2018 ASEE Annual Conference & Exposition, June 24–27, 2018, Salt Palace Convention Center, Salt Lake City, UT

Paper ID #21931

Statics Modeling Kit: Hands-On Learning in the Flipped Classroom

Mr. Eric Davishahl, Whatcom Community College

Eric Davishahl is faculty and engineering program coordinator at Whatcom Community College. His teaching and research interests include developing, implementing and assessing active learning instructional strategies and auto-graded online homework. Eric has been a member of ASEE since 2001. He currently serves as chair of the Pacific Northwest Section and was the recipient of the 2008 Section Outstanding Teaching Award.

Mr. Russell Pearce, Whatcom Community College

Russell Pearce has worked at Whatcom Community College since 2012, giving lab support to physics, engineering, and geology, as well as teaching the occasional physics class. Russell's interests include creating hands on learning tools that encourage active engagement for Physics and Engineering students, and developing classroom demonstrations that confront student misconceptions and create classroom conversation.

Todd R. Haskell, Western Washington University

Todd Haskell is a cognitive scientist interested in learning and the development of expertise, especially in STEM fields. He is currently Associate Professor of Psychology at Western Washington University. In previous projects Dr. Haskell has worked on understanding how chemistry novices and experts navigate between macroscopic, symbolic, and small particle representations, and how pre-service elementary teachers translate an understanding of energy concepts from physics to other disciplines.

Mr. Kordel J. Clarks, Student Representative - Whatcom Community College

Kordel Clarks is an Engineering Student who plans to graduate from Whatcom Community College in June of 2018 and plans to transfer to the University of Washington. He is pursuing a degree in aeronautical engineering. He is also a recipient of WCC's SEECRS (STEM Excellence through Engagement in Collaboration, Research and Scholarship) program and a member of Washington's All-Academic Team.

Statics Modeling Kit: Hands-on Learning in the Flipped Classroom

Abstract

The Statics Modeling Kit is a low cost and flexible modeling system designed to support active learning in engineering statics. The kit consists of a physical model representative of a three-dimensional Cartesian coordinate system constructed from plastic pegboard panels along with a collection of components that students use to build, manipulate and analyze models of textbook-style homework problems. Student groups use the kit to explore statics concepts through a series of exercises that connect typical mechanics analysis tasks such as sketching free-body diagrams, mathematical manipulations in vector notation, and numeric computations to physical representations that students can explore to develop their conceptual knowledge. The pedagogy underlying these activities applies the theory of representational competence to provide learning experiences that target conceptual understanding within a problem-solving context. The design of the kit renders most dimensions and select force types readily apparent by inspection, allowing students to focus their time and mental effort on interpretation, application of relevant statics concepts, and analysis. Modeling activities in this flipped classroom implementation support the majority of statics topics including vector operations, concurrent force systems, moments, equivalent systems, support models, rigid-body equilibrium, and friction, all with an emphasis on three-dimensional geometries.

Student feedback on the modeling exercises indicates that the models and associated curriculum provide an engaging context for group discussion and problem solving. Students report their experience with the physical models as supportive of skill development in visualizing vectors, understanding vector notation, and interpreting three-dimensional geometry information communicated by traditional textbook-style problem figures. Many students cite the modeling activities as key to developing their understanding of fundamental statics concepts such as free-body diagrams, moments and support models. Small increases in class time allocation and completion incentives over two successive terms that further leveraged the modeling curriculum as part of the overall course design resulted in significant increases in student survey responses regarding the effectiveness of the activities. Classroom sessions that feature the modeling kit feature lively discussion within student groups and provide numerous teachable moments for the instructor to use a model to demonstrate and explain a key concept or nuance to small groups of students.

Introduction

Statics instructors would like their students to develop an understanding of concepts such as vectors, forces, free-body diagrams, and moments that are prerequisite for success in follow-on courses and fundamental to engineering practice in several disciplines. We work to help our students learn to apply these concepts appropriately in various problem solving and design contexts. Streveler [1] summarizes the importance of conceptual knowledge to engineering problem solving and identifies conceptual knowledge as “critical to the development of competence in engineering students and in practicing professionals." Our work to design learning activities that emphasize conceptual knowledge; however, can run counter to students' desire to focus on reproducing problem solving procedures presented to them in worked examples by the instructor and/or in the textbook. Litzinger [2] examined student analysis strategies and found that even the highest-performing students do not consistently apply conceptual knowledge within their problem solving strategies, instead relying on memorized relationships and procedures. Steif [3] also identifies the tendency of students to rely on memorization and algorithmic problem solving in statics. Students may perceive problem solving methods as separate from conceptual knowledge and perhaps more immediately relevant to their ability to complete homework problems and perform on exams.

Prior Work

Physical models are widely regarded as a useful focal point for student engagement in active learning, an instructional strategy that leads to learning gains across STEM disciplines [4]. Several authors including [3], [5], [6], [7], [8], [9] have identified hands-on learning with physical models and manipulatives as a useful approach to address gaps in conceptual understanding and serve other purposes in the mechanics classroom. These authors cite the potential of manipulatives and modeling activities to help students feel and visualize force interactions, link theory to students' prior knowledge, practice with modeling assumptions, and provide context to develop other engineering skills such as design and measurement. The growing inventory of activities on the hands-on mechanics website (www.handsonmechanics.com) [8] along with the popularity of the Hands-On Demonstrations session that the Mechanics Division sponsors at the ASEE Annual Conference serve as further evidence of this trend.

Theoretical Basis

The theory of *representational competence* provides a useful framework for considering how experiences with physical models can support students' development of conceptual knowledge. Representational competence refers to the fluency with which a subject expert can move between different representations of a concept (e.g. mathematical, symbolic, graphical, pictorial, etc.) as appropriate for learning, communicating or problem solving [10]. This fluency contrasts with the tendency of novice learners to compartmentalize knowledge and limit their use of representations to the specific contexts in which they are introduced. There are several applications of this construct in the science education literature. Steiff [11] identifies representational competence in chemistry as important to developing true conceptual understanding and as key to knowledge transfer across contexts. Klein [12] finds that high performing physics students "used representations consistently and changed flexibly between different representations. In contrast, low performing students failed to incorporate representational strategies in their problem solving approach." Pande [13] identifies representational competence as a marker of domain expertise across multiple STEM disciplines and suggests the development of a cognitive model for how students develop this attribute as important for further research. Pande notes that the representations used to think about conceptual knowledge can be internal (e.g. visualizations, thought narratives) or external (e.g. drawings, formulas, physical models). This review suggests that external representations such as physical models provide two main benefits: (1) they offload some cognitive demands to the external world, and (2) they create opportunities to think in ways that might not be possible using internal representations alone.

We propose applying the construct of representational competence to how we think about developing conceptual knowledge in statics instruction. Contemporary statics instruction is replete with multiple external representations such as photorealistic figures, free-body diagrams, formulas, numbers, physical models, computer simulations, and narrative. Figure 1 illustrates multiple representations used to think about the force interaction at a pin support.

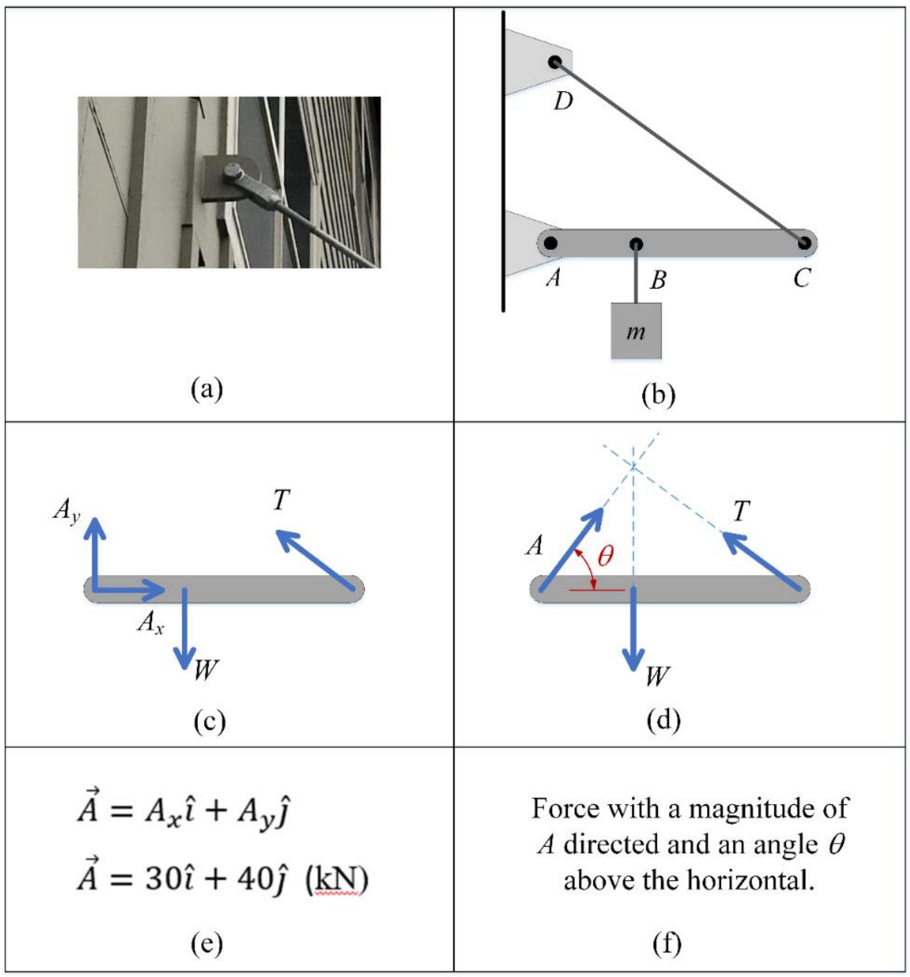


Figure 1. Examples of multiple representations used to consider the force reaction at a pin.

These representations include: (a) a photo of a structural connection appropriately modeled as a pin; (b) a problem figure with a schematic representation of a pin support; (c) a free-body diagram showing the reaction exerted by the pin in Cartesian components; (d) an alternate free-body diagram showing the pin reaction as a vector that forms a concurrent force system with the forces *T* and *W*; (e) the symbolic and numeric mathematical representations of the Cartesian components of the reaction; (f) a narrative description of the pin reaction. When Steif [3] asserts that a "deep understanding of Statics lies in being able to relate the symbols (forces and couples) to the interactions between bodies which they represent…," he is expressing this idea of representational competence as a key indicator of conceptual understanding.

Steiff [11] articulates the potential of physical models "to improve students' skills related to identifying information implicit in two-dimensional diagrams, translating among representations, and predicting the effect of spatial transformations." Physical models offer a useful external representation because they offer multiple modes of interaction with a concept (e.g. tactile manipulation, visual from multiple perspectives) and can serve as an anchor for students to use in correlating multiple abstract representations.

Our Approach

The current project follows the intervention recommendation articulated by Litzinger [2] that learning activities targeting conceptual knowledge may be effective at building that knowledge along with complementary analysis skills if embedded within a problem-solving context. We apply the construct of representational competence by guiding students through the application of multiple abstract representations as they work with physical models to complete problem-solving oriented tasks. We hypothesize that introductory-level problems (e.g. the problem figure shown in Figure 1b above) are the appropriate target for these model-based activities because they integrate efforts to deepen conceptual understanding with quantitative analysis similar to what students will practice in homework. These problem-types are free of the higher-level cognitive demands of more difficult problem solving or modeling assumptions associated with real world applications. We have developed a flexible modeling system we call the "Statics Modeling Kit" (abbreviated SMK) that can be adapted to a variety of problem types that we use to implement this pedagogy.

Consider the generally accepted learning progression for developing mechanics concepts as used by Hibbeler [14] to contrast how our approach fits in to the learning process compared to prior work. This learning progression consists of four stages summarized as follows:

1. The text motivates a topic with discussion and photos of relevant real-world applications.
2. The text formally develops and/or derives relationships with abstract diagrams, symbols and mathematical manipulation.
3. The presentation proceeds to basic applications with introductory problems and examples using abstract representations similar to the example shown in Figure 1b.
4. The text uses more complex systems and structures to develop higher-level analysis skills and to integrate new concepts with prior topics, often with more concrete (less abstract) representation of real world applications.

Prior work using physical models as described in [3], [5], [6], [7], [8], and [9] falls loosely into three categories: (a) demonstrations or activities designed to communicate relevance and/or excite students about a new topic; (b) manipulatives that isolate a concept outside of a problem solving context; or (c) models of real-world objects and structures. The first two categories (a and b) generally support the beginning of the learning progression (stages 1 and 2). The last category of activities target more advanced concept applications and support the end of the learning progression (stage 4). The SMK approach described here introduces a fourth category of model-based learning activities that targets stage 3 in the learning progression.

We use the SMK to construct models that are physical embodiments of the introductory problems (e.g. the example above in Figure 1b) in which modeling assumptions (support models, simplified geometry abstractions) are already made, thus targeting stage 3 of the learning progression and integrating this practice with an emphasis on developing basic concepts using multiple representations. The rationale, inspired by the science education literature on representational competence discussed above, is to provide students physical representations of the abstractions used to construct these problems so they can develop a better conceptual understanding of these systems rather than just memorizing problem solving procedures. As implemented and described further below, the SMK activities generally provide the first example application(s) of the relevant concept(s) after students encounter new topics through pre-class reading assignments. Students can directly relate these models to the problems they see in examples in the reading and homework assignments.

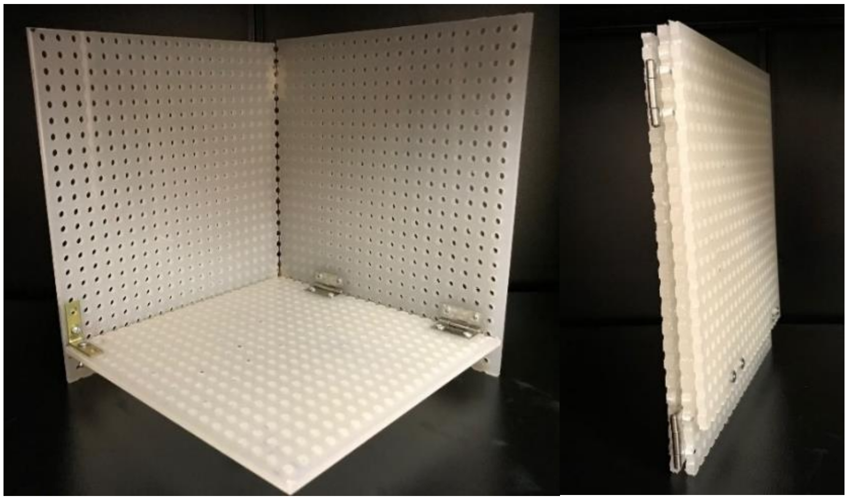
The SMK approach offers several other benefits that can facilitate the inclusion of physical modeling activities in any statics classroom environment. The system has relatively low cost, requires little storage space, and is portable. The design of the system facilitates geometry and force measurements without the need for instrumentation. In addition to reducing the overall cognitive load of the activities, this feature of the SMK makes for efficient use of class time because students can gather most system dimensions by inspection rather than taking measurements. The use of a consistent set of components in the approach throughout the term further lowers class-time overhead because students do not need to become familiar with new equipment for each new activity. Lastly, we designed the kit to facilitate quick modification of the models by the students or by the instructor as they engage in conceptual exploration.

Statics Modeling Kit

SMK Hardware

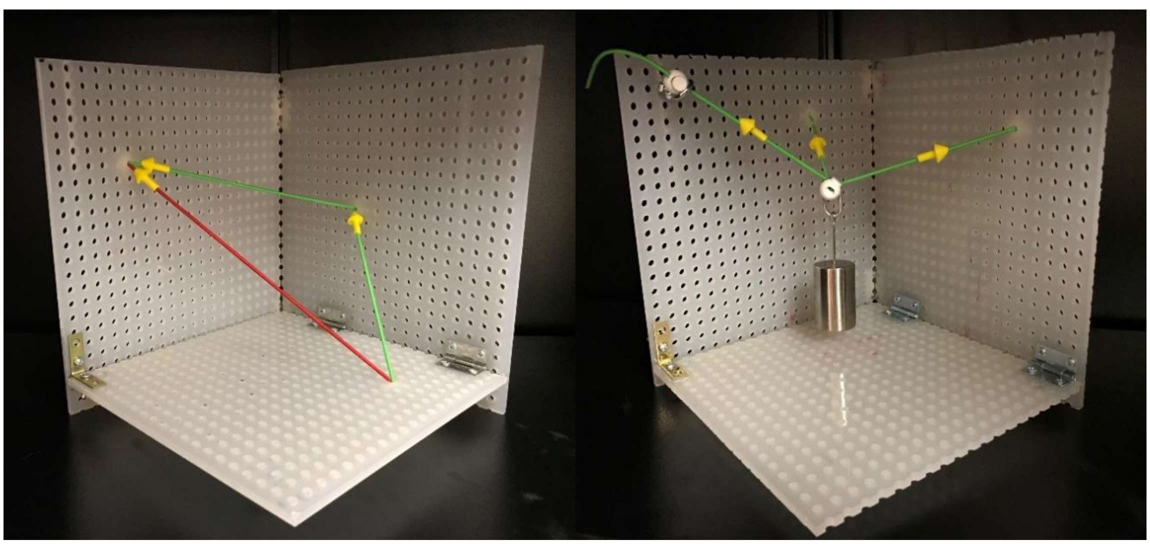
The SMK consists of a set of three orthogonal pegboard panels (see Figure 2 on the next page) along with a collection of 3D printed parts, elastic cord, and standard hardware components. We construct models of statics problems from the hardware and 3D printed parts and install them on the pegboard. The orthogonal panel arrangement serves as a physical embodiment of a 3D coordinate system with the panels modeling the three Cartesian planes and the inside corners representative of the coordinate axes commonly provided for a 3D homework problem. The panels are cut from polypropylene perforated sheet with 1/4-inch diameter holes on a 1/2-inch grid that facilitates quick measurements of dimensions of the models. The material is available in 48 inch × 32 inch × 1/4 inch sheets through Grainger (www.grainger.com). The use of plastic panels allows students to write on the model with standard dry-erase whiteboard markers. Students can read the spatial coordinates of any connections to the panels by counting the holes and record those coordinates directly on the model. Hinges connect the planes in an arrangement that folds into a compact size (approximately 10.5 inch × 12 inch × 2 inch) for ease of transport and storage when not in use as shown in Figure 2b.

Elastic shock cord and cord locks provide a convenient representation of vectors. Paracord Planet sells the 1/8 inch diameter cord in 50-foot lengths in a variety of colors through Amazon.com. We designed one-inch long arrow “beads” that students clip to the cords to indicate direction and represent Cartesian unit vectors. Students can easily adjust the tension in the cords to develop a feel for the magnitudes of the forces involved in their analysis. Figure 3 illustrates this approach with two example models. Figure 3a depicts a vector addition model used during the first week of class with exercises we describe in detail below. Figure 3b shows a 3-D concurrent force system that students analyze in week 2.



(a) **(b)**

Figure 2. The Statics Modeling Kit platform - open for use (a) and folded for storage (b).



(a) **(b)**

Figure 3. (a) 3-D vector addition activity (SMK1) from week 1. (b) 3-D concurrent force system activity (SMK2) from week 2.

Table 1 summarizes the parts, vendors, and costs required for the construction of eight coordinate system models with associated cord and locks. The total cost comes to $303.23, or $37.90 per model not including tax and shipping.

Table 1. Parts list for construction of the pegboard system that serves as the SMK platform.

Item	Dimensions	Price/Unit	QTY	MFGR	Vendor	Cost
Polypropylene perf sheet	48"x32"x0.25"	$79.00/sheet	2	Direct Metals	Grainger	$158.00
Hinges	1.5"	$2.34/(2-pack)	16	Everbuilt	Home Depot	$37.44
Corner braces	1-1/2"	$2.67/(4-pack)	2	Everbuilt	Home Depot	$5.34
Flat head machine screws	#10-24 x 1/2"	$1.18/(8-pack)	10	Everbuilt	Home Depot	$11.80
Wing nuts	#10-24	$1.18/(4-pack)	4	Everbuilt	Home Depot	$4.72
Shock cord	1/8"	$9.99/ (50' of Cord)	3	Paracord Planet	Amazon	$29.97
Cord locks	4mm hole size	$13.99 (Pack of 20)	4	Paracord Planet	Amazon	$55.96
			Total cost for 8 pegboard systems			**$303.23**

Development of the modeling activities we describe below involved additional purchases of readily available and inexpensive components such as S-hooks and small carabiners from the local hardware store. We designed models of objects, pin supports, ball-and-socket joints, and other structural modeling components using CAD (Onshape) and printed them in ABS plastic on a Stratysys F170 3D printer.

SMK Curriculum and Pedagogy

We introduced the first SMK activities into the fall 2016 and winter 2017 sections of our statics course. This initial implementation was a period of continual adjustments based on informal feedback from students and instructor observations. We also developed and introduced new models and activities during this rollout period. The most recent course offerings in fall 2017 and winter 2018 featured an identical set of models and activities though with some slight modifications in implementation that we discuss in the Classroom Implementation section below. The fall 2017 and winter 2018 implementations are the source for the data analyzed later in this paper and featured SMK activities using five different models with concept emphasis summarized in Table 2 on the next page. Note the topic coverage generally follows the conventional sequence adopted by most statics textbooks.

Worksheets prompt students to draw diagrams, define variables, perform calculations and engage in tactile manipulation of the models to emphasize key concepts and express those concepts using multiple representations. Following is a description of how we use the SMK1 and SMK4 models as examples of the pedagogy intended to support students' development of representational competence. We use SMK1 on the second day of class to introduce 3D vectors along with the SMK system itself. We supply each student group with the pegboard system with a single vector installed similar to the red vector shown in Figure 3a. The associated worksheet includes a pictorial view of this same vector as shown below in Figure 4. In addition to providing a second representation that students can correlate to their model, the pictorial view communicates the axis orientation students will use for calculations.

Table 2. Summary of SMK activities for fall 2017 and winter 2018.

Model	Description	Concepts Emphasized
SMK1	Vectors represented by elastic cord and arrow beads (See Figure 3a).	Position vectors Vector addition Cartesian components Unit vectors Coordinate direction angles Tension in ropes and cables Force pairs (Newton's 3rd Law)
SMK2	Mass suspended from three cords (See Figure 3b).	Free-body diagrams Tension in ropes and cables Concurrent force system Unit vectors Force equilibrium
SMK3	Cross-shaped beam with hanging masses supported by a ball-and-socket connection and two cables	Moments and right hand rule Interpreting moment vector components Couples and static equivalence 2D vs 3D representations of force system Moment equilibrium
SMK4	Beam loaded with hanging masses and supported by a pin support and an extension spring (See Figure 5).	Spring forces Support models Couple moment reactions in a support Free-body diagrams 2D rigid body equilibrium Three-force members
SMK5	Platform supported by a hinge and a cord with a (potentially) sliding block.	Support models Free-body diagrams 3D rigid body equilibrium Impending motion analysis Slipping vs. tipping

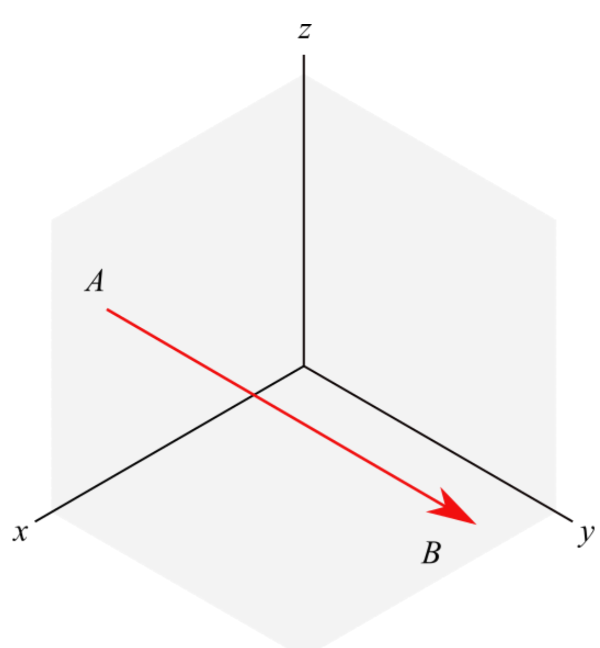


Figure 4. Pictorial representation of the supplied vector model in SMK1.

Each team receives a slightly different vector geometry. We also give students a collection of green elastic cord and locks as well as the unit vector beads. Students complete the following exercises during the first SMK1 session that lasts approximately 30 minutes:

1. Using the pegboard grid to read coordinates for the start and end points of the given vector and writing this position vector ($\vec{\mathbf{r}}_{AB}$.) in Cartesian components.
2. Building a model of the *x*, *y*, and *z* components of $\vec{\mathbf{r}}_{AB}$ with lengths of green cord and using the unit vector beads to show tip-to-tail vector addition.
3. Sketching $\vec{\mathbf{r}}_{AB}$ and the tip-to-tail vector addition of its Cartesian components on an isometric grid in two different axis orientations (*z*-axis vertical and *y*-axis vertical).
4. Removing the green cord that models the vector components and building a new model of the vector addition equation, $\vec{\mathbf{r}}_{AC} + \vec{\mathbf{r}}_{CB} = \vec{\mathbf{r}}_{AB}$, where point *C* is at a location students choose in the *y*-*z* plane. Figure 4a above depicts an example of the resulting model.
5. Sketching this vector addition on given coordinate axes in the worksheet.
6. Computing the magnitudes of $\vec{\mathbf{r}}_{AB}$, $\vec{\mathbf{r}}_{AC}$, and $\vec{\mathbf{r}}_{BC}$ and comparing their results to length measurements of the model vectors.

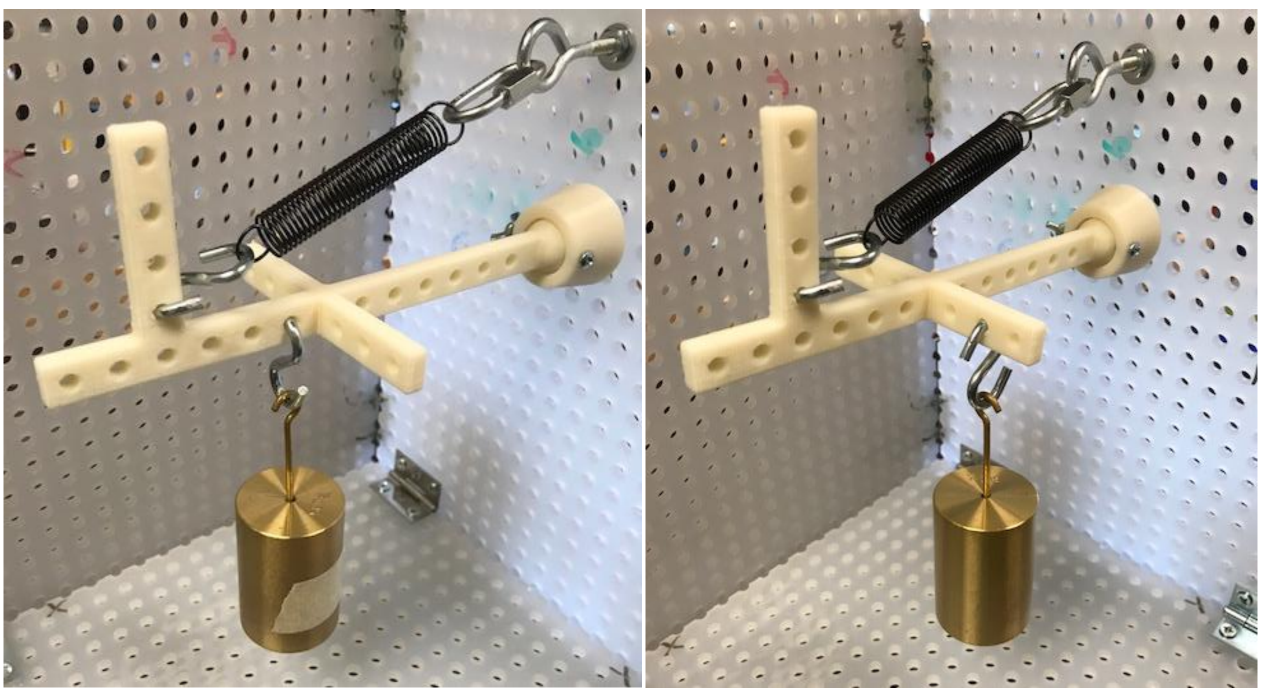
The second SMK1 session takes place on the following class day (the third meeting of the term) and uses this same model to explore multiple approaches for communicating vector direction in 3D. Students also use Cartesian unit vectors determined from position vectors to compute force vectors that represent tensions in ropes and cables. This session features the following activities and requires roughly 45 minutes for most students to complete the following exercises:

1. Computing the unit vectors $\hat{\boldsymbol{u}}_{AB}$, $\hat{\boldsymbol{u}}_{AC}$ and $\hat{\boldsymbol{u}}_{CB}$ and noting how the arrow beads that represent these vectors can move along the line of action while still communicating direction.
2. Assuming the tension in cord *AB* is 3 pounds, computing the force vector in Cartesian components, $\vec{\mathbf{F}}_T$, that represents the tension force in *AB* and expressing this as the force pulling on the attachment at *A*.
3. Writing the tension force vector pulling on the attachment at B, again in Cartesian components.
4. Using the scalar components of unit vector $\hat{\boldsymbol{u}}_{BA} = -\hat{\boldsymbol{u}}_{AB}$ to compute the angles θ_x, θ_y and θ_z that $\vec{\mathbf{F}}_T$ (as acting on *B*) makes with the x, y, and z axes respectively.
5. Using a protractor to measure θ_x, θ_y and θ_z and compare to the calculated result.
6. Sketching $\vec{\mathbf{F}}_T$ on a new set of axes with origin at point B and showing the three direction angles.
7. Computing the spherical coordinate angles that give the direction of $\vec{\mathbf{F}}_T$ acting on *B* and drawing a second sketch communicating the vector direction using this alternate approach.

Note how these activities instruct students to use multiple representations (e.g. narrative, graphical, symbolic, and numeric) of a vector in three dimensions as they work with a physical model to anchor their understanding of what each representation communicates. As described previously, we hypothesize that these exercises will help students develop fluency in using and moving between these various representations to think about statics concepts and apply them appropriately in problem solving contexts.

Figure 5 shows the beam and spring system model identified as SMK4 in Table 2. Students use this model to explore support models and rigid body equilibrium through a series of exercises including the following:

1. Determining whether they should analyze the system in two or three dimensions.
2. Drawing a free-body diagram of the beam and using an equilibrium analysis to determine the spring constant.
3. Diagraming the force system to show that the force reaction in the pin, the force in the spring, and the weight of the hanging mass must intersect at a point and form a concurrent force system.
4. Examining the stability of the system by removing the pin (machine screw and wingnut) in the support such that it behaves as a ball-and-socket.
5. Moving the hanging mass out of the plane of the beam-spring system and analyzing the effects of this change on the reactions developed in the pin support. See Figure 5b.
6. Analyzing the nut and washer connection between the eyebolt and the hole in the perforated panel, proposing the appropriate support model, drawing the free-body diagram, and computing the reactions.



(a) **(b)**

Figure 5. Example model SMK4 for rigid-body equilibrium activities. The photo in (b) shows how students move the mass out of the plane of the beam-spring system and give rise to a couple moment reaction in the support.

Figure 6 depicts the CAD model of a new frame analysis activity we are developing with targeted implementation starting fall 2018. We have included this figure here to illustrate several of the principles guiding our design of these models. This assembly features two pin supports, a pulley, and a pin-slot connection to help students explore the force interactions at connections in a frame. The model can assume multiple configurations and includes several connection points for loads that students will apply to the structure using combinations of elastic cord, hanging masses and/or springs. The design of the model components illustrates the care we have taken to maintain the ease with which students can quantify the geometry by reading markings on the model and/or counting holes that are aligned with the grid on the pegboard panels. This approach aims to minimize the time and cognitive load required for measurements and allow students to focus on conceptual and quantitative analysis. For example, the "2" engraved on the top beam in the assembly indicates that the first vertical mark is exactly two inches from the pin in the support, with subsequent markings along the slot exactly one inch apart.

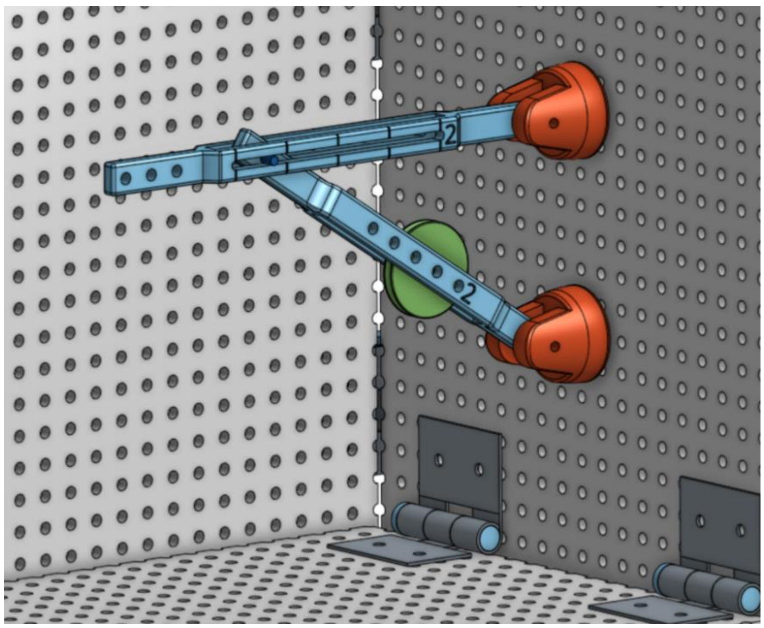


Figure 6. Design for new SMK model for frame analysis with planned implementation in 18-19.

Classroom Implementation

The SMK activities described above account for approximately one third of classroom time on average (the course has three 85-minute meetings per week) during the first six weeks of a ten-week term. We follow a traditional sequencing of statics topics starting with vector analysis and moving to rigid body equilibrium with friction by week six. Class size ranges 20-30 students per section. We divide students into groups of three or four and provide one SMK system to each group. Each student works with their group to complete their own worksheet that guides them through a mix of concept exploration and analysis as described above. These activities occur within a flipped classroom approach that also features mini-lectures and demonstrations, instructor-led example problems, group problem solving on shared whiteboards, and formative assessment using ABCD questions following pedagogy demonstrated by Prather [15]. As we have developed additional SMK activities, we have generally substituted them for the whiteboard problem solving in the overall mix of class activities, thus keeping the overall fraction of class time devoted to active learning approximately constant. Students prepare for each class session by completing an example calculation and reflective writing assignment based on assigned reading from two open educational resources (OERs) [16], [17]. To illustrate this approach we will next describe how the SMK1 activities outlined above fit into the first week of class sessions. The second class meeting begins with a series of ABCD questions assessing student comprehension of the reading reflection assignment on position vectors and Cartesian components. The question prompts provide context for clarifying small group and full class discussion. This series of questions can take up to 45-50 minutes due to the time required to introduce and practice the ABCD question pedagogy and the class discussion stimulated by each question. The balance of class time (30-35 minutes) on the second day is devoted to the SMK1 activities. Students complete a reading reflection assignment introducing Cartesian unit vectors and coordinate direction angles before the next class meeting. The first half (approximately 45 minutes) of this third class session is devoted to the balance of the SMK1 activities covering unit vectors, tension forces, and multiple representations of vector direction in 3D. The second half of this class meeting features two whiteboard problems with more advanced applications of the concepts students explore using SMK1. Note there is no class time devoted to a formal presentation of the material in this flipped-classroom approach. We have found that this heavy time investment in a variety of active learning strategies during the first week of class sets a tone for the quarter promoting a dynamic and collaborative classroom environment that features a mix of student-centered learning activities.

Course assessments include weekly auto-graded online homework assignments, two multi-week team projects (analysis of a bicycle hand brake and design/construction of a balsa wood model truss bridge), two midterm exams and a final exam. The fall 2017 section included weekly 20-minute quantitative quizzes that we removed for winter 2018 to free up some more class time for the SMK activities.

Classroom Dynamics

Class sessions that feature the modeling kit feature lively discussion between students as they work through the modeling activities. One example illustrating the nature of the student discussion is in students' response to a prompt in the SMK4 activity where we ask them whether moving the hanging mass out of the plane of the beam-spring system means that the structure becomes an inherently three-dimensional problem (see Figure 5). Students are generally able to recognize that this change causes a couple moment reaction to develop in the pin support but struggle to identify when this might matter or to use the concept of equivalent systems to explain in detail how this moment reaction is a representation of the contact force interactions between the pin and the hole. Teams rarely come to consensus on these points and seek help from the instructor to clarify when the couple in the support might be significant. The diversity of pace among the student groups seems to allow adequate time for the instructor to work with each group separately right when they are ready to engage deeper in the concept of modeling support reactions, rather than addressing the class as a whole before many students have fully engaged with this aspect of the activity. The presence of the model at each group's table provides the instructor with ready access to useful demonstrations to illustrate the relevant principles. When not being called over by student groups for help, the instructor circulates the room and engages individual groups with leading questions to maintain students' focus on developing conceptual understanding in addition to the mathematical analysis tasks.

Time and Incentives for Activity Completion

As noted above, student groups work at different paces. It is impractical to allocate enough class time for all groups to complete all of the activities, as doing so would leave the faster groups with nothing to do. There were a few differences between the fall 2017 and winter 2018 implementations in how we allotted class time and provided incentives for students to complete unfinished activities outside of official class meetings. For fall, student groups who did not finish the activity were encouraged to return during open lab times that provided access to the models. Our anecdotal observations indicate few students followed this recommendation. Part of the reason may be that students were not required to complete the worksheet for credit. All students received full participation credit (accounting for less than 5% of their overall course grade) for the activities if they were present and engaged. Students also had no verification as to whether their worksheet responses were correct. Students were encouraged to come to office hours for clarification or to have their work checked, but we did not provide example solutions for the worksheets. That said, our observations in working with student groups during class was that, for SMK1 through SMK3, most students were able to accurately perform most of the calculations most of the time and received enough help from the instructor (when needed) during class time to do so. This observation was not true for SMK4 and SMK5. Less than half the class completed these activities during class time and few returned outside of class for additional time with access to the models.

We made a few modifications for winter 2018 to provide increased incentive and more time for worksheet completion. We found increased class time for the SMK activities by removing the weekly quizzes from the course as noted above. The increased class time was particularly important for the activities associated with SMK4 and SMK5, which most students were now able to complete during class. We increased the incentive to complete worksheets by requiring completion for the participation credit, but we did not increase the percentage that this credit contributes toward the overall course grade. This change seemed effective as we observed a notable increase in the number of students coming outside of class to work with the models. We did not check student calculations for accuracy, but we did provide example worked solutions and responses to the conceptual question prompts. We made these available to students through the course learning management system (Canvas) generally 1-2 days after the class sessions that featured the associated activities. We should note here that each student group generally had different numbers for their calculations because we intentionally provided slightly different versions of each model. Our thinking is that this practice encourages inter-team collaboration at a conceptual and symbolic level rather than simply comparing numerical answers. This approach also means that students have different numbers than the provided solutions, again a practice intended to encourage student thinking at the symbolic and conceptual level.

Results and Discussion

Student Demographics

Due to the relatively small size of the engineering program where this work is occurring, it is not practical at this time for us to run concurrent statics sections with and without SMK use to conduct a controlled experiment to evaluate the effectiveness of the models and associated pedagogy in insolation from the other active learning strategies we implement. To date we have relied primarily on student feedback surveys and student performance on the Concept Assessment Test in Statics (CATS), formerly called the Statics Concept Inventory [18], [19], to assess the effectiveness of the SMK activities. Table 3 presents some demographic data and indicators of student preparation for the fall and winter sections. The underrepresented minority (URM) category includes all non-white and non-Asian students (including multi-racial) based on student reporting of race/ethnicity. ENGR 100 and 101 are two different versions of our Introduction to Engineering course that features significant emphasis on academic and problem solving skill development. The two-tailed t-tests we applied to determine significance of the differences in GPA and STEM credits completed returned values of $p = 0.506$ and $p = 0.095$ respectively. There is no statistically significant difference in the preparation of the two student populations.

Table 3. Demographics data for the statics sections in which we implemented the SMK feedback survey and Concept Assessment Test in Statics (CATS).

Section	Fall 2017	Winter 2018
Number students enrolled	29	25
Percent URM	23%	31%
Percent female	24%	12%
Percent first generation	24%	36%
Percent with prior completion of ENGR 100/101	17%	24%
Average STEM GPA	3.33	3.20
Average successful STEM credits completed	44.8	34.5

Feedback Survey Results

Table 4 presents survey prompts and responses on the anonymous feedback survey we administer near the end of the quarter (8^{th} week) after students have worked through all of the SMK activities. The survey uses a standard Likert scale of 1 = Strongly Disagree, 2 = Somewhat Disagree, 3 = Neutral, 4 = Somewhat Agree, and 5 = Strongly Agree. We administer the survey outside of class as a Google form that students access through a link in Canvas. We provide a negligible number of participation points as incentive for survey completion. Since the survey is anonymous, we award the points to the class as a whole based on the response rate. Even though the points available are negligible in terms of the final course grade, this approach of leveraging peer pressure seems effective for achieving high response rates and we have seen no evidence of individual students making multiple submissions.

Table 4. Aggregate feedback survey results for fall 2017 and winter 2018.

	Fall 2017 (N = 25)			Winter 2018 (N = 23)		
Survey Prompt	**Mean**	**SD**	**# 5's**	**Mean**	**SD**	**# 5's**
SMK activities helped me…						
1. Understand vector notation and use it properly.	3.88	1.07	10	3.91	1.14	9
2. Interpret figures for 3D problems on homework and exams.	3.88	0.95	9	4.04	0.95	9
3. Visualize vectors in 3D.	4.32	0.79	12	4.35	0.91	12
4. Understand force equilibrium.	3.60	1.02	6	4.09	0.93	10
5. Understand support models.	3.68	1.16	6	4.30	0.80	11
6. Conceptualize moments in 3D systems.	3.88	1.21	10	3.87	1.30	9
7. Understand moment equilibrium.	3.56	1.27	7	3.91	0.93	8
8. Develop my free-body diagram skills.	3.36	1.20	5	4.00	1.06	11
SMK activities provided…						
9. An effective context for discussing statics concepts with my classmates.	3.96	0.96	9	4.00	1.22	12
10. Opportunities for the instructor to explain statics concepts in detail.	3.68	1.09	7	3.96	1.16	10
Overall Response Mean	**3.78**	**1.11**	**-**	**4.04**	**1.06**	**-**

The quantitative student feedback on the SMK activities is generally positive for both sections with some significant increases for winter 2018. We applied a one-tailed t-test to these results to check our hypothesis that the steps described above to increase the allotted class time and incentives for worksheet completion would result in students perceiving the activities as more effective. The increases in the average response for prompt 4 ($p = 0.048$), prompt 5 ($p = 0.019$), and prompt 8 ($p = 0.03$) as well as the overall response mean ($p = 0.004$) were all significant. Note that the number of students responding with a 5 increased by 67%, 83%, and 120% for prompts 4, 5, and 8 respectively. There were no significant decreases in the mean response for any of the prompts. Recall from the discussion above that the increased class time was an effort to allow enough time for students to complete the worksheets using SMK4 and SMK5, both of which emphasize support models, equilibrium, and free-body diagrams (see Table 2), the three concepts associated with significant increases in the feedback survey. The significant increase in the perceived effectiveness of the activities as a whole (i.e. the overall response mean) indicates an overall favorable reaction to us further leveraging the SMK in the course lesson plan. Students likely also appreciated the opportunity provided in the winter section for them to compare their work to example solutions. It is also possible that the slightly smaller number of students in winter, which yielded six student groups instead of eight, further contributed to the higher feedback responses because of more frequent instructor engagement with each group during class sessions.

A follow on question in the survey asks students to identify any specific concept(s) for which the "SMK activities were a key contributor" to their learning. Selected responses to this prompt include:

- "SMK activities helped me understand how to solve problems in 3-D. I got a better understanding of how to get the components of a vector; and thus, I understood vector notation better."
- "The SMK helped with the concept of moments because it gave a physical model to discuss ideas with group members."
- "I found the SMK for Rigid body equilibrium and support model is really helpful. Thank to this SMK when we take out the screw in the ball, I can clearly visualize and distinguish fix support and pin support."
- "The modeling kit allowed me to better visualize how 3D vectors (cables) pulling on a beam. Visualization is key with this class, whatever helps with that is a useful task in my opinion."
- "The SMK activities made it much easier to visualize different support models, and helped us develop a better understanding of moments. Using the SMK's was also very helpful for visualizing forces in 3-D. Having unit vector arrows that we could clip onto supports also made it much easier understanding of what exactly the unit vectors represent in our calculations."
- "In the beginning of the quarter I was having a difficult time with 3-D problems and using the modeling kits really helped me visualize how forces were acting in different directions. I think being able to analyze a physical model was very helpful in understating almost all of the concepts we've discussed in this class."
- "Concepts of Moment. Moment is quite a new concept to some of us and SMK activities really help me to understand the concept. If the modeling activities is absent, I may be having a hard time imagining what is really happening with the new concept."

The qualitative student feedback on the SMK activities has been almost universally positive with several students identifying these activities as key to developing their understanding of important concepts such as vectors, moments, and support models. Student feedback also indicates they view the activities as helpful for developing some 3D spatial visualization skills such as interpreting figures and visualizing vectors. Overall, students reported that the modeling activities were helpful and that they provided an effective context for engaging with their classmates and the instructor as they worked to learn the material. We also solicited suggestions for improvement. The general sentiment of the responses was a desire for even more class time devoted to these activities and notes from a few students acknowledging the activities as potentially valuable to many students, but not a good fit for their particular learning preferences. Student feedback requesting solutions to the worksheets prompted us to provide them for the winter 2018 section.

CATS Results

We started administering the CATS in fall 2017. We administer the assessment at the beginning of finals week and award a small number of participation points as incentive for student completion. We also recommend students use the CATS as one measure to self-assess their readiness for the final exam. Note that we do not administer the CATS as a pretest following the recommendation of Steif [19] that the assessment offers "negligible information as a pre-test". The CATS results can be viewed as an instrument to assess the extent to which students are developing transferable knowledge and representational competence, because much of the language, diagrams, and question wording on the CATS are significantly different in style compared to the resources we use for this statics course. It is important to reiterate here that the course includes several other activities in which students engage with statics conceptual knowledge (e.g. ABCD questions, mini-lectures, homework problems, projects), so we cannot conclusively attribute learning gains in any area to work with the SMK. Nonetheless, concept subscore performance provides some objective evidence regarding the effectiveness of the models that we can track over time and use to assess the impact of changes to the models and/or associated curriculum. Table 5 presents student performance on the CATS concept subscores for fall 2017 and winter 2018.

Table 5. Student performance (mean fraction of questions answered correctly) on CATS concept subscores. Refer to Table 2 for descriptions of each SMK activity.

	Fall 2017 (N = 22)		**Winter 2018 (N = 23)**		**Related**
CATS Concept Subscore	**Mean**	**SD**	**Mean**	**SD**	**SMK**
A. Free-Body Diagrams	0.59	0.39	0.65	0.35	2, 4, 5
B. Newton's 3rd Law	0.50	0.36	0.42	0.37	1
C. Static Equivalence	0.53	0.33	0.48	0.34	3
D. Roller Joint	0.76	0.30	0.64	0.29	None
E. Pin-in-slot Joint	0.74	0.35	0.74	0.34	None
F. Loads at Surfaces with Negligible Friction	0.47	0.34	0.35	0.30	None
G. Representing Loads at Connections	0.77	0.29	0.61	0.36	4, 5
H. Limits on Friction Force	0.48	0.30	0.52	0.36	5
I. Rigid Body Equilibrium	0.50	0.31	0.39	0.35	4, 5
Overall Mean	**0.59**	**0.22**	**0.53**	**0.19**	-

None of the differences between the fall and winter CATS results are significant. A two-tailed t-test applied to comparisons of each of the subscores as well as to the overall mean score yields a minimum p-value of $p = 0.11$ for all comparisons (subscore G). One difference of note between the two terms is the response rate. For fall, 22 students completed the CATS out of 27 students who completed the course, for a response rate of 81%. In contrast, for winter, all 23 students who completed the course also completed the CATS for a response rate of 100%. The final exam grades for the five students who did not complete the CATS in the fall were 53, 73, 46, 83, and 39 respectively. Given the fall final exam average of 80, it is clear that these five students were among the lower performing students in the course and that the overall CATS results for fall 17 may be artificially high as it does not include the responses of these five students.

One consistent pattern in the results from both sections is that students scored highest on concepts A, D, E, and G, all of which concern types of engineering connections and related free-body diagram skills. The results discussed previously in Table 4 indicate that the fall section students did not attribute as much credit for their development of free-body diagram skills to the SMK activities compared to other concepts, but the winter section did credit the SMK activities. No SMK activities correlate directly to the connection types assessed in D and E; however, we do emphasize the general process of applying physical reasoning to the analysis of a connection in SMK4 and refer back to this activity later in the course as we introduce frame analysis. Perhaps students are succeeding at transferring the conceptual knowledge they gained through the SMK activities to other types of connections that those activities did not specifically emphasize. Overall, the CATS results do not tell us much at this point regarding the impact of the SMK activities compared to other aspects of the course. We hope to use this data as a baseline for comparison as we study the effects of modifying the SMK curriculum and/or increasing the number of modeling activities for future course sections.

Conclusion and Future Work

The Statics Modeling Kit (SMK) offers promise as a learning tool to help students build the understanding of mechanics principles necessary for effective problem solving and eventual competence in engineering practice. The kit offers a flexible platform and approach that can target specific conceptual learning goals while facilitating measurements students can use to perform numerical calculations. Student feedback to date is favorable with many students identifying the modeling activities as key to their learning of specific statics concepts. Small increases in class time allocation and completion incentives from fall 2017 to winter 2018 that further leveraged the modeling curriculum as part of the overall course design resulted in significant increases in student survey responses regarding the effectiveness of the activities.

Going forward we plan to develop additional modeling activities around the topics of contacting bodies, distributed loadings, frame analysis, internal forces and moments, and centroids. As we add new SMK activities in the future, we will be able to use the 2017-18 CATS results presented here as baseline data to evaluate the impact. We plan continued experimentation with the specific nature and mix of exercises (e.g. sketching, manipulations, calculations) that we incorporate into the learning activities. As we are satisfied that certain modeling activities have reached "maturity" we plan to license all this work with Creative Commons and upload the associated worksheets, CAD files for 3D printed parts, and plans for construction of the pegboard system to the hands-on mechanics website at www.handsonmechanics.com.

We also hope to conduct more focused research into the effectiveness of these modeling activities in supporting student progress on course outcomes and in developing a targeted assessment for representational competence in statics. We are furthermore interested in exploring whether the effectiveness of the SMK curriculum varies for different student demographic groups.

References

[1] R. A. Streveler, T. A. Litzinger, R. L. Miller and P. S. Steif, "Learning Conceptual Knowledge in the Engineering Sciences: Overview and Future Research Directions. Journal of Engineering Education," vol. 97, p. 279–294, 2008.

[2] T. Litzinger, P. Meter, C. M. Firetto, L. J. Passmore, C. B. Masters, S. R. Turns, G. L. Gray, F. Costanzo and a. S. E. Zappe, "A Cognitive Study of Problem Solving in Statics," *Journal of Engineering Education,* vol. 99, pp. 337-353, 2010.

[3] P. S. Steif and A. Dollár, "Reinventing the Teaching of Statics," *International Journal of Engineering Education,* vol. 21, no. 4, pp. 723-729, 2005.

[4] S. Freeman, S. Eddy, M. McDonough, M. Smith, H. Jordt and M. Wenderoth, "Active learning increases student performance in science, engineering, and mathematics," *Proceedings of the National Academy of Sciences,* vol. 111, no. 23, pp. 8410-8415, 2015.

[5] A. Dollár and P. S. Steif, "Learning modules for statics," *International Journal of Engineering Education,* vol. 22, pp. 381-392, 2006.

[6] B. D. Coller, "An experiment in hands-on learning in engineering mechanics: statics," *International Journal of Engineering Education,* vol. 24, pp. 545-557, 2008.

[7] J. A. Mejia, W. H. Goodridge, W. J. Call and S. D. Wood, "Manipulatives in Engineering Statics: Supplementing Analytical Techniques with Physical Models," in *American Society for Engineering Education 123rd Annual Conference Proceedings*, 2016.

[8] R. Welch and J. L. Klosky, "An Online Database and User Community for Physical Models in the Engineering Classroom," in *Proceedings of the 2006 ASEE Annual Conference and Exposition*, Chicago, IL, 2006.

[9] J. C. Bruhl, J. L. Klosky and J. P. Hanus, "Let's Break Stuff! A Refit of the Mechanics Sequence of Courses to Inspire Student Inquiry," in *Proceedings of the 2017 ASEE Annual Conference and Exposition*, Columbus, OH, 2017.

[10] R. B. Kozma and J. Russel, "Multimedia and Understanding: Expert and Novice Responses to Different Representations of Chemical Phenomena," *Journal of Research in Science Teaching,* vol. 34, no. 9, pp. 949-968, 1997.

[11] M. Steiff, S. Scopelitis, M. E. Lira and D. Desutter, "Improving Representational Competence with Concrete Models," *Science Education,* vol. 31, no. 3, pp. 344-363, 2016.

[12] P. Klein, A. Müller and J. Kuhn, "Assessment of representational competence in kinematics," *Physical Review Physics Education Research,* vol. 13, no. 1, p. 010132, 2017.

[13] P. Pande and S. Chandrasekharan, "Representational Competence: Towards a distributed and embodied cognition account," *Studies in Science Education,* vol. 107, no. 2, pp. 451-467, 2016.

[14] R. C. Hibbeler, Engineering Mechanics: Statics, 14th ed., Pearson, 2015.

[15] K. Gorman, "Ed Prather (U. of Arizona) on Using Colored Cards in the Classroom. Clip from "Effective Active Learning Strategies for STEM Classes"," 2014. [Online]. Available: https://www.youtube.com/watch?v=onhLr127gWo.

[16] J. Moore, "Adaptive Map Digital Textbook Project: Engineering Statics," [Online]. Available: http://adaptivemap.ma.psu.edu/.

[17] K. Gramoll, "Multimedia Engineering Statics," [Online]. Available: https://ecourses.ou.edu/cgi-bin/ebook.cgi?doc=&topic=st.

[18] P. S. Steif and J. A. J. A. Dantzler, "A Statics Concept Inventory: Development and Psychometric Analysis," *Journal of Engineering Education*, vol. 94, p. 363–371, 2005.

[19] P. S. Steif and M. A. Hansen, "New Practices for Administering and Analyzing the Results of Concept Inventories," *Journal of Engineering Education*, vol. 96, p. 205–212, 2007.