Redesigning the Calculus Curriculum for Engineering Students

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Introduction

This paper provides an overview of year two in our calculus redesign project, marking the second step in a three-year plan to make over the calculus sequence. In our previous paper [0] we discussed the motivation for the Calculus curriculum redesign at our institution, as part of a greater effort to improve and innovate the calculus sequence in many universities in the US. The Mathematical Association of America, MAA, with support by NSF, has published several reports [1], [2], [3], on Calculus curriculum renewal across the first two years in college. The leading trends are clustered in three areas:

- Calculus re-sequencing [4], [5]
- Active learning methods [2], [6], [7], [8]
- Applications from Engineering & Sciences [9], [10], [11], [12].

Based on these suggestions, our goal is to create three "Engineering Math" tracks, each tailored to different skill levels. The hope is that careful customization will enable all students to complete the calculus sequence (including both single-variable and multivariable) in two semesters. These tracks are named *Core* Engineering Math I and II, Engineering Math I and II, and *Honors* Engineering Math I and II.

The *Honors* track was launched in 2016-2017, and is currently in *Phase Two*, semester two of its second year. The *Core* track is in *Phase One*, the second semester of its first year of implementation. As an ongoing project, we continue to investigate the impact of the redesign on student learning. In this paper, we present the main features of the *Core* track implementation and examine some preliminary results. Our results include a comparison of student performance on common final exam problems taken by students in both the traditional sequence and also the Core and Honors sequences. Furthermore, the changes and improvements made for the *Honors* track will also be discussed.

Inspiration for the Core track

Our university's school of engineering assumes that most (if not all) incoming students should begin with Calculus II. In addition, there is an increasing number of students who begin their college math with multivariable calculus. In the Fall 2017 semester, less than 15% of our incoming students began with Calculus I. As a result of this trend away from Calculus I, most of our degree programs now assume that students complete calculus by the end of their first year. Unfortunately, much of the second year schedule is structured around this assumption, which can create difficulties for students that took Calculus I in the fall semester of the previous year. (Note that these students must take three semesters of calculus and are therefore unable to complete the calculus requirement by the end of their first year.) These systemic forces result in an unhealthy dynamic that generates many negative feelings and poor decisions: (1) incoming students that enroll in Calculus I are often afraid of falling behind their peers, while (2) many incoming students that struggle with Calculus II will not move to Calculus I because they are also afraid of falling behind.

It is our opinion that many more of our students would benefit from beginning with Calculus I rather than Calculus II. Unfortunately, this is not likely to happen any time soon, for many of the reasons described above. As a consequence, we have responded to the problem by developing a new two-semester sequence for students needing to begin with Calculus I. We hope to eliminate the worries/pressures related to “falling behind their peers” and even some of the stigma that students can sometimes feel. (Engineering school can be a very competitive place!)

This new sequence includes the most essential elements of single-variable and multivariable calculus. We have kept many topics often skipped in high school (e.g., hyperbolic trigonometric functions, related rates, optimization, Newton’s method, work, hydrostatic force), but we have also scaled back by eliminating some less essential topics (e.g., trigonometric substitution). In addition, many topics have been streamlined. For example, our coverage of trigonometric integrals has been reduced to the forms that students will encounter most in their future engineering courses, and we cover infinite series with a focus on applications of power series. In this context, convergence tests are treated as a means to an end, not an end in itself [13] [14].

The Main Features of the Core Engineering Math Implementation

The Core Engineering Math track is intended for students with a strong pre-calculus background and some exposure to calculus, but who are not quite ready for Calculus II. Prior to this year, these students would have taken Calculus I in the first semester and ultimately needed three semesters to complete the calculus sequence. Calculus I does not count as a required math course, and it is not even considered a pre-requisite for Calculus II, so students have to treat it as an unrestricted elective. With the new Core Engineering Math sequence, students can complete single-variable and multivariable calculus in two semesters (one four-hour course each semester).

How do we cover the same material in less time?

- Our traditional Calculus I course begins with three weeks of pre-calculus review, but the Core sequence skips pre-calculus entirely. We are able to do this because we recruited students for the Core track based on performance on a pre-calculus assessment. We created this pre-assessment over the summer and administered it to all first year students during the first week of class.
- We also streamlined the coverage of integration techniques (for example, trigonometric substitution is omitted). The coverage of sequences and series was also adjusted. Now, applications and approximation methods are emphasized more than convergence tests.
- Our traditional calculus sequence covers parametric equations prior to sequences and series at the end of Calculus II. Then vectors are introduced at the beginning of Multivariable calculus. In the Core sequence, parametric equations are moved to the second semester and covered immediately before vectors, resulting in a more seamless transition.
- Finally, the traditional four-hour courses include a number of “discussion” sessions which allow for student questions. The Core sequence contains far fewer discussion sessions and expects students to attend office hours more frequently.

Cause for Concern

The final exams for Calculus I and Core Engineering Math I included 17 common problems. In order to maximize grading uniformity, each common problem also received a common grader.

We hoped to demonstrate that students in the Core track would perform just as well in comparison to the traditional track. In Figure 1, we show the results for the topics where the Core track and the traditional track differ by more than 10 points. As you can see, when there is a large difference, it is the Core track that generally performed better. An exception is the implicit differentiation topic. Figure 2 shows the results for the other common problems where the difference between the traditional track and core track is within 10 points

We are pleased to have learned that students in the Core track are able to succeed within this new two semester format. However, averages for students in the traditional Calculus I track reinforce our concern that most of them will not excel in a streamlined two semester calculus sequence. The traditional Calculus I course covers less material at a slower rate, but the students are less able to master the material when compared to students in the Core track. In comparison, the first semester of the Core track covers all of the Calculus I material, plus integration techniques, some applications of integration, and sequences and series

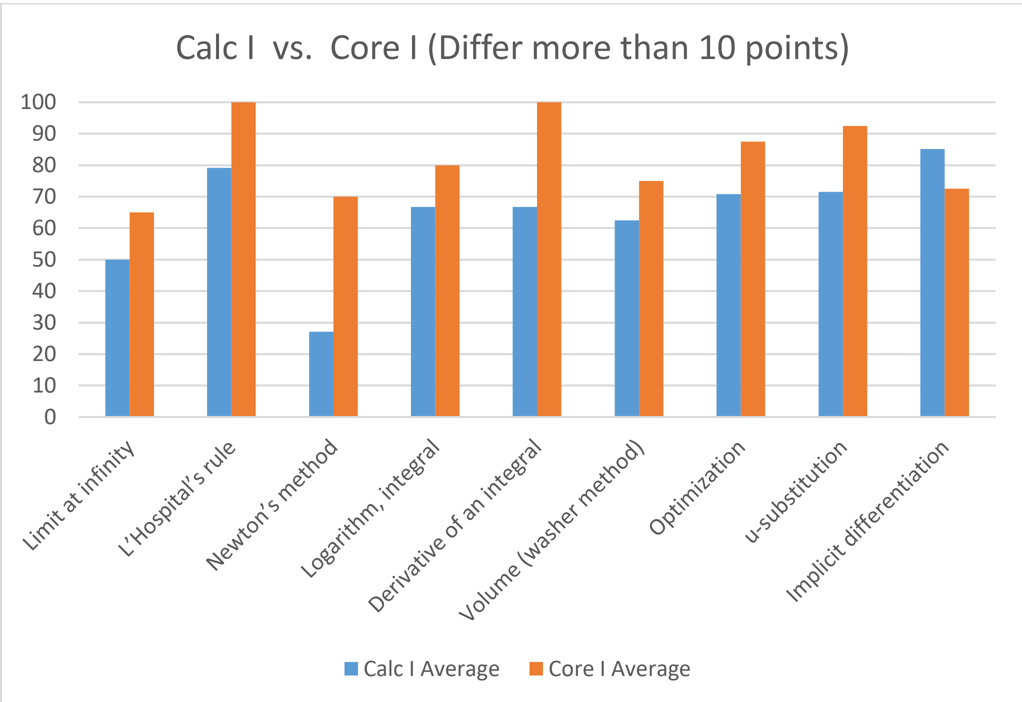


Figure 1

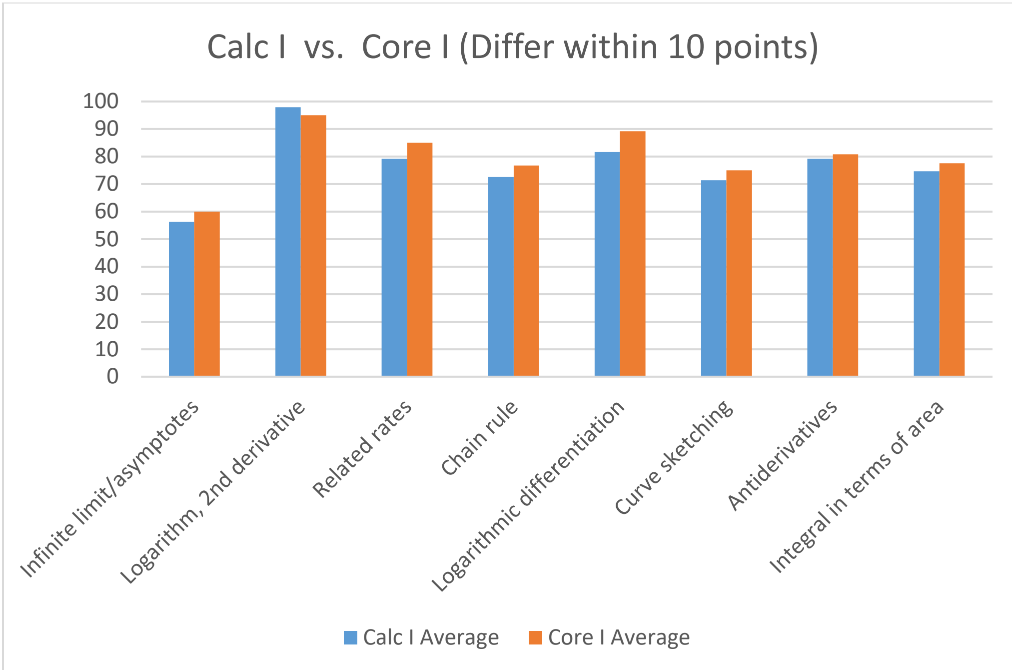


Figure 2

Improvements and Changes for the Honors Engineering Math Track

The Honors Engineering Math track is intended for students with the strongest math background who would typically begin with Multivariable Calculus. These students have successfully completed Single Variable Calculus II through an AP Calculus BC course (with a score of 5 on the AP exam) or through another college. The Honors track is a two-semester calculus sequence with three primary goals:

- Address common gaps in Calculus I and II skills.
- Cover Multivariable calculus topics more thoroughly than possible in a single-semester course.
- Cover additional challenging topics and projects.

We reported on Phase I of the Honors Engineering Math implementation in 2017. Honors Engineering Math is now in the second semester of Phase II. Here we provide an update on what we have learned.

The structure of the course remains similar to the first year [1-our paper]. Several changes have been incorporated based on students' feedback such as:

- Reducing the group size for collaborative group work from four to three. Students in larger groups feared falling behind in the discussion when not part of the "main line of thought".
- Adding a written homework component to help students strengthen their skills in communicating mathematics.

- Devoting a portion of class time every week for students to discuss group projects (in addition to the time spent outside of the class).
- Employing more student presentations in class.

Another important change made this semester has been to ensure all assessments match the teaching approach. For example, all exams consist of three parts: (1) concept questions, (2) essential problems and (3) application of ideas. Deeper conceptual understanding is helping students to tackle unconventional problems and apply what they learned in real-world problems.

To measure whether the changes have made a positive impact, we compared the students' ratings for the class's impact on their skills and attitudes. In both the Fall 2016 and Fall 2017 post-course surveys, the response rate was 19 of 26 students. The following results (Figure 3) show a clear improvement in all areas. In particular, "appreciating different perspectives", "communicating mathematical ideas in writing with clarity" and "persistence in solving problems" were the areas where we see the most improvement.

1: no gains	2: a little gain	3: moderate gain	4: good gain	5: great gain

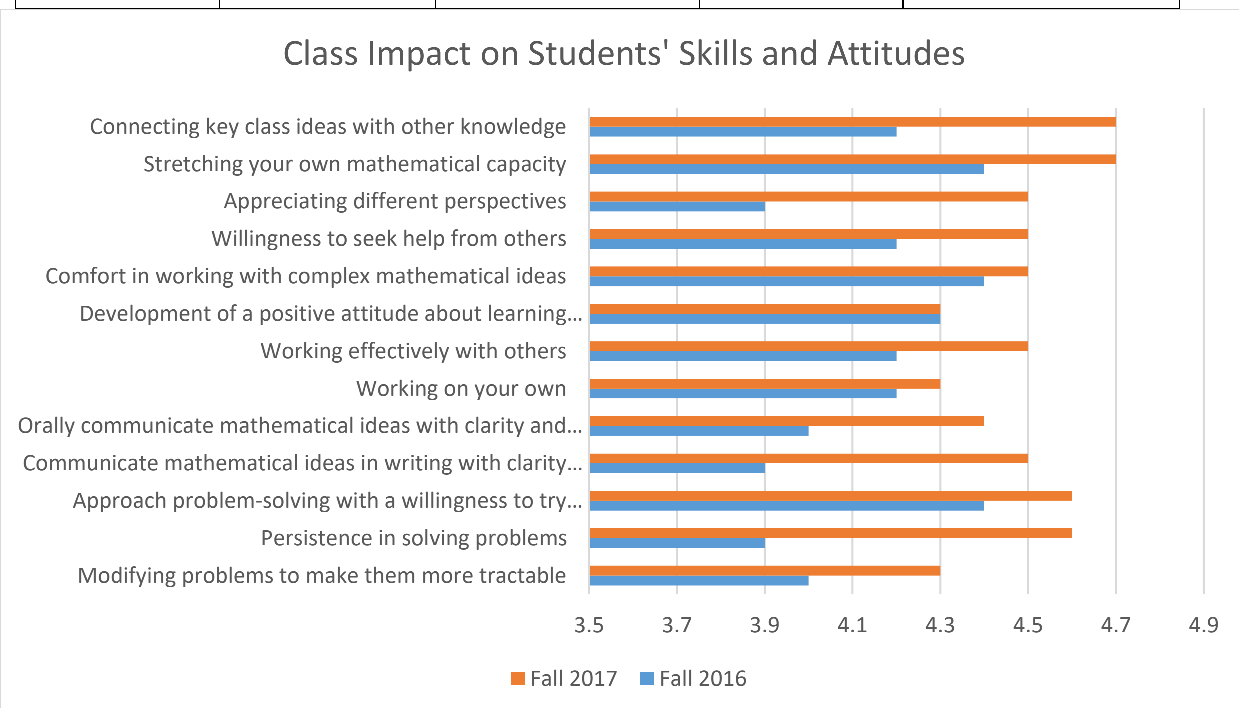
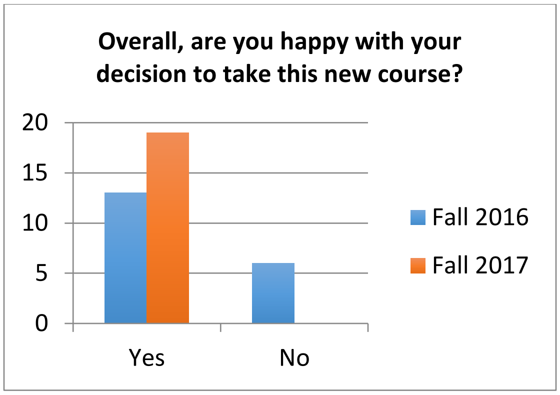


Figure 3

We also asked the following two questions in both Fall 2016 and Fall 2017. In Fall 2016, 13 out of 19 students indicated they were happy with their decision to take this new course, while in Fall 2017, all 19 students said they were happy with their decision and would recommend this class to other students.



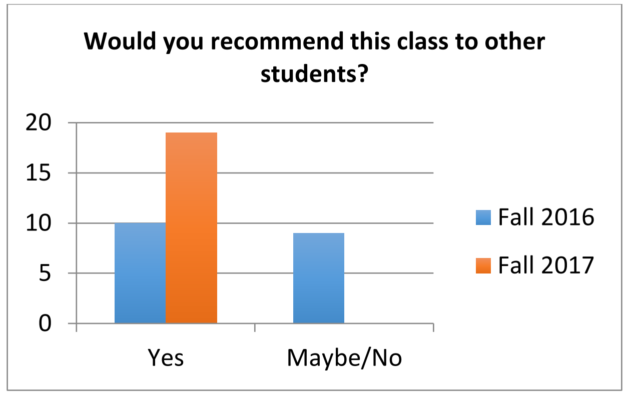


Figure 4

Figure 5

Despite the positive improvements we observed in the overall class impact from the student perspective, the question remains: Does our redesign of the curriculum and use of active learning increase student performance? In Fall 2017, we created some common core multiple-choice "concept questions" for use in our calculus final exams. The following table lists the average scores for all the common topics shared by the various calculus classes. (Calc I, Calc II, and MV Calc indicate the three semesters of the traditional calculus sequence, while Core I is Core Engineering Math I, and Honors I is Honors Engineering Math I.)

Topic Covered:	**Calc I**	**Calc II**	**MV Calc**	**Core I**	**Honors I**
Newton's method	27.1			70.0	92.3
Simpson's rule		54.4		30.0	76.9
partial fractions		98.9		100.0	100.0
integration by parts		86.8		50.0	88.5
hydrostatic force		52.1			73.1
divergent series		90.6		80.0	76.9
Taylor polynomial (cos)		53.9		25.0	46.2
Taylor polynomial (root)				50.0	76.9
dot and cross product			89.7		90.4
quadratic surfaces			89.1		90.8
vector equation of a line			87.9		92.3
partial derivatives			90		92.3

Below we show visually how students performed on the set of common final exam problems.

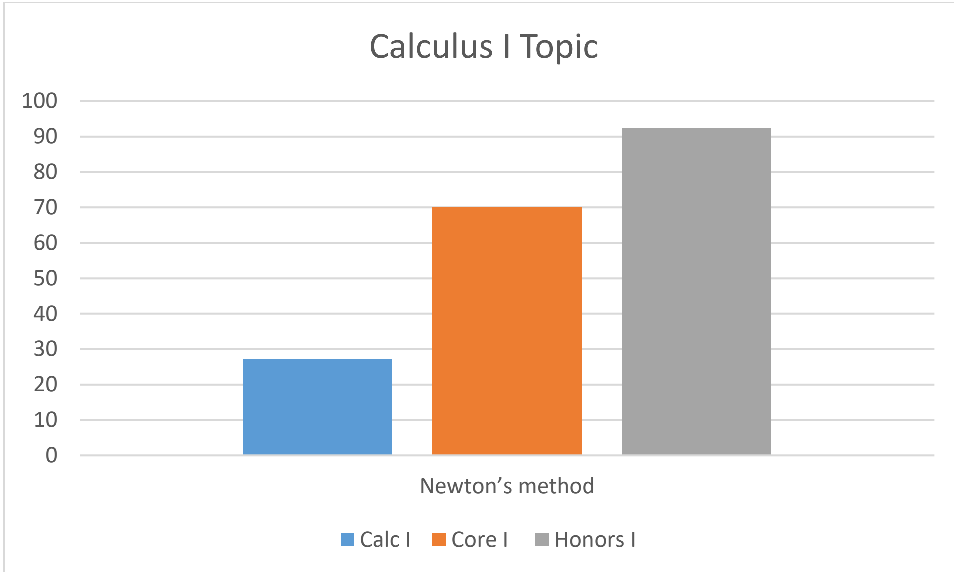


Figure 6

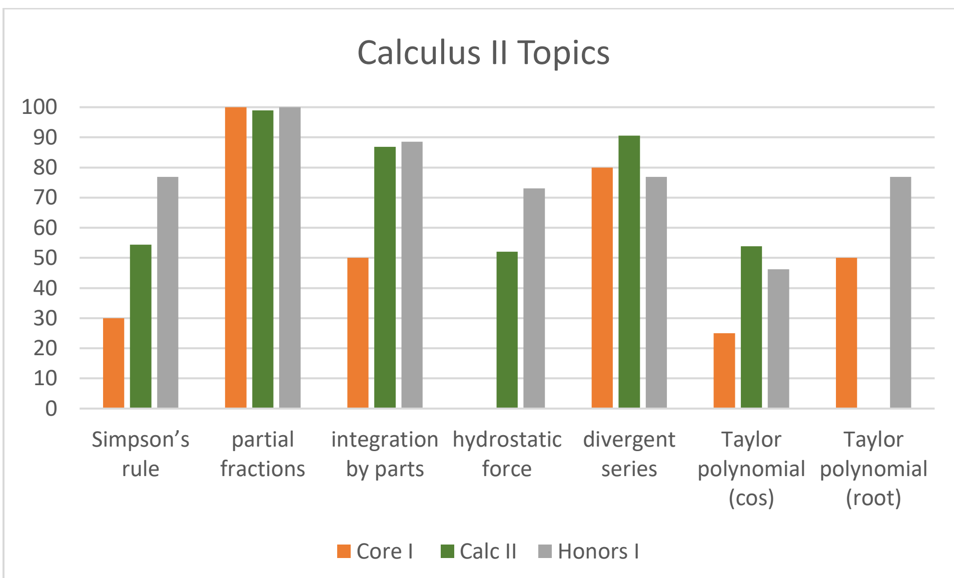


Figure 7

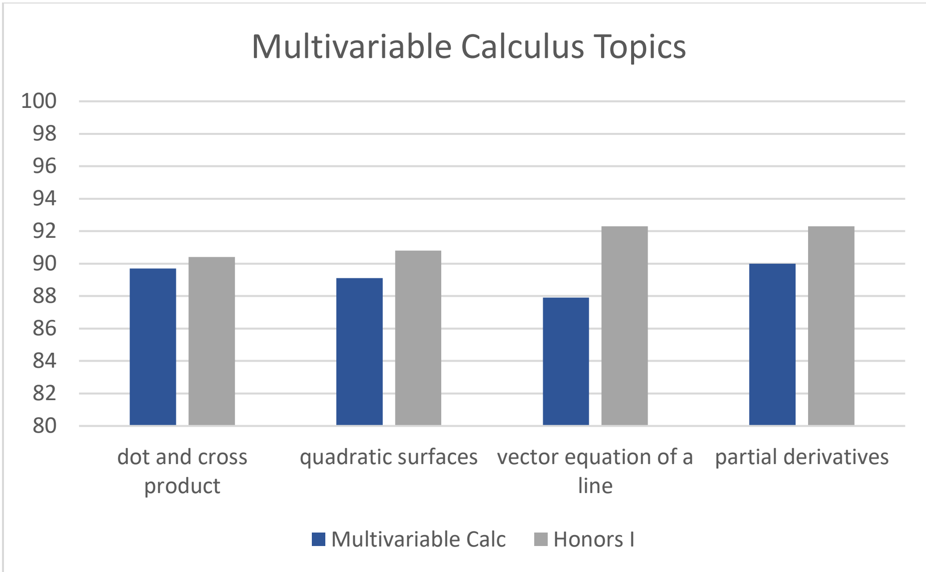


Figure 8

In Figures 1, 2, 6, 7, and 8, we observe that

- Honors track students are best prepared in general, and tended to perform better than the other tracks for the common problems in all areas.
- Core track students are generally better prepared than the traditional Calculus I students and performed better for the Calculus I topics.
- In general the preparation of the Calculus II students lies between the Core track and the Honors track and their performance on the Calculus II topics tended to lie between as well.

It is important to note that the students who participated in the Honors Engineering Math sequence in the previous academic year (2016-17) did not experience a dip in their overall GPA from the first to second semester that is typical of our first-year students. This difference is statistically significant. Our theory is the students' improved academic performance is due to their remaining together as a cohesive and highly functioning unit for both semesters.

Conclusions

Although we are pleased with the outcomes for our students in our two new tracks, it remains to be seen if we can successfully implement all aspects of our plan. For example, it is unlikely that we will be able to completely eliminate the three-semester sequence for the least-prepared. Also, our institution is considering other changes to the curriculum that might impact the feasibility of maintaining several different tracks.

Our project was intended to better meet the needs of engineering students at our institution. To a large degree, performance is related to preparation. We have found benefits and successful outcomes when students are well-matched for both the Core and the Honors tracks based on their preparation. Perhaps one of our most important takeaways is that small classes of first year students that remain together for both semesters can create a bond that benefits the students both in calculus and in their other engineering classes.

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