

## 2nd Law Analysis of a Rankine Cycle Using the Wicks Cycle as the Ideal Standard

**Frank Wicks**

Mechanical Engineering Department  
Union College  
Schenectady, New York, 12308

### Abstract

For the last century thermodynamics books have said and thermodynamics professors have taught that the Carnot cycle is the ideal or most efficient possible engine that can operate between a hot and a cold reservoir. The corresponding Carnot cycle efficiency is  $E_{\text{Carnot}} = (T_{\text{hot}} - T_{\text{cold}}) / T_{\text{hot}}$  where the temperatures are in absolute.

A decade ago the author recognized that with respect to the Carnot cycle, students were learning a classical equation that they did not understand. Also the Carnot cycle standard was widely misapplied by practicing engineers and also by the National Council of Examiners for Engineering and Surveying which prepares the Professional Engineering licensing exams.

The important fact that is not recognized by the Carnot cycle is that the source of the high temperature heat for virtually all heat engines are the products of combustion of a fuel, which are finite in size and in heat capacity. Thus, heat is not released at a single temperature, but over the entire temperature range from the maximum combustion product temperature  $T_{\text{max}}$  down to the ambient temperature which is the infinite and constant temperature environment  $T_{\text{cold}}$ . Accordingly, the efficiency of the ideal fuel burning engine has been defined and derived as the Wicks cycle efficiency with  $E_{\text{Wicks}} = 1 - T_{\text{cold}} \ln(T_{\text{max}} / T_{\text{cold}}) / (T_{\text{max}} - T_{\text{cold}})$ .

Prior papers have shown the importance of understanding that the Wicks cycle rather than the Carnot cycle represents the ideal fuel burning engine. These papers have been of practical as well as academic importance. They have explained that while the Stirling engine may be a method to implement the Carnot cycle, a fuel burning Rankine cycle is more efficient than a fuel burning Stirling engine. These papers have also used the Wicks cycle standard to show why a combination of internal and external combustion engines is more efficient than either one separately.

This paper will use the Wicks cycle as the ideal standard for evaluating a fuel burning Rankine cycle. It will use a 1<sup>st</sup> law technique to calculate efficiency as the product of the boiler efficiency and the cycle efficiency. It will then use a 2<sup>nd</sup> law method to obtain the same result, but also showing the lost efficiency related to each irreversible process.

*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright 2001, American Society for Engineering Education*

The irreversibly processes are identified as heat transfer over a temperature difference in the condenser and the water heating, boiling and superheating sections of the boiler along less than ideal turbine and pump and the stack loss. An important idea is that in terms of the first law analysis the boiler with a typical efficiency of 90 % appears to be quite efficient, the 2<sup>nd</sup> law analysis shows that most of the lost efficiency is related to the boiler because of transferring heat over large temperature differences along with the stack loss.

It is finally shown that the actual efficiency is the Wicks cycle efficiency minus the sum of the lost efficiencies for each process, and that the appropriate 2<sup>nd</sup> law efficiency of the Rankine cycle should be defined as the ratio of the actual and Wicks cycle efficiencies.

## 1. Introduction

For the last century text books have taught and students have learned that the Carnot cycle efficiency which is defined in the 1824 publication by Nicolas Leonard Carnot entitled “Su Puissance Motrice du Feu” or “On the Motive Power of Heat” represents as ideal engine and the efficiency is defined by equation 1 where  $W_{ideal}$  is the ideal work,  $Q_{hot}$  is the high temperature heat that flows to the engine,  $T_{hot}$  is the temperature of the hot reservoir from which the high temperature heat flows and  $T_{cold}$  is the temperature of the cold reservoir to which the low temperature heat is rejected.

$$\text{Eff}_{carnot} = W_{ideal}/Q_{hot} = (T_{hot} - T_{cold})/T_{hot} \quad (1)$$

The Carnot efficiency is the correct standard if the high temperature reservoir releases all heat at the same temperature and the cold reservoir receives heat at a constant temperature. However, the heat source for virtually all engines are the combustion products of the fuel that be recognized as a finite size in size and thus releases temperature over the entire temperature range from the maximum flame temperature  $T_{hotmax}$  down to the infinite size environment and thus constant temperature cold reservoir  $T_{cold}$ .

Using the definition of efficiency, the 1<sup>st</sup> law and the 2<sup>nd</sup> law the author derived an efficiency for the ideal fuel burning engine called the Wicks cycle in terms of  $T_{hotmax}$  and  $T_{cold}$  which was published in reference 1 and is presented in equation 2.

$$\text{Eff}_{wicks} = 1 - T_{cold} * \ln(T_{hotmax}/T_{cold}) / (T_{hotmax} - T_{cold}) \quad (2)$$

If the maximum flame temperature is 3500 R and the ambient temperature is 500 R the Carnot cycle equation incorrectly yields an efficiency of 85.7 %, whereas the Wicks cycle equation correctly shows the ideal efficiency to be 67.6%.

*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright 2001, American Society for Engineering Education*

Along with defining the efficiency of the Wicks cycle and comparing it with the Carnot cycle, this seminal paper (ie reference 1) also showed it could be implemented as a three process gas turbine type cycle but with as isothermal compressor, constant pressure combustion or heat addition and then a reversible adiabatic turbine with exhaust at both the initial ambient pressure and temperature  $T_{cold}$ .

It was also shown that this ideal Wicks cycle is impractical because of the difficulties in building an isothermal compressor and the very high pressure ratio required. For  $T_{hotmax}$  of 3500 R and  $T_{cold}$  of 500 R a pressure ratio of 907 would be required. Thus if the inlet pressure is 14.7 psia the compressor discharge would be 13,340 psia. Thus, the Wicks cycle defines a standard that is not attainable with a single cycle, but could be considered as a bottoming cycle for a combined cycle plant.

The Wicks cycle analysis also helps explain why a combination gas turbine and steam turbine cycle can have an efficiency of more than 50 % while each cycle alone is 35 % efficient.

## 2. Analysis

The purpose of this paper is to apply the Wicks cycle to perform a 1<sup>st</sup> and 2<sup>nd</sup> law analysis of fuel burning steam Rankine cycle of 67.6 % as shown in the introduction. The rate of high temperature heat released is the product of flow rate and heat capacity and the 3000 R temperature difference between  $T_{hotmax}$  and  $T_{cold}$ . The corresponding  $Q_{hot}$  is 750 Btu/hr and the ideal work is this heat times the Wicks efficiency or 507 Btu/hr.

The analysis is based upon the combustion of .05 lbm/hr of fuel with .95 lbm/hr of air to produce 1 lbm/hr of combustion products with a constant heat capacity of .25 (Btu/lbm F) with a maximum flame temperature of 3500 R and with an ambient temperature representing  $T_{cold}$  of 500 R or 40 F.

The boiler has a 90 % 1<sup>st</sup> law efficiency which is the high temperature heat recovered to the high temperature heat released. Thus the temperature going to the stack is 800 R. The steam Rankine with superheat. The high side pressure is 800 psia and the superheat temperature is 900 F. The condenser operates at 100 F. The points that are defined in the cycle are 1) superheated steam to turbine, 2) mixture to the condenser, 3) saturated water to the pump, 4) compressed liquid from the pump, a) point in the boiler where boiling starts and b) is the point in the boiler where the boiling ends and the superheating starts.

The turbine and the pump are each 85 % efficient compared to reversible adiabatic machines. Thus 2I is the discharge from an ideal or isotropic turbine and 4I is the discharge from an ideal pump.

*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright2001, American Society for Engineering Education*

A property table that is made much easier and accurate by the use of computer based property tables is developed and presented in Table I.

Table I  
Rankine Cycle Property Table

Point	T(F)	p(psia)	h(Btu/lbm)	v(ft <sup>3</sup> /lbm)	x (%)	s(Btu/lbm R)
1	900	800	1455.6	.964	superheat	1.6408
2I	100	.9504	913.9	285.5	.8158	1.6408
2	100	.9504	995.16	312.9	.8941	1.7849
3	100	.9504	68.04	.01613	0	.12962
4I	100.28	800	70.42	.016092	comp liq	.12962
4	100.7	800	70.84	.016093	comp liq	.13037
a	518.4	800	309.7	.02087	0	.711
b	518.4	800	1119.3	.5691	1.0	1.416

Next an energy balance is required between the combustion products on the hot side of the boiler and the H<sub>2</sub>O in the form of heating water, boiling and superheating which shows that the 1 lbm/hr of combustion products can produce a Rankine cycle flow rate of .48765 lbm/hr. The temperature of the combustion products will drop from 3500 R to 3000.3 R in the superheater and to 1655.6 R in the boiling section and to the previously defined 800 R to the stack and ultimately the stack gasses will cool to the ambient 500 R or 40 F.

A 1<sup>st</sup> law process and cycle table based upon the .48765 lbm/hr flow rate is presented in Table II.

Table II  
First Law Process and Cycle Table

Process	#-#	Q(Btu/hr)	m*(hout-hin) (Btu/hr)	W(Btu/hr)
Turbine	1-2	0	-224.5	224.5
Condenser	2-3	-451.88	-455.81	0
Pump	3-4	0	1.36	-1.36
Water Heating	4-a	213.9	213.9	0
Boiling	a-b	336.18	336.18	0
Superheating	b-1	124.92	124.92	0
Full Cycle		223.05	0	223.05

$$\text{Effcycle} = W_{\text{cycle}} / Q_{\text{hot}} = 223.05 / 675 = 33.06 \%$$

*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright 2001, American Society for Engineering Education*

The net efficiency is the product of the boiler efficiency  $\text{Eff}_{\text{boiler}}$  which is 90 % and the cycle efficiency of 33.06 %

$$\text{Thus, } E_{\text{net}} = \text{Eff}_{\text{boiler}} * \text{Eff}_{\text{cycle}} = 29.75 \% \quad (3)$$

The same fuel burning cycle can also be evaluated by a 2<sup>nd</sup> law method in which each irreversible process is identified and then multiplied by the ambient temperature to obtain irreversibility or lost work and divided by the total heat released by the combustion products to obtain the lost efficiency for each cycle. Accordingly, the ideal or Wicks cycle  $\text{Eff}_{\text{wicks}}$  should equal the  $\text{Eff}_{\text{net}}$  of the actual cycle and the sum of the lost efficiencies  $\text{Eff}_{\text{lost}}$  as presented in a rearranged form in equation 4.

$$\text{Eff}_{\text{net}} = \text{Eff}_{\text{wicks}} - \text{Eff}_{\text{lost}} \quad (4)$$

Next each of the irreversible processes are identified and a 2<sup>nd</sup> Law process table is developed to show the entropy production rate, rate of lost work and lost efficiency is developed. Processes are irreversible either if there is heat transfer across a temperature difference as in the condenser, water heating, boiling, superheating and stack loss and the turbine and pump which are adiabatic processes that are not ideal.

The entropy production rate for heat exchange processes is the rate at which the cold side gains entropy minus the rate at which the hot side loses entropy. The entropy production rate in the turbine and the pump is the flow rate times the entropy increase from inlet to exhaust or discharge. The resulting analysis is presented in Table III.

Table III

2<sup>nd</sup> Law Analysis for Each Process Including Stack and Total

Process	delta S (Btu/hr R)	T <sub>cold</sub> *delta S (Btu/hr) or Irreversibility or W <sub>lost</sub>	Eff <sub>lost</sub> = W <sub>lost</sub> /Q <sub>hot</sub>	Percent of Total (%)
Turbine	.0729	35.145	.04686	12.39
Condenser	.0983	48.415	.064553	17.07
Pump	.0003654	.183	.0002439	.06
Water Heat	.10120	50.60	.067467	17.84
Boiling	.19502	97.51	.13001	34.39
Superheat	.07107	35.535	.04738	12.53
Stack Loss	.03250	16.23	.02167	5.75
Total	.56728	283.638	.3781	100.00

Converting the  $\text{Eff}_{\text{lost}}$  as a fraction to a % yields  $\text{Eff}_{\text{lost}} = 37.81 \%$

*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright 2001, American Society for Engineering Education*

Now the validating test is whether the 1<sup>st</sup> and 2<sup>nd</sup> law methods yield the same result. The 2<sup>nd</sup> law method equation is equation 4.

$$\text{Eff}_{\text{net}} = \text{Eff}_{\text{wicks}} - \text{Eff}_{\text{lost}} = 67.57 \% - 37.81 \% = 29.75 \% \quad (4)$$

This is the same net efficiency that was obtained by multiplying the boiler and the cycle first law efficiencies as shown in equation 3 and repeated here.

$$\text{Eff}_{\text{net}} = \text{Eff}_{\text{boiler}} * \text{Eff}_{\text{cycle}} = 90 \% * 33.06 \% = 29.75 \%$$

### 3. Discussion

Thus the method has confirmed the same net efficiency which is the ratio of the net work to the heat input by both the 1<sup>st</sup> and 2<sup>nd</sup> law methods. An obvious question is “Why bother to do it again by the 2<sup>nd</sup> law method if you already have the same answer from the 1<sup>st</sup> law method”?

The reason to do the 2<sup>nd</sup> law method is that it gives more information in terms of where the inefficiencies are. The 1<sup>st</sup> law method shows a boiler efficiency of 90 % which seems pretty good. However the 2<sup>nd</sup> law method shows that 70.51% of all of the lost efficiency is boiler related because of the irreversibilities of transferring heat over large temperature differences in the processes of water heating, boiling and superheating along with the stack loss.

Thus, the 2<sup>nd</sup> law method shows that this is an inherent efficiency limitation of a fuel burning combined cycle, and suggests the now well known combined cycle as the method of obtaining the highest practical efficiency.

The alternative method for explaining the advantage of a combined cycle is that the internal combustion engine or Brayton cycle is good because it utilizes high temperature heat well but bad because it exhausts at a moderately high temperature. In contrast the steam Rankine cycle is bad because it degrades heat over a substantial temperature difference in the boiler, but is good because the condenser rejects low temperature heat at near the temperature of the ambient river. Thus, the combination of the two cycles via an interfacing heat exchanger provides about 50 % more power than either cycle separately.

This method also demonstrates important concepts for the thermodynamics students that are not or are not well presented in the text books. Books present the Carnot cycle efficiency but they should also present the Wicks cycle efficiency with the explanation of the conditions for each of these idealized cycles should apply.

*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright 2001, American Society for Engineering Education*

#### Reference:

1. Frank Wicks, “The Thermodynamic Theory and Design of an Ideal Fuel Burning Engine”, Intersociety Energy Conversion Engineering Conference, Volume 2, pp 474-481, Boston, 1991.  
*Proceedings of the 2001 American Society for Engineering Education Annual Conference & Exposition  
Copyright 2001, American Society for Engineering Education*
2. Frank Wicks, “A Combined Cycle with Gas Turbine Topping and Thermodynamically Ideal Gas Turbine Bottoming”, Intersociety Energy Conversion Engineering Conference, pp 3.43-3.49, San Diego, 1992.
3. Frank Wicks, “Synthesis and Evaluation of a Combined Cycle with no Steam nor Cooling Water Requirements”, Intersociety Energy Conversion Engineering Conference, pp 2.105-2.110, Atlanta, 1993.
4. Frank Wicks, “2<sup>nd</sup> Law Heating Systems and the Development of an Electricity Producing Condensing Furnace, ASEE Proceedings, pp 2019-2026, 1994.