

A study of the Efficacy of Free-body Diagrams for the Solution of Frame-Type Mechanics Problems with Increasing Difficulty Level

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Abstract

The intent of this study was to conduct a quantitative assessment of students' free-body diagrams (FBDs) using a predefined rubric to assess accuracy, and to determine the influence of increased difficulty level on the efficacy of these FBDs. Using final exams from a first-year statics course, the difficulties that students encountered when solving rigid body problems were analyzed for two independent cohorts of engineering students in two different years. The specific problems considered were frame type problems which included multiple members (including a two-force member), different distributed loads, and different member axial alignments. Quantitative results show that when members becomes less aligned with the axes, students tend to have more difficulties in choosing and setting up problems in their free-body diagrams. In addition, it was noted that the errors produced in the FBDs for the inclined frame result in more errors in the set-up of the equations as compared with the straight frame problem. Our results support the idea that difficulty level of a problem plays an important role in the effectiveness of FBDs. Suggestions on how to improve student learning in light of the errors produced on FBDs is also presented.

Background

Mechanics problems involving forces acting on or within a body at equilibrium are governed by the equations $\sum \vec{F} = 0$ and $\sum \vec{M} = 0$. Generation of the solution to the problems requires determination of the forces involved, writing the equations, and then solving them. To help develop these equations, visual representations known as Free Body Diagrams (FBDs) are used. Formally, a FBD is a diagram of an isolated particle or rigid body which may include body forces, surface forces, and couple moments. It may also include both the magnitude and the angle of the force and any relevant dimensions on the locations of the applied forces and the coordinate system used [1],[2]. The idea and use of FBDs to aid in the development and solution of mechanics problems is not new and is a standard tool in first year level mechanics course. A literature review into the effect of using FBDs reveals that students who drew correct FBDs were more likely to solve problems correctly [3]-[5] and found that drawing inaccurate FBDs led to more incorrect solutions [5] and/or led to failing the course [3]. Alternatively, other researchers noted the opposite result; that the quality [6] or correctness [7] of the FBDs did not impact student performance significantly. Furthermore, similar studies noted that students who created high quality FBDs were likely to produce low quality equations [8]. A potential explanation for some of these apparent contradictions is that for simple problems with few forces/moments a student may be able to write down the equations with ease. However, as the number of forces increases, the cognitive load [7] on the student (due to the difficulty level of the problem) increases, making it difficult for the student to setup the equations directly from the problem description. In attempt to take into account problem difficulty, non-deformable bodies may be categorized as being either particle or rigid body where the particle classification is generally less involved than the rigid body and multiple rigid body problems. The main difference between the

two is that the particle in equilibrium reduces to a concurrent force system and neglects the influence of spatial placement of forces and reduces the problem to the solution of $\sum \vec{F} = 0$ whereas for the rigid body two vector equations ($\sum \vec{F} = 0$ and $\sum \vec{M} = 0$) must be satisfied. The type of forces acting on the body is not specific to a particle or rigid body with the exception of distributed loads and a couple moments which cannot be associated with particles. In addition, rigid bodies can consist of a single body or of multiple connected bodies. For multiple connected bodies the solution to the equations becomes more involved as the maximum number of equations required to solve the problem is 6N for a 3D problem and 3N for a 2D problem (where N is the number of bodies involved). Depending on the problem, these equations may be highly coupled or uncoupled. A review of the literature on FBDs shows that most of the studies focus on problems involving particle equilibrium [3]-[14]. In the studies which dealt with rigid bodies, Shryock and Haglund [15] focused on questions involving the drawing of FBDs only. In another opinion based paper, Lane [16] discussed different approaches taken in drawing FBDs and the placements of the forces. Finally, Litzinger et. al. [17] studied students' FBDs and equilibrium writing skills for single rigid bodies. The authors found that weak students typically produced "poorer" FBDs and equations as compared with strong students. It was, however, also noted that both groups typically did not write down the moment equations [17]. None of the studies found on rigid bodies included distributed loads.

The literature review showed that there is a clear lack of evidence-based quantitative data on the efficacy of FBDs for problems of varying difficulty level. It is not well known if or where the boundaries for a potential cognitive overload exist, or what causes it for the different types of problems of varying difficulty. The goals of this paper are to quantitatively study the effectiveness of using FBDs for problems which include multiple connected rigid bodies of varying difficulties, develop insight into the effect of varying different elements of the same problem type, and to find common errors that students have when dealing with problems involving these complex problem types.

Methodology

The questions studied are shown in Figure 1. Here, students from two separate years were given a single frame type problem in static equilibrium (either the straight or inclined frame) with varying point and distributed loads and were asked to find the reaction forces at the pins. The questions were part of two independent final exams for a first year engineering mechanics statics course. Topics taught included: vectors, moments and distributed loads, particle and rigid body equilibrium (including trusses, frames, machines, internal forces), surface forces (including springs, pulleys, friction), centroids, and area moments of inertia. Teaching methods [18] as well as types of homework [19] are known to have an impact on student learning. In addition, the clarity and organization of a problem also are known to effect a students' ability to solve problems [20]. These variables were controlled in our study by keeping the classroom teaching methods and assessments similar between instructors (similar teaching styles, similar lecture examples and notes). While teaching, both authors followed a "strong" teaching approach [21]. Emphasis was placed on the importance of the problem solving process in solving equilibrium problems including: drawing complete and neat FBDs (which included calculation of any variables needed in the FBD), generating the equations based on these visual representations while incorporating any auxiliary equations (e.g. W=mg), and solving the system of equations.



Figure 1: Questions used for Exam 1-Straight Frame (Left) and Exam 2-Inclined Frame (Right)

Assignments and seminars in the course reinforced this solution methodology by assessing the students' problem solving process and final answer as opposed to their final answer only. For solving frames specifically, students were taught to look at a variety of ways to set up and solve the problem: both as a whole and by breaking the problem down into parts. It is important to note that the results of the study may be specific to the teaching method described above. Differences between the two questions studied are summarized in Table 1 which include: an increase (from

Туре	Description	Straight	Inclined			
Members	Number	3	3			
	Number of two force members	1	1			
	Not aligned with axes	1	2			
Support	Number	2	2			
Reactions	Туре	Pin, Roller	Pin, Roller			
	Not aligned with axis	0	1			
Connections	More than two members at a pin	0	0			
Point Forces	Number	1	0			
	Unaligned with member	0	0			
	Require auxiliary equations	0	0			
Distributed Loads	Number	1	1			
	Polynomial Order	0	1			

Table 1: Comparison of the different elements in each Frame problem

the straight to the inclined frame) in number of members not aligned with the traditional horizontal and vertical axes, an increase in number of support reactions not aligned with the axes, an increase in the polynomial order of the function describing the distributed load used, and a decrease in the number of applied point forces present. It should be noted that the questions used for this study were taken from a pool of existing exams and were not specifically designed for this study. One characteristic of frame analysis is that there may be several ways to solve the problem depending on the choice of FBDs and the choice of the solution method for the resulting equations. An example of solving the straight frame problem is shown in Appendix 1. Here the entire frame removed from its supports is chosen as the first FBD with three equations determined by $\sum M_A = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The second FBD chosen was the horizontal section resulting in three equations using $\sum M_C = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. A comparison of the equations and the unknown variables for each solution for the two frame problems considered in this study is given in Table 2, which provides the number of terms in each equation and the dependencies of the equations. A comparison between the straight and inclined frame for this solution methodology shows that in general there are more terms and dependencies in the equations for the inclined frame compared to the straight frame. Both methods provide similar

			Straight Frame		Inclined Frame					
Body	Equations	Terms	Dependent on	Result	Terms	Dependent on	Result			
	$\sum M_A = 0$	3	_	Ey	4	_	Е			
Full	$\sum F_x = 0$	2	_	Ax	2	Е	Ax			
	$\sum F_{y}=0$	3	Ey	Ау	4	Е	Ау			
Horizontal	$\sum M_{c}=0$	3	Ey	В	4	Е	В			
	$\sum F_x = 0$	2	В	Cx	3	B, E	Cx			
	$\sum F_{y}=0$	4	B, Ey	Су	5	B, E	Су			

Table 2: Example of one solution process for each frame problem

results for the solution methodology determined. It should be noted that if the full frame and the vertical member were used to develop the FBDs and equilibrium equations, then the solution methodology (terms and dependencies) would be similar. However, if the vertical and horizontal members were chosen for the equilibrium analysis then the equations would be highly coupled and would involve more work for the student to solve the equations. The two questions used for the study were assessed using the rubric given in Appendix 2. The criteria in the rubric focus on determining deficiencies in both individual FBDs and the resulting equilibrium equations. The questions in the rubric are typically straightforward with exception to Rigid Body / Point mix up and Axis direction which are shown in Figure 2. Here, the rigid body / point mixup is when a student mistakenly transforms a rigid body problem into a concurrent force particle problem. Other challenges that a student can have is choosing a coordinate system which may result in a system of equations that is coupled and more difficult to solve. This is not directly an error but may result in more errors being made.



Figure 2: Diagram showing two different assessments of FBD as used in the rubric

Results

Results of the study are shown in Tables 3 and Figures 3–6. The sample sizes for the straight and inclined frame problems were 141 and 150 FBDs, respectively. As expected, on average, students performed better on the straight frame (14.6 / 20) problem as compared with the inclined frame problem (12.8 / 20). In order to begin to understand the efficacy of FBDs in the frame problems, the errors were classified and are presented in Figure 3. These



Figure 3: Comparison between errors in the FBDs and the equations in the two frame questions

results show that for the straight frame problem 48% of the sample size had errors in the equations. Of those who had errors in the equations, 22% of them were attributed to errors in their FBDs. For the inclined frame problem, 55% of the students had errors in the equations of which 60% were due to errors in the FBDs. The errors and omissions in the students' FBDs are

	Percentage [%]							
Description	Straight frame	Inclined frame						
Axis missing	50	63						
Dimension missing	33	55						
Force missing or extra	19	34						
Angle incorrect	2	34						
Angle missing	13	28						
Force magnitude / label missing	8	9						
Force magnitude / label incorrect	10	7						
Dimension incorrect	1	7						
Body missing	9	5						
Force incorrect direction	0	1						

shown in Table 3 for both the straight and inclined frame problems. For both problems the significant errors (>10%) included: not drawing a coordinate system (axes) for vectors, missing

Table 3: Comparison between different types of errors found in the FBDs of the problems

dimensions, missing or extra forces, and missing angles (geometry). In addition, for the inclined frame problem, incorrect angles were a source of significant error. Next, errors that were made in FBDs which resulted in errors in the equations, were considered. Figure 4 indicates that there were only four potential causes for these errors for both problems: incorrect or missing angles, errors caused by misapplication of a point force (either in direction or magnitude), incorrect conversion of the distributed load into a point force (or completely absent), and incorrect or missing dimensions. This includes all of the significant errors reported in Table 3 with the exception of the missing coordinate axes. For the straight frame problem, point forces were found to cause the majority of the errors in the equations (60%). This was attributed to not



Figure 4: Comparison between different errors found in the FBDs in the two frame questions which resulted in errors in the resulting equations

recognizing the two force member or point load, and missing pin reaction components. Furthermore, the conversion of the distributed load to a point force caused 33% of the errors, and finally incorrect dimensions caused 20% of the errors. For the inclined frame problem, the most significant error was due to incorrect angles (52%), then the point forces (26%), and finally the conversion of distributed loads and incorrect dimensions (both 12%). It should be noted that, in terms of the number of people that made errors (in Figure 4), each type was comparable for both problems except for incorrect or missing angles which was significantly higher for the inclined frame problem. Next, results from the study of the frequency of errors found in the equations which were not associated with the FBDs is shown in Figure 5. The significant errors include: the incorrect setup of equations, propagation of errors due to incorrect calculations previously done (ex. angles and distances), dependencies on results from previous equations (i.e. solved reaction forces), and mathematical errors due to solving the equations incorrectly. For the



Figure 5: Comparison between errors found in the equations which were not caused by the FBDs

straight frame problem the incorrect setup of the equations accounted for the most frequent error (90%). This includes errors due to missing moment arms (38%), incorrect signs in the moment equations as well as other errors (33%) such as missing terms. The next most frequent error was due to incorrect calculation dependencies (25%). Very few of the solutions used incorrect variables from previously solved equations (8%) or solved incorrectly (10%). For the inclined frame 65% of the solutions were setup incorrectly (moment arms missing 26%, sign error on moment arm 13%). Here, however, there was an increase in the number of errors caused by incorrectly solving pin reactions and then using them in subsequent equations (23%). Finally, Figure 6 provides insight into how students solved the different frame problems through the FBDs that they chose to solve the problem. For the straight frame problem, the majority of students chose full frame FBDs (86%) as their first choice in the equilibrium analysis and then chose either the horizontal (62%) or the vertical member (62%) as their first choice. For the inclined frame, however, students typically chose the horizontal member as their first choice



Figure 6: Comparison between different members drawn in the two frame questions

(83%), the full member as their second choice (75%) and then the vertical member as their third choice. It should be noted that some students chose all three members to attempt to solve the problem. In addition, for the inclined frame, problems solved using only the vertical and horizontal frame typically resulted in equations that the students were unable to solve (due to the highly coupled equations) or resulted in fundamental assumption errors (omission of the two force member).

Discussion and Conclusions

Results show that there were clear differences between the straight and inclined frame problems studied. The results indicated that only 22% of the errors in the equations derived for the straight frame problem were attributed to errors in FBDs in comparison to the inclined frame where 60% of the errors in the equations were attributed to errors in FBDs. This supports the idea that the effectiveness of using a FBD increases as the difficulty of the problem increases. Differences between the problems show that the inclined frame has more force components not easily aligned with traditional horizontal and vertical axes which resulted in added complexity (i.e. more angles in the FBDs, increased terms in the equations, and the coupling of some equations). This was confirmed by the percentage of incorrect angles used in the FBDs which was substantially higher as compared with the straight frame. It should be noted that other differences such as the increased polynomial order of the distributed load did not cause substantial difficulties for the solution of the two problems. This may suggest that members or supports aligned in a problem may be a good indicator of problem difficulty whereas a change from 0th order to 1st order polynomials for distributed loads may not be a significant cause of difficulty for students. The body breakdown and equation set up for problems involving multiple connected bodies was also seen to cause increased difficulty for the students solving the inclined frame problem. The most direct route for solving both problems was to choose the full frame and then either the vertical or horizontal frame using traditional vertical and horizontal axes. For the

straight frame problem, students typically chose this solution methodology. For the inclined frame, however, students had some difficulty determining the most direct way to solve the problem which may have increased the cognitive load of the problem for them significantly. A choice of vertical and horizontal frame for the FBD often left students with equations that they could not solve.

The current study suggests that for the frame problems considered, students can be helped in two ways: with the calculation of quantities used in FBDs and with ways of solving equations when the procedure is not apparent. With respect to helping students with the calculations, worksheets could be created to help students determine angles in problems where the angle is not readily apparent to them. In addition, worksheets could be created to help them calculate distances to points both on and off bodies (as well as the proper choice of points to take moments about). These exercises could be given to students at the beginning of the term when their workloads are light. In addition, a worksheet to help students with the determination of rotation directions and with forces at angles could be constructed to help them with difficulties found in the construction of moment equations. It should be noted that it was difficult to determine whether the errors were actual errors or simple mistakes. Because of this it would also be beneficial for the students if part of the problem assessment was for students to validate their calculations. Often in class students are advised to check solution calculations but they are not rigorously assessed to confirm this part of the problem solving process and they may opt not to do so. This could be done using: back-of-the-envelope calculations, checklists, or alternative equivalent solution methods

In addition to the above, student learning can be improved in through scaffolding questions [22] which involves increasing the difficulty level of a problem steadily until a student becomes proficient at solving problems in the topic being taught. This, however, can be a challenging task for an instructor as students in the classroom may learn at different rates and require different needs at different times. In addition, instructor expertise may interfere with their ability to predict student performance [23]. One potential way to scaffold problems in mechanics is to use Computer Adaptive Testing [24] where software to chooses the next question for the student based on their success in solving the previous question. In order for a computer to accomplish this, however, it must be able to estimate each problem's difficulty level to determine the student's next problem to solve. The results of the current study provide quantitative data which could be used to help develop algorithms to determine a problem's difficulty level for frame type problems.

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Appendix 1 – Example solution to the Straight frame problem

Appendix 2 – Rubric used for assessing the free-body diagrams

UUID:	Year	
Question Mark:	Question	
Exam Mark:		

Торіс	Description	Body	y #1				Body	#2			
General	FBD Missing	no	yes				no	yes			
Forces / Moments	Forces Missing/Extra	no	1	2	3+	all	no	1	2	3+	all
	Magnitude/Label missing	no	1	2	3+	all	no	1	2	3+	all
	Magnitude/Label incorrect	no	1	2	3+	all	no	1	2	3+	all
	Angle Incorrect	no	1	2	3+	all	No	1	2	3+	all
	Angle missing	no	1	2	3+	all	no	1	2	3+	all
	Incorrect direction	no	1	2	3+	all	no	1	2	3+	all
Body	Body Missing	no	yes				no	yes			
	RB / Point Mixup	no	yes				no	yes			
Axis	Axis Missing	no	yes				no	yes			
	direction	Incor	poor		good	cor	Incor	poor		good	cor
Dimensions	Missing Dimensions	no	1	2	3+	all	no	1	2	3+	all
	Incorrect Dimensions	no	1	2	3+	all	no	1	2	3+	all
Equations	Missing	no	yes				no	yes			
	Errors in Equations	no	yes				no	yes			
	Errors due to Incorrect FBD	no	yes				no	yes			