Aspects of Mathematical Thinking

Aspect	**Definition/Description**
Knowledge Base	Cognitive Resources: Mathematical Content Knowledge
Problem Solving Strategies	Global or local strategies learned from mathematics courses
Use of Resources	Social Resource: Peers, Experts Material Resources: textbooks, time, computers Use of Resources: metacognitive processes such as planning and monitoring
Beliefs and Affects	Beliefs about mathematics and one's mathematical ability, Feelings towards mathematics, Emotions or feelings experienced
Mathematical Practices	Activities or actions that mathematicians engage in, or activities that involve mathematics.

Because design is a practice that is both common to and integral to all branches of engineering, design is a prime context for considering how engineering students engage in mathematical thinking. Additionally, design problems and projects often give engineering students an opportunity to integrate and apply the content knowledge they have learned in their mathematics, science and engineering courses. While there are many different prescriptive models of design that engineering students may be exposed to through engineering textbooks or in their course lectures, most of these models have many similarities. Atman and Bursic[2] conducted a content analysis of the design process presented to students in seven engineering textbooks, and found that most models include ten activities: identifying a need, defining the problem, gathering information, generating ideas, modeling solutions, performing feasibility analysis, evaluating solutions, making decisions, communicating design decisions, and implementing a solution (see Table 2). In this synthesized prescriptive model, it is clear that mathematics is important for modeling solutions and evaluating solutions, and mathematics is also often associated with performing the feasibility analysis. It is unclear, however, from the prescriptive model, what the role of mathematics plays in the other seven design activities.

Table 2: Design Activities[2]

Code	Activity	Definition/Description
N	Identify Need	Identify basic needs (purpose, reason for design)
PD	Problem Definition	Define what the problem really is, identify the constraints, identify criteria, reread problem statement or information sheets, question the problem statement
GATH	Gather Information	Search for and collect information
GEN	Generate Ideas	Develop possible ideas for a solution, brainstorm, list different alternatives
MOD	Modelling	Describe how to build an idea, measurements, dimensions, calculations
FEAS	Feasibility Analysis	Determine workability, does it meet constraints, criteria, etc.
EVAL	Evaluation	Compare alternatives, judge options, is one better, cheaper, more accurate
DEC	Decision	Select one idea or solution among alternatives
COM	Communication	Communicate the design to others, write down a solution or instructions
IMP	Implementation	Produce or construct a physical device, product or system

This paper addresses the question of how mathematics is used by engineering students throughout the design process—that is, in each and every design activity. One of the most prevalent methods used to study engineering design is verbal protocol analysis[12, 13, 2]. In this research, we use the results from the ethnographic study of engineering students working on their senior capstone design project to inform the analysis of verbal protocols collected from students thinking aloud while attending to a three-hour design task.

Methods

In this section we present the methodologies used in the two studies, the Capstone Study and the Lab-based Playground Study. A summary of the two studies is provided in Table 3.

Study one: Capstone design project

The capstone study used a combination of qualitative methodologies to investigate engineering students' use of mathematics during one of their first real-world design projects. One team of five industrial engineering students allowed the investigator to observe their team meetings, individually interview each team member and analyze their work related to their capstone project. In addition, four engineering students (representing Aeronautics and Astronautics, Chemical Engineering and Materials Science Engineering) participated in interviews. In this section, we present a brief overview of the methodology used to collect and analyze the data from the Capstone Study. The findings from the interview portion of the study as well as a more detailed description of the Capstone Study methodology are presented elsewhere[14, 15, 16].

Table 3: Comparison of the Two Studies

	Study One: Capstone Design Project	Study Two: Lab-based Playground Task
Method	Observations of 22 team meetings Interviews with 4 team members and 4 additional engineering students	Verbal Protocol Analysis
Task	2-quarter-long design project: re-design a supply chain system and design a satellite distribution center for an industry client; gather information by talking to the client, other company employees and the greater community	3-hour-long design task: design a playground for a fictitious neighborhood; gather more information about the problem from the task administrator
Client	Industry sponsor with a real need	Task administrator with a hypothetical need
Participants	5 IE seniors (observation & interviews) 4 additional engineering seniors: 2 Aeronautics & Astronautics, 1 Chemical Engineering, 1 Materials Science and Engineering	IE Slice: 7 IE seniors, 2 IE freshmen
Collaboration	Team-based projects	Individual performance

During the capstone design project, the Industrial Engineering students worked on an industry-based senior design project at their own pace as they integrated and applied the mathematics, science and engineering material they learned through their coursework. The capstone project mimicked a real-world job experience, was self-paced and relatively independent, and may be considered to be similar to the types of informal learning opportunities that an engineer might encounter on-the-job.

The data from the Capstone Study include audio recordings and field notes that were collected during the team meetings as well as copies of intermittent design documents (copies of sketches, notes, spreadsheets and papers that the team members created in the course of the project). The field notes from the interviews and observations were analyzed using a constant comparison method[17, 18, 19]—different uses of mathematics and mathematical thinking were noted and grouped into "themes." The themes (different mathematical thinking activities) were then further grouped as they were mapped to the five aspects of mathematical thinking. The interview responses provided a baseline for what "counted" as mathematical thinking—for example, some of the strategies that the students discussed may not be unique to mathematical thinking, but they were included as a "mathematical thinking theme" because the students reported learning the problem solving strategy from a mathematics course.

Study two: Lab-based Playground task

For the laboratory based study, 24 senior and 26 freshman engineering students were asked to think aloud while attending to a three-hour design problem as part of a previous study[20]. The students were asked to design a playground for a fictitious neighborhood while attending to a number of design constraints—the playground needed to be safe, include three different activities, accommodate twelve children, remain outdoors all year long and comply with the Americans with Disabilities Act. The study participants were able to ask the task administrator for information (e.g. the cost of lumber, the average weight of a six year-old child) during the design session but did not have access to pictures of playgrounds, a computer or any other people. A subset of this data—the data from the seven industrial engineering seniors and two industrial engineering freshmen who participated in the study—were analyzed for mathematical thinking. The original study (and the larger data set) is presented elsewhere[20, 21].

The lab-based task provides a contrasting perspective on how engineering students use mathematics. In this case, the students worked individually as they designed a playground for a fictitious client. Participants could ask the task administrator for information, but could not actually visit the job site. The task was focused on design process rather than prompting the students to synthesize and apply all of the material learned in the undergraduate curriculum. While the Capstone Study provided rich insights into students' use of mathematical thinking in the context of an engineering design task, the Lab-based Playground Study provides a clearer picture of students' cognitive processes, and specifically the cognitive processes associated with design thinking and mathematical thinking.

Data for the lab-based playground study include audio and video recordings of each participant as they thought aloud as well as copies of all of the sketches, lists, notes and responses the participants created during the design study session. The audio recordings were transcribed, segmented and coded for design activity for the original study[20]. In applying the coding scheme from Table 2, the design activities "Identify Need" and "Implementation" were not included because "need" had already been identified in the task and the students were unable to implement playground designs. The transcripts for the nine industrial engineering students were also analyzed for mathematical thinking: the transcripts were further coded using the mathematical thinking themes that emerged from the Capstone Study data and then analyzed and coded for new themes that emerged from the Lab-Based Playground Study transcripts.

Findings

In this section, we present an overview of the main findings from the Lab-Based Playground Study as well as selected findings from the Capstone Study.

Ways in which engineering students engage in mathematical thinking

Table 4 presents the main findings from the Capstone Study and the Lab-Based Playground Study —a list of the ways in which students engaged in mathematical thinking, arranged according to aspect of mathematical thinking. The first column presents the five aspects of mathematical thinking that are described in Table 1, the second column presents the themes (mathematical thinking activities) that emerged from the capstone and playground data, and the third column presents the findings that are specific to the playground study—a checkmark in the third column indicates that the particular theme was a type of mathematical thinking activity used by at least one of the nine study participants. Many of the themes that were evident in the Capstone Study were also evident in the Playground Study. The three themes in the second column that are presented in bold typeface were evident in only the Playground Study data. Examples of how students engaged in each of the mathematical thinking activities in the second column, as well as descriptions of each of these activities can be found elsewhere[14,15,16].

Table 4: Themes from the Capstone Study Data and Playground Study Data

Aspect	Theme (Emerged from Capstone Study and Playground Protocols)	Playground Protocols
Knowledge Base	Content Knowledge	v
	Knowledge of How to Use Tools	
	Mathematical Terminology	
Problem Solving Strategies	Start from Fundamentals	
	Guess & Verify	v
	Transform the Problem	v
	Simplify	
	Separate the Problem into Smaller Problems/ Decompose the Problem	v
	Take it Step by Step	v
	Make Use of What You Have	v
	Consider Multiple Approaches	v
Use of Resources	Tools (e.g. Excel, Simulation)	v
	Server	
	Textbooks	v
Resources:	Other Books	
	Time	
	Meeting Spaces	
	Peers: Teammates, Other Classmates	
	"Experts": TAs, Instructors, Industry Partners	
Monitoring use of resources:	Planning	v
	Choosing the Right Approach	v
	Applying Mathematics (knowing when & how to)	
	Discerning What's Relevant	
	Monitoring Progress	v
Beliefs and Affects	Mathematics is Only About Content Knowledge	
	Mathematics is a Tool	
	Mathematics is a Form of Thinking	
	Mathematics is Useful/ is Not Useful	v
	Precision is Necessary/ is not Necessary	v
	It is Better to Have to Have too Much than to Have Not Enough	
	Mathematical Truth	
	Likes and Dislikes Regarding Mathematics	v
	I am Good/I am not Good at Mathematics	v
	Surprise Over a Mathematical Result	v
Mathematical Practices	Having a Mathematical Perspective	v
	Using a Mathematical Vocabulary	v
	Using Numbers to Communicate	
	Creating and Using Visualizations	
	Using Numbers to Justify	v
	Mathematizing	v
	Dealing with Uncertainty	v
	Estimating	v
	Interpreting Numbers	v
	Being Precise	v
	Creating and Using Mathematical Models	v
	Calculating/ Crunching Numbers	v
	Checking if Results Seem Reasonable	

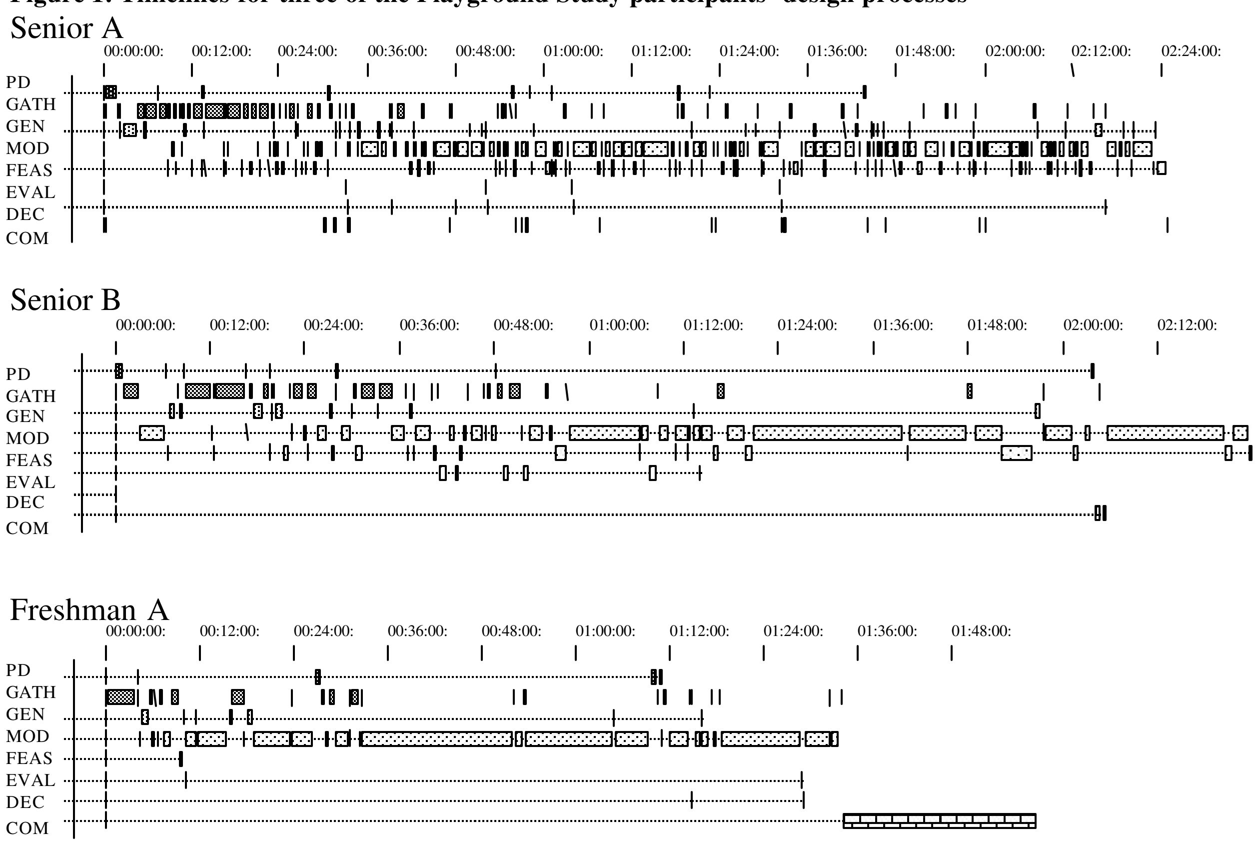
The themes in bold typeface were evident in the Playground Study data but not the Capstone Study data.

Mathematical thinking in an engineering design process

The goal of the original Playground Study was to characterize engineering students' design processes, and compare the processes of senior engineering students to the processes of freshmen. Findings from this study can be found elsewhere[20, 21]. Students' design processes can be characterized numerically: the total time a student spent on the problem, the amount of time a student spent in each of the eight design activities and the number of times a student transitioned between different design activities. Design processes can also be characterized graphically:

through the use of timelines (the timelines were created using MacSHAPA, software developed by Sanderson et al. [22]). Figure 1 presents timelines for three of the study participants. For each timeline, there are eight lines—one for each of the eight design activities. A “tick” on the line indicates that the study participant spent some amount of time in a particular design at a particular point in time; across the top of the timeline, timestamps mark points that are 12 minutes into the design session 24 minutes into the session, etc. Wide “ticks” indicate that the participant spent a relatively large amount of time in a particular design activity while skinny ticks indicate that the participant spent a brief amount of time in a particular design activity before transitioning to a different activity. By looking at students’ design processes depicted in timelines, we can see whether a student tended to transition frequently between design activities (as was the case with Senior A) or spend large amounts of time in an activity (as was the case with Senior B and Freshman A). We can also note when (chronologically) a participant engaged in a particular design activity—for example, Senior A engaged in Gather Information more frequently during the first thirty minutes of the design process, but did occasional Gather Information later in the design process. For the purposes of this study, the timelines provide additional context for understanding the design processes that students engaged in while also engaging in mathematical thinking.

Figure 1: Timelines for three of the Playground Study participants’ design processes[21]



The data from the Playground Study was analyzed for both design activity and mathematical thinking activity. Because each segment—each individual thought—was coded for both design and mathematical activities, we are able to examine how students engaged in mathematical thinking as they defined the problem, gathered information, generated alternative solutions, modeled solutions, performed feasibility analyses, evaluated solutions, made design decisions and communicated design solutions. From the analysis, we found that the study participants engaged in different forms of mathematical thinking all throughout their design processes—in each of the design activities as well as throughout their chronological process.

Table 5 shows which specific themes (mathematical thinking activities) were evident in each of eight design activities. Checkmarks indicate that at least one of the nine participants exhibited a particular mathematical thinking theme during a particular design activity at least once during their design process. Checkmarks are used rather than frequencies of how many times a student engaged in a mathematical thinking theme during each design activity because the number of study participants is small. The checkmarks are able to demonstrate that it is possible for a student to engage in a particular mathematical thinking theme during a particular design activity, as has been evidenced in this data. In addition to showing that study participants engaged in particular mathematical thinking themes in many different design activities, Table 5 also indicates that four of the five aspects of mathematical thinking were evident in all eight design activities. It is not too surprising that beliefs and affects was not evident in many design activities; we would expect that students' beliefs about and affects towards mathematics would be more evident in situations where a student is asked to talk about mathematics, such as in an interview setting or in a conversation with peers about coursework.

Summary of findings

In both the Capstone Study and the Playground Study engineering students engaged in mathematical thinking in a number of ways. Beyond using mathematical content knowledge, such as taking derivatives, the students used problem solving strategies they learned from mathematics courses, social and material resources, metacognitive processes such as planning and monitoring, expressed beliefs and feelings and engaged in mathematical practices in two different design contexts.

In both contexts the engineering students engaged in mathematical thinking throughout their design processes. The students used a mathematical vocabulary to think about and talk about different aspects of the task they worked on. Additionally, the students used mathematics as they analyzed data in order to gain more information about the task they worked on, they used analytical thinking in generating alternative solutions and used mathematics in determining dimensions and costs as they modeled their solutions. The students used mathematics to determine the feasibility of solutions, compare multiple solutions and choose a particular solution. Finally, they used a mathematical vocabulary and numerical results in their communication, especially in justifying why they chose a particular solution. While the prescriptive models of engineering are sometimes vague in relation to the role of mathematical thinking in engineering design, the data from this study suggests that mathematical thinking can (and often does) play a role in each aspect of engineering design.

Table 5: Presence of a Mathematical Thinking Activity During a Particular Design Activity

Aspect	Theme	PD	GATH	GEN	MOD	FEAS	EVAL	DEC	COM
Knowledge Base	Content Knowledge	v	v	v	v	v	v	v	v
Problem Solving Strategies	Start from Fundamentals								
	Guess & Verify		v		v	v			
	Transform the Problem	v	v	v	v	v	v		v
	Simplify the Problem								
	Separate the Problem into Smaller Problems/ Decompose the Problem	v	v	v	v	v	v	v	v
	Take it Step by Step								
	Make Use of What You Have		v	v	v	v			
	Consider Multiple Approaches	v	v	v	v	v	v	v	v
Use of Resources	Tools (e.g. Excel, Simulation)	v	v	v	v		v		v
	Textbooks								
Resources:	Other Books								
	Peers: Teammates, Other Classmates								
	"Experts": TAs, Instructors, Industry Partners								
	Budget	v	v	v	v	v	v	v	v
Monitoring use of resources:	Planning		v	v	v	v			v
	Choosing the Right Approach	v	v	v	v	v	v	v	v
	Applying Mathematics (knowing when & how to)								
	Discerning What's Relevant								
	Monitoring Progress	v	v	v	v	v	v	v	v
Beliefs and Affects	Mathematics is only about Content Knowledge								
	Mathematics is a Tool								
	Mathematics is a Form of Thinking								
	Mathematics is Useful/ is Not Useful				v				
	Precisions is/ is Not Necessary				v				v
	It is Better to Have Too Much Than to Not Have Enough								
	Mathematical Truth								
	Likes and Dislikes regarding mathematics								
	I am Good/ I am Not Good at Mathematics		v		v	v			
	Interest/Excitement				v	v			
	Surprise				v	v			
Mathematical Practices	Using a Mathematical Vocabulary	v	v	v	v	v	v	v	
	Using Numbers to Communicate	v	v	v	v	v			v
	Using Numbers to Justify		v	v	v	v	v	v	
	Mathematizing		v	v	v	v		v	
	Dealing with Uncertainty	v	v	v	v	v	v	v	v
	Estimating	v	v	v	v	v	v	v	v
	Interpreting Numbers		v		v	v	v	v	
	Being Precise								
	Creating and Using Mathematical Models								
	Calculating/ Crunching Numbers	v	v	v	v	v	v	v	v
	Counting		v		v	v			
	Checking if a Result Seems Reasonable				v	v			

Conclusion

The results of this study provide insights into how engineering students actually use mathematics, which can inform the way that mathematics is taught to engineering students as well as students at the pre-college level. In particular, the results of these studies suggest that engineering students engage in mathematical thinking in a number of ways. Not only do engineering students use a breadth of mathematical content knowledge as they tackle complex design problems, but they also use problem solving strategies that they have learned from mathematics courses, social resources, material resources and monitoring and planning processes they have learned from mathematics courses and mathematical practices. In designing courses and curriculum for engineering students, it is important to consider all five aspects of mathematical thinking—how engineering students learn mathematical content knowledge as well as problem solving strategies, what resources engineering students have available to them, how they learn how to use these resources, what their beliefs about and affects towards mathematics are (and how these beliefs and affects about mathematics may impact their beliefs and affects towards engineering) and how engineering students learn different mathematical practices.

The findings from this study also highlight the prevalence of mathematical thinking throughout an engineering design process. By recognizing the role that mathematical thinking may play in each aspect of the design process, we might better teach students how to engage in mathematical thinking and transfer the material they have learned in their mathematics courses into their engineering practice. For example, as we coach students to spend time defining a problem, or scoping out a problem, we might prompt them to consider a mathematical element of the problem definition—in addition to societal, environmental and other elements of the problem definition. In prompting students to consider a mathematical element of the problem definition, we might be able to help them realize what mathematical content knowledge will help them later in the design project.

The prevalence of the use of mathematics throughout the design project examined in this research also offers support for the practice of using engineering design as a mechanism for teaching mathematics and science content at pre-college levels. The findings from this study offer additional empirical evidence that mathematical thinking can play a large role in engineering design, suggesting that there are ample opportunities for pre-college teachers to use engineering design problems to motivate and accentuate mathematics learning.

While the research presented in this paper affords new insights into the question of how mathematics is used by engineering students, there are many opportunities for further work. First, the research can be expanded to include other engineering disciplines. The research can also be expanded to include a larger sample of engineering students. It is unlikely that the mathematical thinking activities presented in this paper are an exhaustive list; rather, it is likely that further work would uncover additional mathematical thinking activities. Using additional design tasks as well as additional study tasks would help us to expand the list. Further, a larger sample of study participants would allow us to determine the prevalence of these mathematical thinking activities and allow us to better understand the way that mathematical thinking can impact design thinking. Finally, conducting research on *practicing* engineers' use of mathematics would benefit our decision making process as we determine how to best teach mathematics to engineering students and also provide us with rich examples which we could share with students unsure of the ways that mathematics is used in the "real world."

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