A Visual, Intuitive, and Engaging Approach to Explaining the Center of Gravity Concept in Statics

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Abstract

We all experience Statics in day-to-day life, and have an intuitive knowledge regarding the interaction of forces around us. Despite this, many students struggle with Statics in classroom settings. The equations and mathematics can be overwhelming and frustrating, in part since they are not visual or intuitive enough and are often missing the connection to daily, real-world experiences.

Today’s students have a plethora of distractions available to them. If students feel bored or frustrated with the material, often times they will browse the Internet on their laptops or pull out their phones. They learn differently, more visually and intuitively, and they have short attention spans. To make them pay attention in class, the material and presentation methods should be visually clear, intuitive and engaging.

This paper focuses on a visual, example-based, and engaging approach to teaching a specific Statics concept, namely center of gravity. To teach this concept, the paper shares supplemental materials that can be introduced during instruction. The paper introduces this concept using (1) daily, experience-based examples, such as an elephant balancing on its front legs, (2) examples for finding the center of gravity, such as balancing sticks and brooms, (3) applications, such as the Leaning Tower of Pisa, (4) activities, such as standing from a chair without leaning forward, (5) teasers and puzzles, such as a man standing at a strange angle, (6) “Did you know?” stories such as the world record for balancing golf balls, (7) and changes to the location of the center of gravity due to geometry, such as bending a wire hanger. The point of this approach is to provide students with easy to understand examples that translate bookwork to real life and help in comprehension of the material. When using these examples students tend to understand the concept of center of gravity more clearly. This more intuitive understanding allows them to be successful when introduced to equations and calculations associated with the topic.

The examples have been introduced to 54 Statics students in a regular classroom setting, followed by a comprehensive survey that assessed both the effectiveness of this learning method as well as students’ preferences for learning. Conducted surveys show that (1) students think it is important to learn through hands-on activities and visual examples, (2) many students do not find as much value in traditional lectures, reading the textbook, and PowerPoint presentations as the generations of students before them did, and (3) the visual and engaging presentation of center of gravity helped them understand the concept.
Based on students’ initial feedback, we believe that visual, engaging, and example-based learning is effective for teaching today’s students and that the use of similar methods can be employed throughout the entire Statics course in order to enhance students’ comprehension experience.

It should be noted that this paper is a work in progress. In addition, this method of teaching is meant to be supplemental in nature and not to replace existing textbooks or other teaching and learning methodologies.

Introduction

The problem: learning Statics

The subject of Statics is fundamental to Mechanical, Civil, Aerospace, Marine, and Ocean Engineering fields since it provides methods for solving real life force and moment equilibrium problems. It also serves as the foundation for other courses such as Dynamics and Strength of Materials.

For many engineering students, Statics is the first “real” engineering course they enroll in. Some students underestimate the course load and misjudge how much time they need to spend studying and doing homework. In addition, many experience difficulties grasping concepts, especially those that are introduced to them in class following traditional textbook math with little intuition or visualization. A common complaint from students is their inability to visualize the problems given to them, including the effects of forces and moments acting on objects. We believe that visualization helps students to develop intuition; creation of the “Aha!” moments can help in better understanding of the basic concepts.

Related Literature

Extensive work has been done attempting to find more effective ways of teaching foundational engineering courses. Trying to find out why so many students struggle with engineering mechanics and what can be done about it shows that there is no one single approach [1]. Assessing students prior to the course shows a strong correlation between the strength of their understanding of pre-requisite classes and their ability to succeed in Statics [2], but doesn’t reveal what can be done to improve current performance. Some instructors have acknowledged the difficulty students experience in perceiving integrated concepts and have begun to teach each concept in isolation [3][4] or to utilize concept questions to supplement active learning methods [5]. In one case, a seven-year study was conducted to determine the effectiveness of active learning methods which found a statistically significant improvement in final class grades over passive learning methods in Statics [6].

Different tools have been implemented to assist in the active learning process. Interactive computer games have been utilized to develop proficiency in narrow, specific topics such as centroids and moments of inertia [7]. LabVIEW graphical interfaces have been utilized to assist students in finding deflections, stresses, and shear and bending diagrams [8]. Learning modules with PowerPoints and interactive concept testing have been used to supplement lectures [9].
Many researchers have found success with active learning methods by focusing on utilizing visual and hands-on activities. Hands-on learning has been used in small classrooms with enough success to plan a scale-up to larger lectures [10]. One method that has led to success in student learning is to use intuitive approaches to teach students concepts in a way that easily makes sense, in Statics [11] as well as in other subjects [12] [13].

Another experiment compared two groups of students in which one group received classes focused on hands-on manipulatives to solidify Statics concepts in addition to lectures. The other group only attended traditional lectures. This split continued until the midterm exam. For the rest of the course, the instructor performed in-class demonstrations rather than letting students do it themselves. The researchers used five different assessments to test the effectiveness of the hands-on portion of the class as compared to the hands-off. Two of the assessments were multiple choice concept tests, one at the midterm and one for the final. Two others were problem solving tests where students demonstrated their proficiency using free body diagrams, resolving forces, and more. The last assessment was a Statics Concept Inventory given online the week before the final. The assessments showed that the class as a whole had little difference between the two groups. Mechanical engineering students in the hands-off group performed a bit better than the hands-on students, but the researchers could not prove statistical significance. They did find, however, that the hands-on group of electrical engineering students in the class performed, on average, one standard deviation better on the midterm than the hands-off group did [14].

**Improving Student Understanding of the “Center of Gravity” (CoG) Concept**

One concept in Statics where students tend to struggle is center of gravity and its implications on the interaction of objects in the real world. Center of Gravity is defined as a point at which the total weight of a system can be considered to be concentrated [16]. This paper attempts to improve student understanding of this topic by focusing on intuitive, visual, and engaging examples. The first co-author presented an 80-minute interactive lecture to 54 students on the subject of center of gravity using visual and intuitive daily examples, in-class hands-on individual and group activities performed in class by student volunteers, and puzzles, along with “Did you know?” examples, while minimizing the use of equations. Based on students’ engagement, reactions, and results of the assessment we believe that by using visualization as an introduction to teaching the subject, students are able to grasp concepts more quickly and apply their understanding to future problems.

Two main equilibrium principles that govern Statics, i.e., “net force = 0” and “sum of moments about an axis = 0,” may be viewed as special cases of Newton’s Laws of Motion.

**The Bigger Picture**

This work is part of a larger project which spans multiple subjects and seeks to broaden understanding of STEM topics. This approach is meant to help both teachers and students, allowing for more innovative teaching and enhanced comprehension-based learning. The project recognizes that students learn in diverse ways, and attempts to appeal to those who learn best through visual, intuitive, and engaging methods. It draws upon real-life experiences, as well as
various STEM/STEAM examples and activities, and targets both an understanding and appreciation of basic concepts in STEM, including Physics/Mechanics, Calculus, Statics, Control Systems, Digital Signal Processing (DSP), Probability, Estimation, and Computer Algorithms. The material can be used by teachers and students in the classroom, but it is primarily designed with the long-term goal of large-scale web-based dissemination, targeting those who prefer self-paced and self-learning friendly environments. Examples of Statics concepts that the author and his team plan to develop and integrate into the learning experience include: (a) games, (b) puzzles and teasers, (c) animations, (d) visual and intuitive daily-experiences-based examples, (e) movies and short video clips, (f) demonstrations, (g) hands-on activities (including those based on virtual reality and augmented reality), (h) team and communication exercises, (i) small-scale inquiry-based research, (j) presentations and peer-based teaching/learning, (k) visual click-based expandable e-book, (l) community and social engagement, and (m) challenges beyond the basics.

This paper is organized into focused sections to make it most useful for instructors. The specific sections are (1) Daily, Experience Based Examples, (2) Finding the Center of Gravity, (3) Center of Gravity vs. Center of Mass, (4) Mathematical Review, (5) Applications, (6) Activities, (7) Teasers and Puzzles, (8) “Did you know?” type questions, and (9) Center of Gravity Changes Due to Geometry. Appendix B has additional examples on the center of gravity that involve Dynamics – a topic beyond the scope of this paper. We share these examples just to widen students’ views on the topic.

1. Daily, Experience-Based Examples

By introducing the concept of center of gravity using familiar visual examples, students are exposed to the concept in a meaningful and relatable way. This allows students to connect to the concept prior to delving into the related math. To achieve this, this section shares examples from art, entertainment, sports, and other daily experiences.

A. Elephants Balancing on Front Legs

Some trained elephants can balance on their front legs, as shown in Figure 1. The performer and the elephant together can be considered one stationary system; therefore, the total system’s net force and net moment are zero. Stated another way, the center of gravity of the system is directly above the elephant’s legs so that there is no net moment created.
B. Yoga: Balanced Poses

When yogis balance themselves as shown in Figure 2, they keep one leg on the floor and the other is jutting out. The center of the yogi’s mass is located directly above the supporting leg that is touching the floor. They do not tip over simply because the net moment relative to their planted legs is zero. Any change in the equilibrium will create a moment causing her/him to lose balance.

Figure 2: People practice balance maintenance. The blue dot shows the center of gravity.
C. Carrying Baskets

Figure 3: Person using stick to carry full baskets

Figure 4: Simplified visual of balancing weights depicting a greater mass on the right side

Many people in Asia balance baskets on sticks to carry items more effectively, such as in Figure 3. To do so, the stick has to be positioned so that, relative to the contact point on the shoulder, the net moment created by the two baskets is equal to zero. The baskets do not have to be loaded equally as long as the weight differences are accounted for. Figure 4 gives a simplified version of Figure 3, showing a greater mass on the right side which shifts the center of gravity towards the right. The point of contact shows where the stick is balanced on the shoulder to create a net moment of zero. To consider this example as pure Statics case we need to assume that the shoulder moves at a constant velocity (i.e., speed and direction).
D. Balancing on a Tight Rope

![Figure 5: Sculpture of a man balancing on a tightrope](image)

The key to balancing on a tightrope is to keep the center of gravity directly below the rope. If the center of gravity is not exactly below the rope then the center of gravity creates a moment relative to the tightrope causing the subject to lose balance and fall. Assuming that the center of gravity of the subject is lower than the rope, then that center of gravity will move towards the equilibrium point (lowest height; lowest potential energy) and stay there. Figure 5 is an artistic depiction of a tightrope walker, and in this case the subject is in equilibrium (it is a static case). This case is different from the case of a real person walking the tightrope, since in a dynamic case the subject’s center of gravity is sometimes above the rope, leading to an unstable situation that needs to be stabilized by the walker. Skilled tightrope walkers constantly move to compensate for the situation and avoid falling. They usually use a horizontal pole to enhance their stability.

E. Balancing Seesaws

![Figure 6: Seesaw](image)

The seesaw apparatus, like the one in Figure 6, is playground equipment that can further explain the concept of equilibrium. When the net moment of the system is zero, the seesaw is stationary. If one person is heavier than the other, or if one sits closer to the center than the other,
the center of gravity will not be above the fulcrum unless the users compensate for this difference (i.e., by having the heavier person sit closer to the center, or if they are the same weight being equidistant from the center).

F. A Rocking Chair

Rocking chairs, like the one in Figure 7, utilize center of gravity to ensure that whoever is sitting on them doesn’t accidentally tip over. The chair is designed so that it does not tip over because the center of gravity always tends to get to its lowest point (as in the middle Figure 7) even when someone is sits in it. In the left and right images, the center of gravity is higher relative to the center of gravity in the middle image.

![Figure 7: Rocking chair at different positions](image)

2. Finding the Center of Gravity

Being able to locate the center of gravity of a given object is crucial to solving problems related to the subject. The following examples help students learn to do this, starting with simple examples and moving on to something that may not be obvious, i.e., that the center of gravity is sometimes located outside the object. To accomplish this, we provide examples of (a) a stick and a broom, (b) donut, banana, and basketball, and (c) a banana hanging on a thread.

A. Stick and Broom

The center of gravity of a stick with uniform area and uniform density is in the middle, at half the length of the stick. This can easily be tested by balancing the stick on one finger. Here is a simple experiment to find the center of gravity: balance a smooth stick on your outstretched forefingers as shown in Figure 8a below.
Figure 8a: Person finding the center of gravity of a stick. The blue dot shows center of gravity.

Now move your fingers towards each other until they come together. As long as the broom or stick stays balanced at all times, your fingers will end up touching each other right under the stick’s center of gravity mass, and the stick will balance. You can repeat the trick, changing the initial position of your fingers, but the result will always be the same, whatever you use – a ruler, a walking stick, a billiard cue, or a broom like in Figure 8b.

Figure 8b: Person finding the center of gravity of a broom

B. Donuts, Bananas, and Basketballs

The center of gravity of an object does not necessarily have to be within the space that the object occupies. Three examples that illustrate this point are a donut, a banana, and a basketball. The center of gravity of a donut lies in the middle of the hole even though no part of the donut exists in that spot, as shown in Figure 9.
Similarly, the center of gravity of a basketball is at its center, even though the middle of a basketball is hollow. A banana’s center of gravity can also be outside of the banana depending on its curvature.

C. Banana on a Thread

In order to find the center of gravity we can perform the following experiment: Connect a banana to a thread and hang it as shown in Figure 10(a). By doing so, the center of gravity will be directly below the thread (along the yellow line). Now repeat the experiment using another connected point. Again (Figure 10(b)) the center of gravity will be below the thread (along the red line). The intersection of the yellow and red lines is the location of the center of gravity, as seen in Figure 10(c). The center of gravity will always be directly below the line of the thread, regardless of where the line is attached to the banana.
3. **Center of Gravity vs. Center of Mass**

Mass refers to the amount of matter in a given object. This property does not change as location changes – any given object will have the same mass on earth, the moon, mars, or floating in space. Gravity refers to the force exerted on an object by another object due to the value of both masses. The force of gravity is felt between any two small objects but is usually negligible except when talking about the force celestial bodies exert on objects relatively close to them.

Center of Gravity refers to the point at which the total weight of a system can be considered to be concentrated. Similarly, center of mass is the point at which the total mass can be considered to be concentrated. What’s the difference? They are generally at the same point on a body, but if the force of gravity is not the same over the entire body then they are not the same. This is the case in rare cases, e.g., for very tall objects.

As previously mentioned, the force of gravity acting between two objects depends on the mass of each object. This means that the force of gravity (weight) we feel on earth (which is calculated using a gravitational acceleration of approximately 9.81 m/(sec)$^2$) is not the same as the gravitational acceleration we would feel when standing on any other celestial body.

4. **Mathematical Review**

A. **Net Force and Net Moment Equal Zero**

Statics focuses only on objects and structures in static equilibrium. These objects are either at rest or moving with a constant velocity, so their acceleration (either rectilinear or rotational) is zero, and the net acting force (and moment) is zero (recall $\sum F = ma$ and $\sum M = 0$). Because the objects studied are in equilibrium, the sum of moments acting on that object must equal to zero as well. If not, the object will begin to rotate. It is important to note that the sum of forces and the sum of moments are equal to zero regardless of the coordinate system chosen.

B. **Calculating the Location of the Center of Gravity**

In the one-dimensional case the center of gravity of a system can be calculated using the equation $\frac{\sum (mg)x}{\sum (mg)}$, where $mg$ represents the weight of each object and $x$ is the distance of each mass from one end of the system. To further explain this concept, imagine a massless beam of length 6 meters that is supporting three 10kg weights, as shown in Figure 11.
The weights are 1, 3, and 4 meters from the left side of the massless beam. Using \( x = 0 \) to be the left end of the beam, we can calculate the horizontal component of the center of gravity using the above equations.

\[
\sum (mg)x = (10kg) \cdot \left(\frac{9.81m}{s^2}\right) \cdot (1m) + (10kg) \cdot \left(\frac{9.81m}{s^2}\right) \cdot (3m) + (10kg) \cdot \left(\frac{9.81m}{s^2}\right) \cdot (4m) = 784.8Nm
\]

\[
\sum (mg) = (10kg) \cdot \left(\frac{9.81m}{s^2}\right) + (10kg) \cdot \left(\frac{9.81m}{s^2}\right) + (10kg) \cdot \left(\frac{9.81m}{s^2}\right) = 294.3N
\]

\[
\frac{784.8Nm}{294.3N} = 2.667 \text{ m}. \text{ Therefore, the center of gravity is 2.667 meters from the left end of the beam.}
\]

5. Applications

The following are real life applications affected by center of gravity. The examples include (a) the Leaning Tower of Pisa, (b) an overloaded cart, (c) work trucks and their stabilization legs, and (d) an overloaded tow truck.

A. Tower of Pisa

The Tower of Pisa, shown in Figure 12, built in the 13\(^{th}\) and 14\(^{th}\) centuries, has become famous due to its tendency to lean. In class this example can help students relate to center of gravity. Many students have known about this tower for so long that, despite its distinctive feature, they’ve never stopped to think, “Why doesn’t it tip over?” The answer is simply that the center of gravity of the tower is directly above its base.
As the tower has tilted further and further over time, engineers and mathematicians have used their knowledge of center of gravity to slow down the leaning process and stabilize the tower. Stabilization methods included adding counterweights to the side opposite the lean. In 1990, bells were removed from the top portion of the tower. Both of these methods resulted in the tower’s center of gravity moving down and towards the center line. Cables were also cinched around the third level of the tower and anchored hundreds of meters away. Further, the tower was slightly straightened by removing soil from underneath the raised end. The tower was closed to the public for 11 years while these efforts were underway. Discussion of both the tower and attempts to stabilize it is a valuable tool to impart the basic concepts of center of gravity.

B. Overloaded Cart

What happened to the cart in Figure 13? Before the cart is loaded, the donkey stood just fine on its four legs. As more weight is added, the center of gravity shifts away from the donkey. Once the center of gravity is farther back than point of contact between the ground and the wheels, the cart tips clockwise raising the poor donkey. This could have been prevented by designing the cart so that the wheels are farther back, or by hauling less weight behind the wheels. The person loading the cart could have also moved the packages so the center of gravity is closer to the donkey.
Figure 13: An overloaded cart tipped over Source: https://www.johns-jokes.com/safety-at-work-overloaded-cart-008

C. Truck Stabilization Legs

Work truck stabilization legs are often hydraulically operated and act to share the weight of a truck (and the load the truck is supporting) on these legs rather than on the wheel of the truck. It is meant to assure that the center of gravity of the whole system (truck + load) will stay well inside the convex hull of the new structure and will not cause it to tip over. Refer to Figure 14.

Figure 14: Stabilization legs on a firetruck
D. Tow Truck

Tow trucks utilize center of gravity to safely lift vehicles without tipping over. When the center of gravity of the new system (i.e. the tow truck and the load together) are outside the convex hull of the tow truck extended legs, the truck will tip over.

Figure 15 shows a tow truck attempting to pull a car from the water. As the weight of the car was added to the tow hook, the system at work became the tow truck and car combined instead of just the tow truck. This meant the center of gravity shifted, but with the additional weight from the water it shifted more than expected. Unfortunately, the moment created by the car on the tow hook caused the tow truck to tip over and also fall into the river. A larger tow truck was brought to the site to remove the car and the first tow truck from the river, only to also tip over.

Figure 15: A tow truck fails to pull a car from the water. Follow-up tow truck fails to pull the first tow truck from the water.

6. Activities

The following are hands on activities that students can practice in class and also at home. They are in addition to the hands-on examples shown earlier, such as the broom, the stick and the banana experiment. These activities help engage students in the lecture and can help solidifying
the concept. The examples provided include (a) forks balanced on a marker, (b) standing up from a chair without leaning forward, (c) standing next to a wall and sticking a leg out, (d) balancing phones and golf balls, (e) mobile, (f) Jenga, and (g) balancing a water bottle on its edge.

A. Balancing Forks on a Marker

Figure 16 shows two forks and a toothpick attached with some putty, balanced on a marker. This is a simple exercise to try. At first glance it seems like this must be some kind of optical illusion; all the weight seems to be balanced on one side of the toothpick, so why doesn’t it fall off the marker? The key is understanding that because of the way the forks are positioned (so that a large portion of their masses are on the other side of the marker), the center of gravity of the system is still directly below the top of the marker, and the net moment created by the forks, putty, and toothpick is equal to zero.

Figure 16: Forks, putty, and toothpick balance on a marker.

B. Standing Up Without Leaning Forward

Your center of gravity while sitting in a chair is above the seat. Any attempt to raise your upper body will generate a moment (due to your weight) that will prevent such an attempt. In order to stand up, you must first lean forward to move your center of gravity forward.

Figure 17 shows a person standing up from a chair. In Figure 17 (a), the person is sitting in the chair normally and his center of gravity is above the seat. In Figure 17 (b), he leans forward to move his center of gravity forward, and in Figure 17 (c) he can lift himself up.
**Figure 17 (a):** Person sitting in a chair. His center of gravity (blue point) is above the seat.

**Figure 17 (b):** The person leans forward to put his center of gravity (blue point) forward.

**Figure 17 (c):** The person can finally push himself up.
C. Losing Balance with Leg Jutting Out

While standing completely straight next to a wall, try to lift one leg and jut it out to your side. Shown in Figure 18, sticking your leg out to the side shifts your center of gravity so that it is no longer above your supporting leg. This creates a net moment relative to your leg which causes you to lose your balance and fall over. Figure 18(a) shows a person standing against the wall. Figure 18(b) shows him as he sticks his leg out. In Figure 18(c) the person has lost his balance and begins to fall. The red dot in each picture depicts the person’s center of gravity.

Figure 18 (a): The person stands next to a wall

Figure 18 (b): The person tries to stick his leg out. Notice the location of the center of gravity.
D. Balancing Phones

Figure 19 shows balanced phones and golf balls. At first glance, it appears that the structure is unstable and tends to topple. A closer inspection shows a clever use of center of gravity to perform this balancing trick. The top phone together with the red ball is balanced on a white golf ball. The net moment around the top of the white ball equal to zero. Similarly, the net moment on the top of the yellow ball is zero.

E. Balanced Mobile

Mobiles, like the one in Figure 20, are designed to balance utilizing the “net moments equal zero” at different levels. Each rod of a mobile balances the objects hanging below it.
relative to its equilibrium point, i.e. the string above it. Thus, the rod is balanced with regard to the objects hanging from it. At each connection point, the net moment acting at that point equals zero.

![Figure 20: A standard mobile](image)

**F. Falling Jenga**

The popular game Jenga is based around center of gravity. As wooden blocks are pulled out of the tower, the center of gravity shifts. Once the center of gravity of one part of the tower is no longer located above its supporting blocks, the tower will tip over and the game ends. This is shown in Figure 21.

![Figure 21: A Jenga-like set collapsing](image)
G. Balancing a Water Bottle

Can you balance a water bottle on its edge, like the one in Figure 22? At first glance the bottle appears to be normal and empty. However, a closer inspection shows that there is an additional weight in the cap of the bottle. The water bottle can balance on its edge because the net moment about its touching point is zero.

![Figure 22: A water bottle balanced on its edge](image)

7. Teasers and Puzzles

A. A Challenge to the Student: Can You Stand at This Angle?

Try to stand at the angle shown in Figure 23 while keeping your entire body straight. Can you do it?

![Figure 23: A person standing at an angle](image)
Generally speaking, it is impossible to lean back in that manner without falling because your center of gravity won’t be above your legs. In this case there is a trick: the person stands upright on an escalator, but the image is rotated, as shown in Figure 24.

**Figure 24:** The person stands straight the entire time.

### B. Balancing Nails

The aim of this puzzle (shown in Figure 25) is to balance 13 nails on single nail. The key to solving the puzzle is to balance the nails in such a way that the center of gravity of the system appears at some point inside the vertical nail that is placed in the wood (as shown). This also means that the net moment of the system about the central nail has to be zero so that the system becomes stable. The solution is shown in Figure 26.

**Figure 25:** The balancing nails puzzle disassembled
C. The Balancing Bird

The balancing bird, shown in Figure 27, is a classic toy designed around manipulating center of gravity to appear as though it’s defying gravity. How does it work? The wings are extended far enough forward ahead of the beak, and are heavy enough, so that the center of gravity of the balancing bird is actually below the tip of its beak. It can therefore balance on a pillar (as pictured), on the tip of a finger, or anything else. You can easily make a simplified version of the balancing bird, like the one shown earlier in Figure 16.

D. Self-Balancing Toys

Self-balancing toys, such as the one pictured Figure 28 below, work because they’re hollow and light on top but solid and heavy at the bottom. The reason: while the toy is upright, its center of gravity is at the lowest possible point. As the toy is intentionally tipped over by an
outside disturbance, the center of gravity is elevated. As a result, it will move back to its upright stable position.

Figure 28: Visual explanation of self-balancing toys

8. Did You Know?

A. Golf Balls

The Guinness World Record for most golf balls stacked without the use of adhesives is seven, and was set by Lang Martin in 1980. Stacking that many golf balls on top of each other requires careful and precise placement. For each golf ball stacked, its center of gravity must be directly above the point of contact between that ball and the one below it, otherwise it will fall. There is a small margin for error (the width between the golf balls), but as more and more golf balls are stacked this margin gets smaller and smaller. Figure 29 shows four stacked golf balls.

Figure 29: Four stacked golf balls Source: recordsetter.com
9. **Center of Gravity Changes Due to Geometry**

A. **Wire Hangers**

A wire hanger, used for hanging clothes, is an example where the center of gravity can be manipulated by changing its geometry. Figure 30 displays a standard wire hanger with its center of gravity shown. Figure 31 shows that the center of gravity changes as the hanger is bent into different positions.

![Figure 30: A wire hanger with its center of gravity shown](image)

![Figure 31: By bending the coat hanger, the center of gravity shifts](image)

B. **Warped Metal Loops**

The metal loop shown in Figures 32 and 33 is a good example of how the center of gravity changes based on different geometries. If the loop were a perfect circle then the center of gravity would have been in the center of the loop. But when the different pieces are twisted into strange shapes, the center of gravity changes. Below two different centers of gravity are shown.
**Figure 32:** A metal loop bent into two different shapes

**Figure 33:** A metal loop with different locations of the center of gravity (shown as orange dots)

**Assessment**

The methods and examples shown in this paper were implemented in a Statics class taught by a co-author of this paper at Florida Atlantic University. A questionnaire was distributed to students at the end of the lecture in order to gauge how receptive students were to the visual, intuitive, and engaging learning techniques presented in this paper. The responses are based on a class presentation that included the examples in this paper. 54 students filled out the questionnaire. A summary of the questionnaire and raw results can be found in the Appendix.

Figures A.1 and A.2 show the self-identified demographic breakdown of participating students. There is a fair amount of diversity, which is representative of the diversity at Florida Atlantic University and its engineering programs. Most students were 23 years old or younger, but there were a handful of older students, including two older than 33.

Questions regarding student opinions about importance of the lecture or learning in general had five options: very important, important, moderately important, slightly important, and not important. Questions regarding opinions about effectiveness of various presentations or learning methods had the options: strongly agree, agree, neutral, disagree, or strongly disagree.
Students surveyed largely feel that it is important to be introduced to Statics concepts through visual examples and hands-on activities. When asked about the “importance of being introduced to Statics through visual examples,” 37% responded with “very important” and an additional 52% responded with “important”. No student marked “slightly important” or “not important.” To a question regarding the “importance of being introduced to Statics concepts through hands-on activities, 37% responded “very important,” and an additional 43% responded “important.” Only two students responded “slightly important” and none felt that hands-on activities were not important.

Students are mixed on their opinions of the importance of traditional and PowerPoint presentations. When asked about the “importance of learning using traditional presentations,” only 10 students marked either very important or not important. 31% marked moderately important, 26% marked important, and 24% felt that traditional presentations were only slightly important. Answers to the question regarding the importance of learning Statics through PowerPoint presentations were similar.

Students subjectively found that visual presentations helped them understand the concept of center of gravity. Responding to the questions, “visual presentation of center of gravity helped me better understand concepts,” and, “visual/engaging presentation of center of gravity helped me understand the concept,” most students either agreed or strongly agreed. Few felt neutral and no students disagreed or strongly disagreed.

These results, when taken as a whole, show that many students feel that they learn best through visual and engaging approaches to education. They generally are neutral to the idea of traditional and PowerPoint presentations, and prefer to be taught rather than teach themselves.

Conclusion and Future Work

This paper is a work in progress which still needs to be expanded upon. More work needs to be done to collect additional data regarding the effectiveness of these learning methods. Further, numerical examples will be added in the future as a follow-up to these lecture examples. The examples in this paper largely do not include numerical examples in order to avoid intimidating students with deep mathematical explanations, and instead focus on facilitating an understanding of the concepts prior to delving into the math.

This teaching method will also be expanded in the future to include other Statics topics that students struggle with. The next focus is planned to be on free body diagrams. Eventually, the intention is to incorporate these works into a larger body of work that will include multiple topics from STEM courses to assist students in comprehending concepts. Work is already underway building the framework for teaching Calculus along the line of methods exposed in this study.
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References


Appendix

The following figures contain the results from the questionnaire discussed in the assessment section.

**Figure A.1:** Self-identified ethnic distribution

**Figure A.2:** Self-identified age distribution
Figure A.3: Engineering majors

Figure A.4: Student feedback on importance of understanding Statics concepts

Figure A.5: Student feedback on importance of visualizing Statics
Figure A.6: Student feedback on importance of being introduced to Statics concepts through visual examples

Figure A.7: Student feedback on importance of hands-on activities in Statics

Figure A.7: Student feedback on importance of engaging in class exercises to learn Statics concepts
Figure A.8: Student feedback on importance of class engagement in Statics

Figure A.9: Student feedback on importance of traditional presentations for learning

Figure A.10: Student feedback on importance of PowerPoint presentations for learning Statics

Figure A.11: Student feedback on importance of learning Statics through textbook reading
Figure A.12: Student self-assessment of mathematics competency

Figure A.13: Student assessment of effectiveness of visual presentation

Figure A.14: Student feedback on effectiveness of visual/engaging presentation
Appendix B: Beyond Statics

A. Boeing 747 vs. X-29 Fighter Plane

When designing a passenger plane such as a Boeing 747 or a fighter plane such as the X-29, center of gravity plays a major role. Besides differences in design, a passenger plane’s center of gravity is located in front of its center of pressure. This means the plane is stable during flight, but certainly cannot perform sharp maneuvers. On the other hand, some fighter planes (such as the X-29 shown in Figure B.1) have their center of gravity located behind the center of pressure. This makes the plane unstable without a pilot, but stable and maneuverable by highly qualified pilots.

Figure B.1: Boeing 747 passenger plane and X-29 fighter jet

B. Paper Airplane with Paperclips
Paper airplanes can help in showing the effect of center of gravity on stability. Students can make paper airplanes and attach a paperclip either to the front or to the back. When the paperclip is in the front of the airplane it flies very smoothly (it is stable), but if the paperclip is placed on the back of the plane then it flies very poorly (it is unstable), like in Figure B.2. In order for the paper airplane to be stable, the center of mass needs to be in front of the center of pressure.

![Figure B.2: The effect of added mass on a paper airplane](image)

**C. Roll Paper Cups in Different Configurations**

Use two sets of two plastic cups, one set joined at the rim and the other set joined at the bases of the cups. Roll the cups down an incline. What happens? As Figure B.3 shows, the cups joined at the base derail before reaching the end of the incline. The cups joined at the rims, however, stay on track.

When considering the set of cups joined at the rims, the center of gravity of the system is at its lowest when the cup set is at the center of the incline. Any change from that position will cause the center of gravity to elevate, therefore move back to the center.

When the cups are joined at the bases, the system’s center of gravity is highest when the cups are centered on the incline. This means that any minor imbalance will cause the center of gravity to move to a lower point and therefore derail the system from its central path.
Figure B.3: Sets of cups in different configurations roll down an incline.

D. The Fosbury Flop

High jumping utilizes a long flexible pole to help an athlete jump over a horizontal bar. A high jumper named Dick Fosbury introduced a new technique for the high jump at the 1968 Summer Olympics that is almost universally used throughout the sport today. By twisting his body around the pole instead of simply jumping above it, as shown in Figure B.4, he was able to reach higher heights without raising his center of gravity as much.
E. Thrown Keys

When keys on a keychain (system) are thrown in an arc, the center of gravity of the overall system will follow a parabolic path despite the fact that the keys move around the path in a semi random fashion. This is similar to the Fosbury flop in which the path of the center of gravity is determined but the rest of the body can be manipulated around it. Figure B.5 shows the path of travel of a set of keys. Notice that the individual keys are not following that travel path.